Model Insensitive and Calibration Independent Method for
Determination of the Downstream Neutral Hydrogen Density
Through Ly-α Glow Observations

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ABSTRACT

Our knowledge of the various heliospheric phenomena (location of the solar wind termination shock, heliopause configuration and very local interstellar medium parameters) is limited by uncertainties in the available heliospheric plasma models and by calibration uncertainties in the observing instruments. There is, thus, a strong motivation to develop model insensitive and calibration independent methods to reduce the uncertainties in the relevant heliospheric parameters. We have developed such a method to constrain the downstream neutral hydrogen density inside the heliospheric tail. In our approach we have taken advantage of the relative insensitivity of the downstream neutral hydrogen density profile to the specific plasma model adopted. We have also used the fact that the presence of an asymmetric neutral hydrogen cavity surrounding the sun, characteristic of all neutral densities models, results in a higher multiple scattering contribution to the observed glow in the downstream region than in the upstream region. This allows us to approximate the actual density profile with one which is spatially uniform for the purpose of calculating the downstream backscattered glow. Using different spatially constant density profiles, radiative transfer calculations are performed, and the radial dependence of the predicted glow is compared with the observed 1/R dependence of Pioneer 10 UV data. Such a comparison bounds the large distance heliospheric neutral hydrogen density in the downstream direction to a value between 0.05 and 0.1 cc$^{-1}$. 
INTRODUCTION

After Pioneers 10 and 11 (1972-73) and Voyagers 1 and 2 (1978-79) were launched into deep space our understanding of the heliosphere increased dramatically. The data obtained from these deep space probes have led to major advances in our understanding of heliospheric physics, including such open questions as where the solar wind termination shock is located, the heliopause characteristics and the magnitude of the very local interstellar medium (VLISM) plasma and neutral densities.

Yet, despite impressive advances, definitive answers to the above topics remain to be unambiguously determined. There is now increasingly firm evidence for the existence of a solar wind termination shock (Hall et al., 1993) and a heliopause (Gurnett et al., 1993) but no definitive consensus about their precise location has yet emerged. Hall et al., 1993 determined that the solar wind termination shock lies between 70 and 105 AU from the sun in the upstream direction. Gurnett et al., 1993 estimated the distance to the stagnation point to be between 116 and 177 AU. The situation regarding the thermodynamic parameters is also unsettled. For example, both the interstellar plasma and the neutral densities remain to be firmly established. Astronomical observations suggest a "local" interstellar electron density as high as 0.3 cc\(^{-1}\) (Lallement et al., 1994) and 0.44 cc\(^{-1}\) (Frisch, 1994) while Gurnett et al., 1993 found the density near the heliopause to be about ~ 0.04 cc\(^{-1}\) from the 2-3 khz radiation observed in the heliosphere. The large uncertainties in interstellar plasma density translate into an uncertainty in pressure balance at the plasma interface. The large range of plasma densities permits a broad range of heliospheric plasma models, and thus a large uncertainty in the location of the termination shock and the heliopause. The reported values of the neutral hydrogen at "infinity" (the interstellar value) also cover a broad range. Nearly a decade ago Ajello et al., 1987
summarized the reported values of the VLISM hydrogen density determined from remote UV observations and found them to lie between 0.02 and 0.12 cc$^{-1}$. Apart from heliospheric model and UV instrument calibration uncertainties, the limited spatial extent of many of the heliospheric observations have also played a role in keeping this range so uncertain. Recently Baranov and Malama, 1993, 1995 have constructed a supersonic plasma model in which they used a neutral hydrogen density of 0.14 cc$^{-1}$ at "infinity", larger than the upper range given by Ajello et al., 1987. Thus, in the absence of a consensus value of the neutral hydrogen density at "infinity" the present work was undertaken to significantly narrow the acceptable heliospheric and LISM parameters.

The large uncertainties to date in our knowledge of the interstellar medium has led to a variety of plasma models. It is necessary to establish tighter bounds on at least some of the relevant parameters. Recently we have developed a simple approach which is both relatively model insensitive and calibration independent and has been described briefly in Gangopadhyay and Judge, 1995 in an effort to extract from the Pioneer 10 UV data a tighter bound on the downstream neutral hydrogen density. The downstream direction is defined with respect to the incoming flow of neutrals in the heliosphere. The power of this approach has persuaded us to describe it in significantly greater detail.

DESCRIPTION OF THE APPROACH

Two attractive features of the present approach to the reduction of uncertainty in the large distance downstream neutral hydrogen density are plasma model insensitivity and instrument calibration independence. As far as plasma model insensitivity is concerned we rely on the fact that a wide variety of physically reasonable plasma models yield similar downstream hydrogen
density distributions (Gangopadhyay and Judge, 1989; Baranov and Malama, 1993; Osterbart and Fahr, 1992; Fahr et al., 1993, 1995; Fahr and Osterbart, 1995). Our approach will thus yield results which are sensibly insensitive to the details of any particular plasma model used. In Fig. 1 we have plotted our calculated downstream neutral hydrogen density profiles for both a supersonic and a subsonic model. They are quite similar unlike the large distance upstream neutral density profiles (Fig. 2), which are quite different, depending on the plasma model used. There is a large hill-like structure extending hundreds of AUs from the sun in the upstream direction for the supersonic plasma model case. The point of this discussion is that a downstream flat density is a quite reasonable approximation to the large distance downstream density for a variety of models. Even when the downstream profile is not quite flat (Osterbart and Fahr, 1992; Fahr et al., 1993) the maximum departure from flatness for 3 of the 4 shock models shown by them is of the order of $\sim 20\%$ beyond a heliocentric distance of $\sim 50$ AU. The flat density approximation is clearly not strictly correct inside 100 AU where the density is definitely increasing with increasing distance. The downstream hydrogen density at a distance of 30 AU, is only a fraction of the asymptotic value for all models developed to date. Nevertheless, a uniform flat density is a remarkably good approximation for the calculation of the downstream glow as we shall show in the next section. The flat density approximation, however, can not be used for the calculation of the upstream glow.

The most straightforward way to interpret the only downstream data available, i.e., the Pioneer 10 UV data, would be to approximate the downstream density with a flat profile, solve the radiative transfer problem and vary the absolute value of the neutral density until an optimum fit to the data is obtained. Wu et al., 1988 did exactly that and discovered that the best fit to the data could be obtained using an ad hoc spatially flat neutral distribution of density 0.05 cc$^{-1}$. 
Wu et al., 1988 did not incorporate the effect of the interstellar plasma - solar plasma interaction, and they only approximately took multiple scattering into account. The principal uncertainty in their density determination was due to calibration uncertainty. Even today, there is an unresolved calibration difference with Voyager 2 UVS which is a factor of 4.4 larger than the Pioneer 10 UV photometer at Ly-α (Shemansky et al., 1984). The precise difference is uncertain at least partly because Shemansky et al., 1984 did not incorporate a heliospheric shock structure in their earlier analysis of the upstream data. A factor of two increase in the Pioneer 10 glow data would increase the downstream density to 0.1 cc⁻¹. This strongly suggests that the Voyager absolute values are too high by at least a factor of two since a density of 0.1 cc⁻¹ or higher in the downstream region will be shown to be ruled out by the present work. To avoid dependence on calibration the present method utilized the radial dependence of the large distance glow and not its absolute value. So the heart of the present approach is to use the similarity of the downstream neutral hydrogen density distributions for a wide variety of plasma models in the calculation of the backscattered glow, and calculate the radial dependence of the glow as a function of uniform hydrogen density. The radial dependence of the glow depends on the absolute value of the flat density due to optical depth considerations. Hence, a comparison of the calculated radial dependence and the observed dependence places a tight upper bound on the downstream density. The rest of the paper discusses in detail how we limit the range of acceptable downstream hydrogen densities.

RADIAL DEPENDENCE CALCULATION OF THE DOWNSTREAM Ly-α GLOW

The first step in calculating the radial dependence of the expected Ly-α glow is to adopt a suitable hydrogen density. We have performed radiative transfer calculations for three
idealized cases: (i) a uniform heliospheric hydrogen density of 0.05 cc\(^{-1}\), (ii) of 0.1 cc\(^{-1}\) and (iii) of 0.2 cc\(^{-1}\). The radiative transfer code is described in Gangopadhyay, Ogawa and Judge, 1989. The density is taken to be constant throughout the heliosphere in order to demonstrate the effect of the radial glow dependence on hydrogen density. The results are shown in Figure 3. The most significant result is that while the radial backscattered intensity for a constant solar flux deviates from a 1/R dependence at a heliocentric distance \(R > 30\) AU for a hydrogen density of 0.2 cc\(^{-1}\), any such deviation for the lowest density of 0.05 cc\(^{-1}\) occurs for \(R > > 100\) AU. This result provides us with a very powerful tool for bounding the neutral density in the downstream direction valid for a wide variety of heliospheric plasma models, without any calibration uncertainties. The extension of the Pioneer 10 data base to ever greater radial distances will establish a tighter bound on the acceptable downstream density and hence the required upstream filtration.

The deviation from a 1/R dependence for a uniform density can be understood as follows. The backscattered intensity is conventionally divided into two parts: single scattering and multiple scattering. The single scattering component as the name implies includes the contribution due to solar photons backscattered only once from an atom. All other contributions are lumped together as multiple scattering. The single scattering contribution is dominant very close to the source. The multiple scattering contribution becomes the dominant contribution as the optical depth increases. For "moderate" optical depths, \(\tau \leq 1\), the two contributions add up to a 1/R fall off. However, beyond a certain heliocentric distance, \(\tau > > 1\), only multiple scattering terms contribute to the backscattered intensity, resulting in a faster than 1/R decline.

We would like to justify here the use of a flat approximation inside 100 AU where the density varies with distance in all available models, and is highest in the upstream direction.
In the single scattering approximation such an asymmetric density distribution yields a lower backscattered downstream radial glow intensity than the upstream radial glow intensity for distances less than 100 AU from the sun. It is known, however, in the case of a hot model density (Thomas, 1978; Wu and Judge, 1979) that the downstream radial glow intensity becomes almost equal to the upstream intensity beyond a heliocentric distance of ~20 AU (Keller, Richter and Thomas, 1981). This requires that there is a larger multiple scattering contribution to the downstream intensity than to the upstream intensity, since the downstream density is lower and singly scattered photons produce a glow which is less than the upstream glow. This near equal intensity in the up and down stream directions is clearly counterintuitive as more multiple scattering is expected to occur in the high density upstream region compared to the relatively low density downstream region. The explanation for this apparent paradox is that more multiply scattered photons reach the detector in the downstream region than for a detector placed at an equivalent position in the upstream density region. This can be understood if we compare the contribution of specific groups of photons entering a detector in the downstream and upstream regions. These groups of photons are as follows:

1. Singly scattered photons,
2. Multiply scattered upstream photons which get scattered into the downstream detector,
3. Multiply scattered downstream photons which get scattered into the upstream detector,
4. Multiply scattered upstream photons scattering into the upstream detector, and
5. Multiply scattered downstream photons scattering into the downstream detector.

In the low to moderate optical depth region being traversed by Pioneers 10 and 11 and Voyagers
1 and 2 the ratio of multiply to singly scattered photons observed by an upstream detector will be less than that observed by a downstream detector at the same heliocentric distance. Fewer multiply scattered photons reach the upstream detector beyond 20 AU because the higher density and lower mean free path makes it harder for solar photons to reach the region in front of the detector and then scatter into it. The larger mean free path in the downstream region actually allows multiply scattered photons to more easily reach the region in front of the detector and then scatter into it. Thus, there is more multiple scattering observed in the downstream region than in the upstream region. This is confirmed by our Monte Carlo radiative transfer calculation for a self consistent heliospheric plasma and neutral model incorporating a solar wind shock, heliopause and a bow shock (Gangopadhyay, Judge, and Liewer, 1995). It was found that the multiple scattering to single scattering ratio of photons reaching a downstream detector is of the order of \( \sim 4 \) at 30 AU compared to about \( \sim 2 \) for an upstream detector. The multiple scattering photons "fill" in the downstream radial intensity more than the upstream radial intensity in agreement with Keller, Richter and Thomas, 1981 at distances > 20 AU. These additional photons are equivalent to an effective increase in the downstream density close to the sun. It was also found that the bulk of the photons (\( \sim 85\% \)) that contribute to the downstream intensity at 30 AU originated from a region of space greater than 10° from the downstream direction. This explains the relative insensitivity of the downstream backscattered intensity to the downstream density profile and makes it possible to extend the flat density approximation to within \( \sim 20 \) AU of the sun. Thus, we replace the hydrogen cavity by an "effective" flat density for the calculation of the downwind backscattered glow. Clearly the effective density is an upper limit to the real density within 100 AU of the sun.

The effective increase in the downstream density near the sun will exist in all currently
available neutral density models as all have a similar asymmetric upstream-downstream hydrogen cavity surrounding the sun. We need to consider here whether the large differences in neutral density profiles for various plasma models might affect the downstream glow. We will argue here that they do not as the large differences in neutral density profiles for various plasma models occur far from the sun. There is, for example, a broad hydrogen hill in the upstream direction for a supersonic plasma model unlike the density for a Parker type subsonic model or the hot model. The hill is, however, located at a distance greater than 100 AU from the sun and can not play any substantial role in the downstream region because of the very small mean free path in its vicinity. In fact, any photon which scatters near the hill has a mean free path of the order of $\sim 5$ AU for an interstellar density of 0.1 cc$^{-1}$. This is the reason the flat density approximation can be used throughout the heliosphere when one is interested only in calculating the downstream glow.

**COMPARISON WITH OBSERVED PIONEER 10 UV DATA**

The observed backscattered Ly-$\alpha$ glow data obtained by the Pioneer 10 UV detector is plotted in Fig. 4 as a function of the heliocentric distance from the sun. The striking feature of the data is its $1/R$ dependence from 30 AU up to the current position of the spacecraft at $R \sim 60$ AU. It may be remarked here that Pioneer 10 UV data are actually tracking the $1/R$ curve within 15 AU from the sun. A detailed discussion of the $R$-dependence of the Pioneer 10 glow dependence shortward of 30 AU is, however, beyond the scope of this paper. The Pioneer 10 field of view was partially away from the downstream direction in this region. Also, the region close to the sun is extremely complex and its explanation must incorporate a specific plasma model, its associated self consistent neutral density, and include variations in radiation pressure,
variation in the shape and size of the cavity and incorporate the field of view of Pioneer 10 exactly. Thus, in this work we confine ourselves to the large distance (> 30 AU) data.

The large heliocentric distance (> 30 AU) glow data impose the requirement that the interstellar neutral hydrogen density and the heliospheric plasma models should be such that the downstream neutral hydrogen density remains less than 0.1 cc⁻¹. This is because a uniform neutral density as high as 0.1 cc⁻¹ would cause a deviation from 1/R beyond a heliocentric distance of about 50 AU. This information together with the result obtained by Wu et al., 1988 suggests a downstream neutral density in the range 0.05 to 0.1 cc⁻¹.

The lower limit of 0.05 cc⁻¹ is dependent on the use of the Pioneer 10 UV photometer calibration. We exclude the possibility of a lower density as that would require decreasing the sensitivity of the Pioneer 10 UV instrument. Any lowering of the Pioneer 10 UV photometer sensitivity calibration would also require lowering the Jupiter dayglow observed in 1973 during the Pioneer 10 flyby (Judge and Carlson, 1973, Judge et al., 1976; Wu et al., 1995). Since Pioneer 10 observed an extremely dim Jovian Ly-α and H₂ band emissions compared to the extremely bright emissions (about 15 times the Pioneer 10 value) observed by the Voyagers 1 and 2 about six years later, a lowering of the Pioneer 10 calibration would make Jupiter even dimmer in 1973 and further increase the contrast with subsequent observations. Also, based on repeated stellar observations as a given star was observed first at one side of the conical viewing shell and then months later on the other side, and on the inherent pulse counting stability of the Pioneer 10 UV photometer instrument, we find no basis for a downward revision of the preflight calibration. In fact, the only revision that would be acceptable based on Voyager and other Jovian observations is in the upward direction. There is thus no rational basis to propose any change of the Pioneer 10 calibration downwards.
DISCUSSION AND CONCLUSION

The fundamental finding presented in the present work is that an upper limit to the neutral hydrogen density in the downstream direction can be estimated merely from the shape of the downstream glow curve. It is perhaps remarkable that we obtain a very reasonable upper limit even though we use an extremely simple, flat, static, model independent hydrogen distribution, and a radiative transfer multiple scattering code that uses a flat solar line profile and complete frequency redistribution. The current work should be viewed as serving as a useful check to any model dependent result. It is because of our desire to avoid model dependence that an upper limit to the upstream density at "infinity", the interstellar value, was not determined at this time since the neutral hydrogen depletion in the upstream direction is highly model dependent (Fig.2). The large distance (~ 100 AU or more) downstream density is not the interstellar value as all plasma shock models predict a depletion in the downstream density with the interstellar value hundreds of AUs away. We are currently working on a model specific upper limit to the upstream hydrogen density but that will be a topic for a subsequent paper.

The results presented here have bounded the downstream neutral density. The present method in fact promises a tighter limit in the future since the Pioneer 10 spacecraft is expected to continue delivering useful data from ever larger heliospheric distances. The more distant the data the better the accuracy promised by the method. If at any time in the future Pioneer 10 downstream UV data show a deviation from 1/R that would enable us to determine the neutral density to very high accuracy. This improved knowledge of the downstream hydrogen density has a direct impact on the acceptable heliospheric plasma models since nearly all heliospheric plasma models that incorporate interstellar plasma - solar wind plasma interactions predict filtration (Ripken and Fahr, 1983; Bleszynski, 1987; Gangopadhyay and Judge, 1989; Baranov
et al., 1991) of neutral H atoms. Thus, any acceptable heliospheric model must provide enough filtration to limit the downstream density to the range reported here. An improved knowledge of the downstream hydrogen density would further limit the calibration uncertainties reported in the literature and also powerfully constrain the acceptable heliospheric plasma models and the VLISM parameters.

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REFERENCES


FIGURE CAPTIONS

Fig. 1. Variation in the downstream hydrogen density for a Steinolfson supersonic plasma model (o) and a Parker type subsonic model (+) as a function of heliocentric distance. The downstream density remaining significantly below the interstellar value (taken to be one here) is a common feature for a variety of plasma models that incorporate the interstellar plasma solar wind plasma interaction. The interstellar parameters used in the Parker type model used here are: proton density = 0.01 cc⁻¹, proton temperature = 10⁴ °K, proton bulk velocity = 20 km/sec and the termination shock location = 65 AU. The interstellar parameters used in the Steinolfson supersonic model are: proton density = 0.09 cc⁻¹, proton velocity = 26 km s⁻¹ and proton temperature = 10⁴ °K, and termination shock location = 110 AU in the upstream direction. These are representative interstellar plasma parameters. Since the upstream hill feature in Fig. 2 extends to 350 AU we show here and in Fig. 2 density profiles up to 500 AU. The solid lines are best fit curves through the calculated points. The large scatter in the calculated density shortwards of 100 AU in both Figs. 1 and 2 is an artifact of the Monte Carlo method close to the Sun.

Fig. 2. Variation of the upstream density for a subsonic heliosphere plasma model (+) and for a supersonic model (o) as a function of heliocentric distance. The interstellar value is taken to be one. The two densities differ markedly between 200 and 300 AU from the sun. The subsonic model is a Parker type model while the supersonic model is the Steinolfson model already mentioned in the previous figure.

Fig. 3. Variation in the backscattered Ly-α glow times distance as a function of heliocentric distance for a uniform hydrogen density of 0.2 cc⁻¹ (o), of 0.1 cc⁻¹ (x) and 0.05 cc⁻¹ (+). The calculation is performed assuming a flat solar Ly-α profile and a constant neutral
hydrogen temperature of $10^4$ °K. The solid lines are best fit curves through the calculated points. A horizontal line would represent a 1/R profile. The details of the Monte Carlo radiative transfer calculation may be found in Gangopadhyay, Ogawa and Judge, 1989.

Fig.4. Pioneer 10 Ly-α intensity plotted as a function of the radial distance from the sun. The solid line is a 1/R curve normalized at 35 AU. The data have been adjusted for a constant solar flux of $3 \times 10^{11}$ photons cm$^{-2}$sec$^{-1}$Å$^{-1}$. The He 10830 Å data used as a proxy for Ly-α flux is a NSO/Kitt Peak measurement produced cooperatively by NSF/NOAO, NASA/GSFC and NOAA/SEL.