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Todd M. Sokolowski
The Ohio State University
Columbus, Ohio

Barry Hojjatie
University of Florida
Gainesville, Florida

Noel N. Nemeth
Lewis Research Center
Cleveland, Ohio

Kenneth J. Anusavice
University of Florida
Gainesville, Florida

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STRESS AND RELIABILITY ANALYSIS OF A METAL-CERAMIC DENTAL CROWN

Todd M. Sokolowski  
The Ohio State University  
Department of Mechanical Engineering  
Columbus, Ohio

Barry Hojjatie  
University of Florida  
College of Dentistry  
Gainesville, Florida

Noel N. Nemeth  
National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio

Kenneth J. Anusavice  
University of Florida  
College of Dentistry  
Gainesville, Florida

ABSTRACT

Interaction of mechanical and thermal stresses with the flaws and microcracks within the ceramic region of metal-ceramic dental crowns can result in catastrophic or delayed failure of these restorations. The objective of this study was to determine the combined influence of induced functional stresses and pre-existing flaws and microcracks on the time-dependent probability of failure of a metal-ceramic molar crown. A three-dimensional finite element model of a porcelain-fused-to-metal (PFM) molar crown was developed using the ANSYS® finite element program. The crown consisted of a body porcelain, opaque porcelain, and a metal substrate. The model had a 300 N load applied perpendicular to one cusp, a load of 300 N applied at 30 degrees from the perpendicular load case, directed toward the center, and a 600 N vertical load. Ceramic specimens were subjected to a biaxial flexure test and the load-to-failure of each specimen was measured. The results of the finite element stress analysis and the flexure tests were incorporated in the NASA developed CARES/LIFE program to determine the Weibull and fatigue parameters and time-dependent fracture reliability of the PFM crown. CARES/LIFE calculates the time-dependent reliability of monolithic ceramic components subjected to thermomechanical and/or proof test loading. This program is an extension of the CARES (Ceramics Analysis and Reliability Evaluation of Structures) computer program.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>surface area; material-environmental fatigue constant</td>
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<tr>
<td>a</td>
<td>crack half length or radius</td>
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<tr>
<td>B</td>
<td>subcritical crack growth constant</td>
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<td>C</td>
<td>Shetty’s constant in mixed-mode fracture criterion</td>
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<td>g</td>
<td>g-factor</td>
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<td>H</td>
<td>step function</td>
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<td>I</td>
<td>ranking of ordered fracture data in statistical analysis</td>
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<td>K</td>
<td>stress intensity factor</td>
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<td>k</td>
<td>crack density coefficient</td>
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<td>m</td>
<td>Weibull modulus, or shape parameter</td>
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<tr>
<td>N</td>
<td>material-environmental fatigue constant</td>
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<tr>
<td>n</td>
<td>number of cycles</td>
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<tr>
<td>P</td>
<td>applied force</td>
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<tr>
<td>P_f</td>
<td>cumulative failure probability</td>
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<tr>
<td>Q</td>
<td>cyclic fatigue parameter</td>
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<tr>
<td>R</td>
<td>ratio of minimum to maximum effective stress in a load cycle</td>
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<tr>
<td>T</td>
<td>period of one cycle</td>
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<tr>
<td>t</td>
<td>time or thickness</td>
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<tr>
<td>t_a</td>
<td>time-dependent scale parameter</td>
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<tr>
<td>V</td>
<td>volume; crack velocity</td>
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<tr>
<td>x,y,z</td>
<td>Cartesian coordinate directions</td>
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<tr>
<td>Y</td>
<td>crack geometry factor</td>
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INTRODUCTION - BRITTLE MATERIAL FAILURE MODES AND DESIGN METHODOLOGY

Ceramics and glasses are two broad classifications of materials that share the same characteristic trait of brittleness. This deficiency represents a concern when stresses must be borne. For instance, ceramics are used for wear parts (nozzles, valves, seals, etc.), cutting tools, grinding wheels, bearings, electronics, human prostheses, and in gasoline and turbine engine environments where the combination of high temperature strength and relative light weight make these materials attractive. Glasses are used, among other things, as containment vessels, including bottles, light bulbs, and television picture tubes, as well as transmission devices such as windows and optical fibers. Among the many requirements for the successful deployment of these components are the proper characterization of material properties and the use of a mature and validated brittle material design methodology.

Ceramics and glasses lack ductility which leads to low strain tolerance, low fracture toughness, and large variations in observed fracture strength. When a load is applied, the absence of significant plastic deformation or microcracking causes large stress concentrations to occur at microscopic flaws, which are unavoidably present as a result of materials processing operations or in-service environmental factors. The observed scatter in component strength is caused by the variable severity of these flaws and by the behavior of sudden catastrophic crack growth which occurs when the crack driving force reaches a critical value. Since ceramics and glasses fail at a critical flaw, examination of their fracture surfaces can reveal the nature of failure. Fractography of broken samples has shown that these flaws can be characterized into two general categories: (1) defects internal or intrinsic to the material volume (volume flaws) and (2) defects extrinsic to the material volume (surface flaws). Intrinsic defects are a result of materials processing. Extrinsic flaws can result from grinding or other finishing operations, chemical reaction with the environment, or the internal defects intersecting the external surface. The different physical nature of these flaws results in dissimilar failure response to identical loading situations.

The ability of a ceramic component to sustain a load may degrade over time. This is caused by a variety of effects such as oxidation, creep, stress corrosion, and cyclic fatigue. Stress corrosion and cyclic fatigue result in a phenomenon called subcritical crack growth (SCG). SCG initiates at an existing flaw and continues until a critical length is reached, causing catastrophic crack propagation. The SCG failure mechanism is a load-induced phenomenon over time. It can also be a function of chemical reaction, environment, debris wedging near the crack tip, and deterioration of bridging ligaments.

Because of the variable severity of inherent flaws, the nature of ceramic failure is probabilistic and optimization of design requires the ability to accurately determine a loaded component’s reliability as a function of time in service. Consequently, a successful brittle material design methodology must combine the statistical nature of strength-controlling flaws with fracture mechanics to allow for multiaxial stress states, simultaneously active flaw populations (families of flaws), and subcritical crack growth.

The objective of this paper is to demonstrate this design methodology applied to a dental prosthetic molar crown using the NASA developed integrated design computer program, CARES/LIFE (Ceramics Analysis and Reliability Evaluation of Structures LIFE prediction program). With this program, it is possible to identify an optimum geometry and material that will reduce the need for expensive fabrication and testing. The theory and concepts presented in this paper reflect the capabilities of the CARES/LIFE program for time-dependent probabilistic design.

PROGRAM CAPABILITY AND DESCRIPTION

Probabilistic component design involves predicting the probability of failure for a thermomechanically loaded component from specimen rupture data. Typically these experiments are performed with flexural or tensile test specimens of simple geometries. A static, dynamic, or cyclic load is applied to each specimen until fracture. Statistical strength and SCG (fatigue) parameters are then determined from these data. Using these parameters and the results obtained from finite
element analysis, the time-dependent reliability for a complex component geometry and loading is then predicted. Appropriate design changes are made until an acceptable probability of failure has been reached. This design methodology combines the statistical nature of strength-controlling flaws with the mechanics of crack growth to allow for multiaxial stress states, concurrent (simultaneously occurring) flaw populations, and scaling effects. These issues are addressed within the CARES/LIFE program.

CARES/LIFE predicts the probability of failure of a monolithic ceramic component as a function of service time. It assesses the risk that the component will fracture prematurely as a result of subcritical crack growth. The effect of proof testing components prior to service is also considered. CARES/LIFE is an extension of the CARES program. It retains all of the capabilities of the previous CARES code, which include fast-fracture component reliability evaluation and Weibull parameter estimation from inert stress (without SCG contributing to failure) specimen data. CARES/LIFE can estimate parameters that characterize SCG from specimen data as well.

ANSYS® Finite element heat transfer and linear elastic stress analyses are used to determine the component's temperature and stress distributions. The element stress output from ANSYS® is translated into an ASCII character neutral file. The information from the neutral file is then input into CARES/LIFE and the reliability at each element is calculated assuming that randomly distributed volume flaws and/or surface flaws control the failure response. The probability of survival for each element is assumed to be a mutually exclusive event, and the overall component reliability is then the product of all the element survival probabilities. CARES/LIFE describes the probabilistic nature of material strength, using the Weibull cumulative distribution function. The effect of multiaxial stresses on reliability is predicted using the principle of independent action (PIA), the Weibull normal stress averaging method (NSA), or the Batdorf theory. The Batdorf theory combines the weakest link theory and linear elastic fracture mechanics (LEFM). Conventional fracture mechanics analysis requires that both the size of the critical crack and its orientation relative to the applied loads determine the fracture stress. The Batdorf theory includes the calculation of the combined probability of the critical flaw within a certain size range and located and oriented so that it may cause fracture. A user-selected flaw geometry and a mixed-mode fracture criterion are required to model volume- or surface-strength-limiting defects. Mixed-mode fracture refers to the ability of a crack to grow under the combined actions of a normal load (opening mode) and shear load (sliding and tearing modes) on the crack face. CARES/LIFE includes the total strain energy release rate fracture criterion, which assumes a crack will extend in its own plane (coplanar). Out-of-plane crack extension criteria are approximated by a simple semiempirical equation. Available flaw geometries include the Griffith crack, penny-shaped crack, semicircular crack, and notched crack. The Batdorf theory is equivalent to the probabilistic multiaxial theories proposed by Evans and Matsuo.

Subcritical crack growth is difficult to model, because it is a complex phenomenon often involving a combination of failure mechanisms. Existing models usually involve empirically derived crack propagation laws that describe the crack growth in terms of the stress intensity factor at the crack tip plus additional parameters obtained from experimental data.

In CARES/LIFE, the relations describing subcritical crack growth are directly incorporated into the PIA, NSA, and Batdorf theories. Subcritical crack growth is modeled with the power law, the Paris law, and the Walker law for static and constant-amplitude cyclic loading. These laws use experimentally determined parameters which are material- and environment-sensitive. The power law is used to model stress corrosion cracking in materials such as glasses and alumina exposed to H2O. Elevated-temperature slow crack growth of silicon nitrides, silicon carbides, and alumina also follows power law behavior.

Some polycrystalline ceramics are prone to strength degradation associated with mechanical damage induced by cyclic loading. The Paris and Walker laws have been suggested as models to account for this behavior. The Walker equation is functionally similar to the Paris equation with additional terms to account for the effect of the R-ratio (minimum cycle stress to maximum cycle stress) on lifetime.

CARES/LIFE is capable of predicting the change in a surviving component's reliability after proof testing is performed. Proof testing is the loading of all components prior to service to eliminate those which may fail prematurely. The components that survive the proof test will have a lower (attenuated) risk of failure in service. In CARES/LIFE the attenuated failure probability is calculated using the PIA, the Weibull normal stress averaging, and the Batdorf theories. The Batdorf model is used to calculate the attenuated failure probability when the proof test load and the service load are not in line or have different multiaxial stress states. This feature is useful when the proof test does not accurately simulate the actual service conditions on the component. The durations of the proof test and the service load are also considered in the analysis.

Predicted lifetime reliability of structural ceramic components depends on Weibull and fatigue parameters estimated from widely used tests involving flexural or tensile specimens. CARES/LIFE estimates fatigue parameters from naturally flaw specimens ruptured under static, cyclic, or dynamic (constant stress rate) loading. Fatigue and Weibull parameters are calculated from rupture data of three-point and four-point flexure bars, as well as tensile specimens. For other specimen geometries, a finite element model of the specimen is also required when estimating these parameters.

THEORY

Time-dependent reliability is based on the mode I equivalent stress distribution transformed to its equivalent stress distribution at time t=0. Investigations of mode I crack extension have resulted in the following relationship for the equivalent mode I stress intensity factor

\[ K_{eq}(\Psi, t) = a_{eq}(\Psi, t) \sqrt{\Psi} \]

where \( a_{eq}(\Psi, t) \) is the equivalent mode I stress on the crack, \( Y \) is a function of crack geometry, \( a(\Psi, t) \) is the appropriate crack length, and \( \Psi \) represents a location (x,y,z) within the body and the orientation (α,β) of the crack. In the Weibull and PIA models, \( \Psi \) represents a location only. \( Y \) is a function of crack geometry; however, herein it is assumed constant with subcritical crack growth. Crack growth as a function of the equivalent mode I stress intensity factor is assumed to follow a power law relationship

\[ \frac{da(\Psi, t)}{dt} = A K_{eq}^N(\Psi, t) \]

where \( A \) and \( N \) are material/environmental constants. The transformation of the equivalent stress distribution at time of failure, \( t=t_f \), to its critical effective stress distribution at time \( t=0 \) is expressed as

\[ \sigma_{eq}(\Psi, t_f) = \left[ \frac{\int_0^{t_f} \sigma_{eq}^N(\Psi, t) \, dt}{B} + \sigma_{eq}^{N-2}(\Psi, t_f) \right]^{-\frac{1}{N-2}} \]

where
averaging to Volume Flaw Analysis

The probability of failure for a ceramic component using the Batdorf model\textsuperscript{21} for volume flaws is

\[ P_{rv} = 1 - \exp\left[-\frac{1}{V} \int_{0}^{2\pi} \int_{0}^{\pi} \frac{\Omega d\phi}{4\pi} \frac{d\eta_{v}(\sigma_{\alpha})}{d\sigma_{\alpha}} \right] dV \]  \hspace{1cm} (4)

where \( V \) is the volume, \( \eta_{v} \) is the crack density function, \( \sigma_{\alpha} \) is the maximum value of \( \sigma_{\alpha} \) for all values of \( \Psi \), and \( \Omega \) is the area of a solid angle projected onto a unit radius sphere in principal stress space containing all crack orientations for which the effective stress is greater than or equal to the critical mode I strength, \( \sigma_{\alpha} \). The crack density function is a function of the critical effective stress distribution. For volume flaw analysis, the crack density function is expressed as

\[ \eta_{v}(\sigma_{\alpha}(\Psi)) = k_{bv} \sigma_{\alpha}^{m_{v}} \]  \hspace{1cm} (5)

where \( k_{bv} \) and \( m_{v} \) are material constants. The solid angle is expressed as

\[ \Omega = \int_{0}^{2\pi} \int_{0}^{\pi} H(\sigma_{\log_{e} \sigma_{\alpha}}) \sin \alpha d\alpha d\beta \]  \hspace{1cm} (6)

where

\[ H(\sigma_{\log_{e} \sigma_{\alpha}}) = \frac{1}{\sigma_{\log_{e} \sigma_{\alpha}}} \begin{cases} \sigma_{\alpha} - \sigma_{\log_{e} \sigma_{\alpha}} & \text{for } \sigma_{\alpha} > \sigma_{\log_{e} \sigma_{\alpha}} \\ 0 & \text{for } \sigma_{\alpha} < \sigma_{\log_{e} \sigma_{\alpha}} \end{cases} \]

and \( \alpha \) and \( \beta \) are the radial and azimuthal angles, respectively, on the unit radius sphere. The transformed equivalent stress \( \sigma_{\log_{e}} \) is dependent on the appropriate fracture criterion, crack shape, and time to failure, \( \tau_{c} \). Equation (4) can be simplified by performing the integration of \( \sigma_{\alpha} \)\textsuperscript{21} yielding the time-dependent probability of failure for volume flaw analysis

\[ P_{rv}(\tau_{c}) = 1 - \exp\left[-\frac{k_{bv}}{2\pi} \int_{0}^{2\pi} \int_{0}^{\pi} \frac{\Omega d\phi}{4\pi} \frac{d\eta_{v}(\sigma_{\alpha})}{d\sigma_{\alpha}} \sin \alpha d\alpha d\beta \right] \]  \hspace{1cm} (7)

Fracture criteria and crack shapes available for time-dependent analysis are identical to those used for fast fracture analysis in CARES.\textsuperscript{0.6} These fracture criteria include Weibull normal stress averaging (a shear-insensitive case of the Batdorf theory), the total coplanar strain energy release rate, and the noncoplanar crack extension (Shetty) criterion.

For a stressed component, the probability of failure for volume flaw analysis is calculated from equation (7). The finite element method enables discretization of the component into incremental volume elements. CARES/LIFE evaluates the reliability at the Gaussian integration points of the element or, optionally, at the element centroid. Subelement volume is defined as the contribution of the integration point to the element volume in the course of the numerical integration procedure. The volume of each subelement (corresponding to a Gauss integration point) is calculated using shape functions inherent to the element type\textsuperscript{0.6}. Assuming that the probability of survival for each element is a mutually exclusive event, the overall component reliability is then the product of all the calculated element (or subelement) survival probabilities.

\[ P_{es} = 1 - \exp\left[\frac{1}{A} \int_{0}^{A} \int_{0}^{\phi} \frac{\Omega d\phi}{4\pi} \frac{d\eta_{e}(\sigma_{\alpha})}{d\sigma_{\alpha}} \right] dA \]  \hspace{1cm} (8)

The transformed equivalent stress \( \sigma_{\log_{e}} \) is dependent on the appropriate fracture criterion, crack shape, and time to failure, \( t_{f} \). The fracture criteria and crack shapes available for time-dependent analysis are identical to those used for fast fracture analysis. These fracture criteria include Weibull normal stress averaging (a shear-insensitive case of the Batdorf theory), the total coplanar strain energy release rate, and the noncoplanar crack extension (Shetty) criterion.

The finite element method enables discretization of the surface of the component into incremental area elements. CARES/LIFE evaluates the failure probability at the Gaussian integration points of shell elements or, optionally, at the element centroids. The area of each subelement (corresponding to a Gauss integration point) is calculated using shape functions inherent to the element type\textsuperscript{0.6}. Assuming that the probability of survival for each element is a mutually exclusive event, the overall component reliability is then the product of all the calculated element (or subelement) survival probabilities.

\[ P_{ds}(t_{f}) = 1 - \exp\left[\frac{k_{es}}{A} \int_{0}^{A} \int_{0}^{\phi} \frac{\Omega d\phi}{4\pi} \frac{d\eta_{e}(\sigma_{\alpha})}{d\sigma_{\alpha}} \right] dA \]  \hspace{1cm} (9)

Static Fatigue

Static fatigue is defined as the application of a nonvarying load over time. For this case the mode I equivalent stress, \( \sigma_{eq}(\Psi, 0) \), is independent of time and is thus denoted by \( \sigma_{eq}(\Psi) \). Integrating equation (3) with respect to time yields

\[ \sigma_{eq}(\Psi, t_{f}) = \sigma_{eq}(\Psi) \left[ \frac{t_{f}}{B} \sigma_{eq}(\Psi) + 1 \right]^{\frac{1}{N-2}} \]  \hspace{1cm} (10)
Dynamic Fatigue

Dynamic fatigue is defined as the application of a constant stress rate \( \dot{\sigma}(\Psi) \) over a period of time, \( t \). Assuming the applied stress is zero at time \( t=0 \), then

\[
\sigma_{\text{eff}}(\Psi, t) = \sigma(\Psi) t \tag{11}
\]

Substituting equation (11) into equation (3) results in an expression for effective stress at the time of failure

\[
\sigma_{\text{eff}}(\Psi) = \left[ \frac{\sigma_{\text{eff}}(\Psi, t_i)}{(N+1)B} + \sigma_{\text{eff}}(\Psi, t_f) \right]^{\frac{1}{N-2}} \tag{12}
\]

Cyclic Fatigue

Cyclic fatigue is defined as the repeated application of a loading sequence. Analysis of the time-dependent probability of failure for a component subjected to various cyclic boundary load conditions is simplified by transforming that type of loading to an equivalent static load. The conversion satisfies the requirement that both systems will cause the same crack growth. Implicit in this conversion is the validity of equation (2) for describing the crack growth. The probability of failure is obtained with respect to the equivalent static state.

Mencik\(^{23}\) and Evans\(^{56}\) defined \( g \)-factors, \( g(\Psi) \), for various types of cyclic loading, that are used to convert the cyclic load pattern to an equivalent static load. For periodic loading, \( T \) is the time interval of one cycle, and \( \sigma_{\text{eff}}(\Psi) \) is the equivalent static stress acting over the same time interval, \( t_i \), as the applied cyclic stress, \( \sigma_{\text{eff}}(\Psi, t) \), at some location \( \Psi \). The equivalent static stress is related to the cyclic stress by

\[
\sigma_{\text{eff}}(\Psi, t) = \int_0^T \sigma_{\text{eff}}(\Psi, t) \, dt = t_i \left[ \int_0^T \sigma_{\text{eff}}(\Psi, t) \, dt \right] \tag{13}
\]

The CARES/LIFE program uses the maximum cyclic stress, \( \sigma_{\text{max}}(\Psi) \), of the periodic load as a characteristic value to normalize the \( g \)-factor. For a periodic load, over a time \( t_i \), the mode I static equivalent stress distribution is

\[
\sigma_{\text{eff}}(\Psi, t_i) = \sigma_{\text{max}}(\Psi) \left[ \frac{g(\Psi) t_i \sigma_{\text{max}}(\Psi)}{B} + 1 \right]^{\frac{1}{N-2}} \tag{14}
\]

The use of \( g \)-factors for determining component life is an unconservative practice for materials prone to cyclic damage. The Walker equation\(^{17}\), which has traditionally been used in metals design, has been suggested as a model of fatigue damage for some ceramic materials.\(^{10}\) The Walker equation describes the crack growth increment per cycle, \( n \), as

\[
\frac{da(\Psi, n)}{dn} = A \left[ K_{\text{eff}}(\Psi, n) \right]^2 \Delta K_n(\Psi, n) \tag{15}
\]

where

\[
K_{\text{eff}}(\Psi, n) = \sigma_{\text{eff}}(\Psi, n) \sqrt{a(\Psi, n)}
\]

and \( \Delta K_n(\Psi, n) \) represents the range of the stress intensity factor over the load cycle. The subscripts max and min indicate the maximum and minimum cycle stress, respectively. The cyclic fatigue parameters \( A, N, \) and \( Q \) are experimentally determined. The Walker equation reduces to the Paris law\(^{16}\) when \( N \) and \( Q \) are equal in value. The integration of Eq. (15) parallels that of Eq. (2), yielding the cyclic fatigue equivalent stress distribution

\[
\sigma_{\text{eff},0}(\Psi, n) = \frac{\int_0^n \left[ 1 - R(\Psi, n) \right]^2 \sigma_{\text{eff}}(\Psi, n) \, dn}{B} \tag{16}
\]

where \( R(\Psi, n) \) is the ratio of the minimum to maximum cyclic stress, \( n_i \) is the number of cycles to failure, and \( B \) is now expressed in units of stress\(^{7} \times \) cycle. The parameters \( B \) and \( N \) are determined from cyclic data.

Evaluation of Fatigue Parameters from Inherently Flawed Specimens

Lifetime reliability of structural ceramic components depends on the history of the loading, the component geometry, the distribution of pre-existing flaws, and the parameters that characterize subcritical crack growth. These crack growth parameters must be measured under conditions representative of the service environment. When determining fatigue parameters from rupture data of naturally flawed specimens, the statistical effects of the flaw distribution must be considered along with the strength degradation effects of subcritical crack growth. In the following discussion, fatigue parameter estimation methods are described for surface flaw analysis using the power law formulation for constant stress rate loading (dynamic fatigue). Analogous formulations for volume flaws, static fatigue, and cyclic fatigue have also been developed.\(^{10}\)

For the uniaxial Weibull distribution the probability of failure is expressed as

\[
P_{\text{f}}(t) = 1 - \exp \left[ -k_s \int_{\sigma_{\text{max}}}^{\sigma} \sigma_{\text{max}}(\Psi) \, d\Psi \right] \tag{17}
\]

where \( \Psi \) represents a location \((x, y)\) and \( \sigma_{\text{max}} \) denotes the transformed uniaxial stress analogous to \( \sigma_{\text{eff}} \) as defined in equation (13). The Weibull crack density coefficient is given by
The Weibull scale parameter, \( \sigma_{oS} \), corresponds to the stress level where 63.2 percent of specimens with unit area would fail and has units of stress \( \times \) area \( 1^{m_s} \).

The Weibull parameters are usually determined in an inert environment or at a high enough stressing rate such that crack extension from SCG is negligible. Specimens are usually of simple geometry and loading conditions, such as beams or disks under flexure, or cylindrical specimens under uniform uniaxial tension.

The test failure probability can be expressed in terms of the highest stressed point in the specimen, \( \sigma_t \), using the two-parameter Weibull form

\[
P_{ss} = 1 - \exp \left[ - A_s \left( \frac{\sigma_s}{\sigma_{ss}} \right)^{m_s} \right] = 1 - \exp \left[ - \left( \frac{\sigma_t}{\sigma_b} \right)^{m_s} \right]
\]

where \( \sigma_b \) is the characteristic strength of the experimental data and \( A_s \) is the effective area. The Weibull modulus and characteristic strength are estimated from specimen rupture data using techniques such as maximum likelihood and least squares as detailed in references 1, 3, and 25. The effective area, \( A_s \), is defined for the uniaxial Weibull distribution as

\[
A_s = \int_A \left( \frac{\sigma_t(\Psi)}{\sigma_f} \right)^{m_s} \, dA
\]

where \( \sigma_t(\Psi) \) denotes the maximum principal stress distribution. For multiaxially stressed components, the Batdorf technique is used to evaluate the effective area. The analogous formulation for \( A_s \) is then

\[
A_s = \frac{2K_{BS}}{\pi} \int_A \left[ \int_0^{\frac{\pi}{2}} \left( \frac{\sigma_{bl}(\Psi)}{\sigma_f} \right)^{m_s} \, d\alpha \right] \, dA
\]

where the normalized Batdorf crack density coefficient \( k_{BS} = k_{BS}/k_{ws} \) is used to normalize to the uniaxial stress state. Similar expressions for the effective volume, \( V_s \), are determined based on volume flaw formulations.

CARES/LIFE normalizes the various fracture criteria to yield an identical probability of failure for the uniaxial stress state. This is achieved by adjusting the fatigue constant \( B \) as well as \( k_{BS} \). For the uniaxial Weibull model this adjusted value is denoted by \( B_{ws} \) and for the Batdorf model it is denoted by \( B_{ws} \). From the dynamic fatigue equation (12), substituting \( B_{ws} \) for \( B \), \( N_s \) for \( N \), the uniaxial stress \( \sigma_t \) for \( \sigma_{bl} \), and rearranging equation (17) while assuming that

\[
\frac{\sigma_t^2(\Psi,t_{w})}{(N_s+1) B_{ws}} > 1
\]

the median behavior of the experimental dynamic fatigue data can be described by

\[
\sigma_{ws} = A_s \sigma^{1/(N_s+1)}
\]

where \( \sigma_{ws} \) is the median rupture stress of the specimen and \( \sigma \) represents the stress rate at the location of maximum stress. The constant \( A_s \) is

\[
A_s = \left\{ \frac{(N_s + 1) B_{ws} \sigma_{ws}^{N_s+2}}{A_{ef} \left( \frac{1}{1-0.50} \right)^{1/b_{ws}}} \right\}^{1/(N_s+1)}
\]

where

\[
\bar{m}_s = \frac{m_s}{N_s - 2}
\]

The constants \( A_s \) and \( A_{ef} \) are obtained by equating risks of rupture. \( A_{ef} \) is a modified effective area required for the time-dependent formulation. For the uniaxial Weibull distribution, the expression for the modified effective area is

\[
A_{ef} = \int_A \left( \frac{\sigma_t(\Psi)}{\sigma_f} \right)^{m_s N_s} \, dA
\]

where \( \sigma_t(\Psi,t) \) denotes the maximum principal stress distribution. For multiaxially stressed components, the Batdorf technique is used to evaluate fatigue parameters. The analogous formulation for \( A_{ef} \) is then

\[
A_{ef} = \frac{2K_{BS}}{\pi} \int_A \left[ \int_0^{\frac{\pi}{2}} \left( \frac{\sigma_{bl}(\Psi,t)}{\sigma_f} \right)^{m_s N_s} \, d\alpha \right] \, dA
\]

Equation (24) is applicable except that \( B_{ws} \) replaces \( B_{ws} \). The relationship between \( B_{ws} \) and \( B_{ws} \) for a uniaxial load is established by equating the risk of rupture of the Batdorf model with that of the uniaxial Weibull model

\[
B_{ws} = \left\{ \frac{\pi \int_A \sigma_{ws}^{N_s} (\Psi,t_{w}) \, dA}{2 K_{BS} \int_A \left[ \int_0^{\frac{\pi}{2}} \sigma_{bl}^{N_s} (\Psi,t) \, d\alpha \right] \, dA} \right\}^{1/m_s}
\]

As \( N_s \) becomes large, equation (27) approaches unity.
The terms \( A_d \) and \( N_d \) in equation (24) are determined from experimental data. Taking the logarithm of equation (23) yields

\[
\ln \sigma = \ln A_d + \frac{1}{N_d + 1} \ln \sigma
\]

Linear regression analysis of the experimental data is used to solve equation (28). The median value method is based on least squares linear regression of median data points for various stress rates. Another technique uses least squares linear regression on all the data points. A third option for estimating fatigue parameters is a modification to a method used by Jakus. In this procedure, fatigue parameters are determined by minimizing the median deviation of the logarithm of the failure stress. The median deviation is the mean of the residuals, where the residual is defined as the absolute value of the difference between the logarithm of the failure stress and the logarithm of the median value. In CARES/LIFE this minimization is accomplished by maximizing \( N_d (\ln N_d - 1) \) estimated from the data versus the fatigue exponent. CARES/LIFE performs least squares or maximum likelihood Weibull parameter estimation as described by Pai and Gyekenesi to solve for Weibull distribution parameters. The fatigue constant \( B \) is obtained from equations (24) and (27).

**INTRODUCTION - DENTAL CERAMICS**

Recent breakthroughs in materials science and technology have led to improved dental restorative ceramics, metals, and resin-based composites. These breakthroughs have become important since major controversies during this period have centered on allegations of the physiological risks of mercury released from amalgam restorations and environmental concerns regarding waste disposal. In response to the this the Swedish government has proposed the progressive elimination of amalgam as a dental restorative material by 1997. Other nations are expected to follow the Swedish model. Among the materials that have been considered as the most biocompatible alternatives to amalgam are dental ceramics. New ceramic products and technologies have gained considerable acceptance in Sweden even though their safety and efficacy have not been fully established.

Ceramic restorations represent one choice for treatment of small to large areas of tooth breakdown in anterior and posterior teeth. Current ceramic products are designed as feldspathic veneering porcelains for a metal substrate (metal-ceramic or porcelain-fused-to-metal restoration) or as components of an all-ceramic restoration. The properties of feldspathic porcelains depend on their composition, microstructure, residual stress state, flaw characteristics, and surface finish. The compositions of dental porcelains for metal-ceramic restorations are often formulated from feldspar and may contain 37-63 wt% \( \text{SiO}_2 \), 9-17 wt% \( \text{Al}_2\text{O}_3 \), 4-15 wt% \( \text{Na}_2\text{O} \), 5-14 wt% \( \text{K}_2\text{O} \), 0-3 wt% TiO\(_2\), 0-27 wt% ZrO\(_2\), and 4-15 wt% SnO\(_2\). Colorant oxides and opacifiers are added for esthetic purposes. All-ceramic restorations derive their fracture resistance from tougher, higher-strength core ceramics since they lack the high elastic modulus of some metals used as the substructure of metal-ceramic prostheses. Compared with glass ionomer cement and glass-filled resin-based composites that are restricted to lower stress applications, dental ceramics are more durable and less technique sensitive. However, they are more costly and require at least two dental appointments, expect for CAD-CAM restorations that can be machined on-site after an image of the prepared tooth is recorded and transmitted to the computer controlled machining unit. Also, compared with cast-metal or metal-ceramic restorations, all-ceramic restorations are more esthetic, but they are more susceptible to fracture, especially when they are used for reconstruction of molar teeth that are exposed to higher forces. Failure of a ceramic or metal-ceramic restoration may also be caused by debonding, secondary decay at the borders of the restorations, root canal treatment, root fracture, and periodontal disease.

During the past 10 years, advances have been made in the development of ceramics with greater toughness and flexure strength, and of glass-polymer composites with greater wear resistance and higher strength. Although dental ceramics constitute a much more costly alternative than some other materials, they offer the advantages of aesthetics, inertness, durability, low thermal conductivity, and low diffusivity, as compared with amalgams. Compared with resin-based composites, ceramics are not as susceptible to bacterial adhesion and plaque accumulation. While ceramics are generally considered to be inert in the oral cavity, there are certain phenomena associated with their use that may cause significant surface degradation of these materials and subsequent physiological side effects. The survival of these materials as dental restorations depends on their resistance to degradation under the harsh conditions of the oral cavity. These conditions include exposure to a variety of fluids ranging in pH from 2 to 9, thermal cycling between 5 and 55°C, loads of up to 4,345 N as reported by Gibbs et al., attacked by acidiﬁed phosphate ﬂuorides, and abrasion from tooth brushing, professional cleanings, and food substances.

Because of their brittle nature, dental ceramic materials are susceptible to failure from small flaws or cracks under applied tensile stresses. There are several factors which are associated with crack initiation and propagation in dental ceramic restorations, including: (1) shape of the restoration and thickness of ceramic layers; (2) microstructural inhomogeneity; (3) size and distribution of surface and volume flaws; (4) residual and transient stresses and stress gradients induced by polishing and/or thermal processing; (5) the environment in contact with the restoration (stress corrosion fatigue), (6) repeated loadings (cyclic fatigue from debris wedging at crack tips); (7) thermal expansion or contraction differences; and (8) magnitude and orientation of applied loads. The possible interactions among these variables complicate the interpretation of failure analysis observations.

Fairhurst et al. cite several studies showing that dental porcelains exhibit stress corrosion fatigue. These studies have shown: (1) a lower fracture strength when the porcelain specimens were tested in water as compared to testing in air; and (2) a decrease in breaking strength with increased durations of applied load in water. White showed evidence that feldspathic porcelain undergoes mechanical fatigue as well. Additional research is expected to uncover the role cyclic loading has on the strength of these materials over time. As a consequence, failure over time for dental restorations involves accumulated degradation of the material as well as the occasional instance of overloading and accumulated decay of the tooth at the borders of the restoration. Anusavice has reviewed several clinical studies of porcelain-fused-to-metal (PFM) service lifetime reported by various authors.
Results varied from study to study and a clear consensus regarding lifetime did not emerge. Walton et al.\(^1\) reported a mean life span of 5.7 years and porcelain failure was the second most common failure cause for the bridges and crown surveyed. Maryniak and Kaplan\(^2\) reported a survey of practicing dentists in which longevity estimates were made for a single-unit PFM restoration and a comparable all gold unit. The PFM units were estimated to have a mean life of 12.7 years versus 14.7 years for the all gold restorations. For a glass-ceramic dental molar crown (Dicor material), Moffa\(^3\) reported a failure rate of 35% during the first three years. In a similar study Erpenstein and Kerschbaum\(^4\) reported a four-year failure rate of 64%. An earlier study by Linkowski\(^5\) with Cerestore material indicated a four-year failure rate of 19%.

Fracture surfaces of clinically failed all-ceramic crowns were systematically studied by Thompson et al.\(^6\) for Dicor and Cerestore materials. Ten fractured Dicor and 12 fractured Cerestore crowns were retrieved and analyzed. The authors conclude that fracture initiation sites appear to be controlled primarily by the location and size of the critical flaw, and not by the specimen thickness. Also, surface preparation and installation technique were important regarding fracture mode. All Dicor crowns displayed fractographic features indicating failure initiation along the internal surface from surface residing flaws as shown in Figure 1, taken from reference 36. The fracture surfaces were relatively smooth and fracture origins were clearly defined. Etching of the surface prior to resin bonding (cementation) introduces a modification of existing flaws. Cerestore crowns displayed fractographic features indicating failure initiation along the porcelain/core interface, or within the core near the interface. One of the Cerestore crowns had a failure initiation site on the outer surface of the porcelain layer at the cervical margin; another had a failure initiation site located at a large internal pore within the porcelain layer. Cerestore fracture surfaces were rough and the core ceramic contained a large volume fraction of porosity. As such, precise location of critical flaws could not be determined. For other types of crowns, the location of failure varies, including localized chipping as a result of Hertzian stresses from contact loads.\(^7\)

Kelly et al.\(^8\) investigated all-ceramic fixed partial dentures with the goal of correlating laboratory testing with clinically failed units using finite element analysis and the Weibull distribution function. Anusavice and Hojjatie\(^9\) used finite element analysis to model occlusally loaded glass-ceramic crowns with flaws in the cementitious bonding layer. These are among the first studies attempting to correlate modeling with observed clinical failure modes.

Since it has been demonstrated that ceramic dental restorations fail in a classic brittle manner, brittle material design methodology employing reliability analysis can be useful in the analysis, design and optimization of these restorations. To test this hypothesis this study was designed to demonstrate reliability analysis of a PFM molar crown restoration. Although no clinical study regarding failure analysis of metal-ceramic crowns exists, it is reasonable to expect that the ceramic material behaves in a similar manner as described in the previous studies cited above. A key question to be answered was if the reliability analysis based on an idealized model and finite element analysis could reasonably provide further insight on the failure modes and likely locations of critical flaws in these restorations. Also, the step-by-step process by which testing data is translated into reliability predictions of a dental crown is demonstrated. We speculate that in conjunction with imaging technology (scanning an individual's dental topology), finite element analysis and optimization techniques that employ reliability prediction methodology can lead to improved designs.

**EVALUATION OF WEIBULL AND FATIGUE PARAMETERS**

To predict the reliability of a component over time in service the material Weibull and fatigue parameters must be known. Weibull and fatigue parameters are obtained from rupture experiments of many nominally identical flawed specimens. In this study reliability is predicted for two materials: (1) Porcelain B (J. F. Jelenko and Co., Armonk, NY, USA) used to form the exposed outer surface of the crown; and (2) Porcelain O (J. F. Jelenko and Co., Armonk, NY, USA) that is applied directly over the surface of the metal layer to reduce visibility of the metal. Both of these materials are feldspathic porcelains. Fast-fracture rupture data for the body 1 (B) and opaque 4 (O) porcelains was obtained from references (39) and (40) respectively. Fatigue rupture experiments were not performed on these materials in reference (39), however dynamic fatigue experiments were performed by Fairhurst et al.\(^20\) on the Porcelain B material. Specific information regarding the experimental setup and testing environment for fast-fracture is described in reference (39). Two specimen configurations were tested; 3-point flexure bars and piston-on-3-ball loaded circular disks. Only the results for the piston-on-3-ball specimens were used herein, since the 3-point data reported in references (39) and (40) suffered from poor dimensional control of the specimen thickness. Additionally, the 3-point specimen inherently can suffer from machining (chip) flaws along the specimen edges (confounding results by introducing a competing failure mode against the natural material flaws). Two major advantages of the piston-on-3-ball configuration is that specimen edges are not highly stressed and that the 3-point fixture can accommodate slightly warped specimens. The disks had an average radius of 8.0 mm, a thickness of 2.0 mm, a support radius of 5.0 mm, and a radius of the load piston of 0.8 mm. Specimen to specimen dimensional variation was negligible.

In order to sample the inert strength distributions, 10 specimens of each material were ruptured in dry air. Note that sample sizes of at least 30 specimens are desired to reduce uncertainty in the accuracy of the estimated parameters (especially the Weibull modulus m). Detailed fractographic examination of the broken samples to determine the identity of flaw populations was not performed. However, fractography is recommended on all specimens. The Weibull plots of the fracture data, Figure 2(a) and (b), and the outlier test in CARES/LIFE did not indicate any strong curvatures or unusual data. This is an indication that the data is unimodal (derived from only one flaw population). Optical examination of the fracture surfaces did not reveal unusual features such as scratch or machining flaws. For this study the flaws are considered to be randomly oriented and distributed exclusively in the material volume (volume flaws) or on the surface (surface flaws). Fracture strengths from references (39) and (40) were determined at the center of the disk (the highest stressed point) at the instant of failure using the formulation recommended by Marshall et al.\(^41\) and Young et al.\(^42\).
\[ \sigma_x = \frac{3P(1-v)}{4\pi t^2} \left( 1 - 2\ln\frac{a}{r^*} \right) \left( 1 - v \right) \left( -\frac{r^*}{2a^2} \right) R^2 \]

\[ r^* = (1.6t^2 + t^2)^{1/2} - 0.675t \quad \text{for} \quad r^*_a \leq t/2 \]

\[ r^*_a = r^*_o \quad \text{for} \quad r^*_a > t/2 \]

Where \( a \) is the radius of the balls from the center of the disk, \( r^*_o \) is the radius of the piston, \( R \) is the radius of the disk, \( t \) is the thickness of the disk, and \( P \) is the applied force. Figure 2(a) and (b) show Weibull plots of the fracture stresses. The Porcelain O is significantly stronger than the Porcelain B. Note this figure also includes Kanofsky-Srinivasan 90% confidence bands. As indicated in reference (39), equation (29) correlates well with results from 3-dimensional finite element analysis within certain ranges. However, this stress distribution throughout the disk must be known. Because of the loading complexity and presence of multiaxial stress states, the integration is performed numerically in CARES/LIFE using the results from ANSYS finite element analysis. A one-third finite element model of the piston-on-3-ball disk was used to study the specimen's stress distribution. The specimen disk model consists of solid elements (ANSYS element SOLID45) and shell elements (ANSYS element SHELL63) where mesh density is highest at the disk center on the tensile surface. A sufficiently refined mesh is essential for reliability analysis since stresses are exponentiated by the Weibull modulus in the stress-volume (-area) integral. Hence, the accuracy of results is highly sensitive to the accuracy of the stress solution. The solid elements are used for the volume flaw formulations. Shell elements were placed on the surface of the model sharing common nodes with the solid elements. The shell elements were set with a very small (1x10^-6 mm) thickness and with membrane properties only (no bending stiffness) so that a negligible amount of stiffness was added to the problem. The presence of the shell elements yielded the in-plane elemental surface stresses and corresponding elemental areas. Figure 3 is a first-principal-stress plot for the specimen disk. Stresses predicted by FEA were consistently within a few percent of the closed form approximation (equation 29).

The effective volume, \( V_e \), and area, \( A_e \), were calculated with CARES/LIFE using the Batdorf methodology (see equation (21)). Shetty's mixed-mode fracture criterion, and a penny-shaped crack for the volume flaw and a semi-circular crack for the surface flaw (further details are given in refs. 1 and 3). Table 1 summarizes the results from CARES/LIFE of the Weibull moduli, characteristic strengths, effective volumes and areas, as well as the scale parameters for the two materials (as described by equations (18) to (21)). These parameters are used for the subsequent fast-fracture reliability analysis.

Assuming that small crack-like imperfections control the failure, the material strength in multiaxial stress states can be correlated to the effects of mixed-mode loading on the individual cracks. Shetty(61) developed a simple equation describing the ability of a crack to extend under the combined actions of a normal and shear load on the crack face using an empirically determined parameter, \( C \). For a semicircular crack this equation is

\[ \sigma_{eq} = \frac{1}{2} \left[ \sigma_n + \sqrt{\sigma_n^2 + 3.301 \left( \frac{\tau}{C} \right)^2} \right] \]

where \( \sigma_n \) and \( \tau \) are the normal and shear stresses, respectively, acting on the flaw plane. For a penny-shaped crack this relationship is

\[ \sigma_{eq} = \frac{1}{2} \left[ \sigma_n + \sqrt{\sigma_n^2 + \left( \frac{4\tau}{C(2-v)} \right)^2} \right] \]

A value of \( C \approx 0.82 \) approximates the non-coplanar strain energy release rate criterion.

The dynamic fatigue specimen rupture data from Fairhurst et al. was used to obtain fatigue parameters for Jelenko Gingival porcelain B. Fairhurst et al. also determines fatigue parameters for a second material, designated C1/C3 Model porcelain. The parameters, for the C1/C3 Model material, are used for the porcelain O in this study (for the time-dependent reliability analysis). Note porcelain C1/C3 Model and porcelain O have similar Weibull parameters as shown in Table 1 and Table 2. We conjecture that the two porcelains (C1/C3 Model and O) will generally have a similar fatigue behavior because of the similarity of the glass components of the two materials. The C1/C3 Model is a bimaterial consisting of a high leucite-containing component (C1), and the glass component (C3), provided by Jelenko Dental Health Products, Armonk, NY. A mixture of 0.6 C1 and 0.4 C3 was used for the model system. Beam deflection of specimens from this composition matched the beam deflection of the body porcelain (indicating the Young’s modulus of C1/C3 Model is similar to the value for the porcelain O). Further information regarding the material compositions and experimental setup and testing environment is described in reference 28.

Again a piston-on-3-ball specimen configuration was used, except with a disk radius of 6.00 mm, a thickness of 1.00 mm, a support radius of 3.16 mm, and a piston radius of 0.78 mm. Equation (29) was used to determine peak fracture stresses and a finite element model of the specimen was prepared. Each material was tested at five stressing rates (100, 10, 1, 0.1, and 0.01 MPa/s) in water (to simulate the oral environment), and also in a dry (inert) environment. All tests were performed at 37°C. Fifty B and twenty C1/C3 Model porcelain specimens were tested at each stressing rate. One hundred B and twenty C1/C3 porcelain specimens were tested in fast-fracture. Individual specimen fracture stresses were not available, however reference (28) gives median rupture stress values at each stressing rate, as reproduced in Figure 4(a) and (b). Figure 4 also shows median regression line and the 90% prediction band of the scatter in the data estimated from CARES/LIFE. Fortunately, the data reported...
by Fairhurst et al.\textsuperscript{(28)} are sufficient to utilize the parameter estimation capabilities of CARES/LIFE (see equations (28), (26), (24), (21), and (19)). Using these results and finite element analyses of the specimen, the Weibull and fatigue parameters were calculated and listed in Table 2. The values listed in Table 2 were calculated based on specific specimen geometry, loading, crack shapes, and the fracture criterion; hence, the results are not directly comparable to reference 28 (which did not consider these factors). Also, the difference in the fatigue exponent \( N \) from this study and that of Fairhurst et al.\textsuperscript{(28)} is associated with regression being performed on median values versus regression performed on all fracture stresses.

### RELIABILITY ANALYSIS OF A PFM MOLAR CROWN

The ANSYS\textsuperscript{\textregistered} Revision 5.1 FEA program was used to create the model of the ceramic-metal dental restoration and analyze various load cases. Although a molar crown can be analyzed by an axisymmetric model \textsuperscript{\textsuperscript{34}}, a three-dimensional model needed to be created\textsuperscript{\textsuperscript{30}} to analyze stresses in the crown subjected to a nonsymmetric loading condition. A three-dimensional finite element model of a mandibular second molar crown was developed and the overall dimensions were selected to be consistent with the values identified by Wheeler\textsuperscript{(40)} for an average molar crown. It was assumed that the natural enamel in the prepared areas was completely removed.

The model of one half of the PFM crown is composed of four materials. Figure 5 is a cross-sectional view of the this 3-dimensional model. At the center is the pulp chamber (modeled as a void). The dentin layer is the remaining part of the original tooth, to which the restoration is affixed. Directly in contact with the dentin is a thin layer of metal, since the cement layer was ignored with this model. A 200 \( \mu \)m nickel-chromium alloy coping acts as a support to alleviate stress build-up within the ceramic. The metal layer is covered by two layers of ceramic materials. The outer ceramic is porcelain B (J.F. Jelenko and Co., Armonk, N.Y.) and the inner ceramic is known as porcelain O (J.F. Jelenko and Co., Armonk, N.Y.). We have assumed that residual stresses (which are induced in the ceramic due to contraction differences between the metal, Opaque 4, and Porcelain B layers and nonuniform distribution of temperature during high aspect processing) were negligible. Failure or debonding in the metal layer was not investigated (perfect bonding was assumed). A cement layer, which is used to bond the metal to the dentin was not considered. The materials were assumed to be homogeneous, linearly elastic, and isotropic.

The model is composed of 9504 volume elements and 1685 surface elements. The volume elements are ANSYS\textsuperscript{\textregistered} SOLID45 3-D structural solid 8 node brick elements, all of which are full bricks (none are collapsed to prisms or tetrahedrons). The surface elements are ANSYS\textsuperscript{\textregistered} SHELL63 elastic shell elements which are all collapsed to three-node triangles. Mesh density was concentrated in areas of high stress so accuracy could be maintained. The elastic modulus values for porcelain B, porcelain O, Ni-Cr alloy, and the dentin were 68.0, 75.0, 210.0, and 18.6 GPa, respectively, and the corresponding Poisson’s ratios were 0.28, 0.28, 0.33, and 0.31, respectively. The effect of the pulp region on stress was considered to be negligible. General guidelines for material thickness were followed such that the body porcelain thickness was generally 1.05 mm thick, the opaque porcelain was 0.35 mm thick, and the Ni-Cr alloy was held constant at a thickness of 0.20 mm. The overall height of the tooth-crown combination was 9.1 mm, and the width at the root was 6.00 mm.

The use of a three-dimensional model allowed concentrated direction loading on one cusp of the crown to be examined (rotational or twisting loads were not considered). A simple fixed support is assumed at the bottom (root) portion of the dentin and the plane of symmetry is fixed with respect to out of plane displacements and rotations. The top surface of the crown had gradual height undulations varying by 0.50 mm to simulate the four cusps.

To study surface stresses, shell elements were placed on selected surfaces sharing common nodes with the solid elements. The outer surface of porcelain B had shell elements placed on the areas surrounding the load and the opposing side of the restoration. The areas which are shared by the metal layer and porcelain O also had shell elements applied to them. These surfaces were expected to experience the highest stresses for the various load cases investigated. The interface between the two porcelain materials was not examined since they have similar elastic modulus. The shell elements were set with a very small thickness (1x10\textsuperscript{-4} mm) and with membrane properties only (no bending stiffness) so that a negligible amount of stiffness was added to the problem. Note that the capability to selectively add shell elements enabled interior surfaces to be analyzed and planes of symmetry to be ignored (not erroneously recognized as a surface). The presence of the shell elements yielded the in-plane elemental surface stresses and corresponding elemental areas.

Two-dimensional finite element stress analysis of ceramic crowns for anterior teeth indicate that the loading orientation is a major cause of tensile stresses\textsuperscript{(40)}. As a result three loading cases were used to evaluate the stress distribution within the restoration. The first case was a load of 300 N applied perpendicular to the face of one cusp. The second load case was a 300 N load applied 30 degrees from perpendicular directed toward the center of the tooth. The third load case was a 600 N load applied vertically at the peak of the cusp. The loads were distributed over the surface of 10 elements which made up an area of roughly 0.5mm\textsuperscript{2} (one square millimeter for a whole 360 degree restoration). Direct forces were applied to the nodes of these elements. All three load cases were investigated for fast-fracture reliability. The first load case was also investigated for cyclic loading by assuming it approximated a chewing action.

Figure 6 shows first-principal stress plots for the three load cases. All the load cases indicate highest stresses are in the metallic layer. Maximum stress within the metal layer was 73 MPa, which is well below the 690 MPa yield strength of the alloy. As shown in the plots, the high elastic modulus of the metal alleviates stress build-up in the porcelain materials, especially at the opaque-metal boundary. In fact stresses in the opaque porcelain near the metal layer, although tensile, are quite low in magnitude (maximum stress was 26 MPa). Certainly, debonding and metal fatigue resulting in stress redistribution and allowing the oral environment to contact with the opaque porcelain can drastically affect the integrity of the restoration, but these failure modes were not considered. Peak stress for the porcelains consistently occur in the porcelain B surrounding the load. This is an expected result of the Hertzian type of loading. Obviously other loading distributions may reduce or increase this localized stress. For the loading situations used in this study the Hertzian
stress are believed to be realistic and capable of inducing material damage. Whether this damage would remain as localized cracking, chipping, or result in failure of the restoration is beyond the scope of this study. Suffice it to say that even localized cracking or chipping could necessitate repair or replacement of the crown, and therefore is an undesirable situation we consider equivalent to a "failure." Overall, while the highest porcelain stresses occur on the porcelain B surfaces, there is also a significant distribution of tensile stress throughout both porcelain volumes. This situation is significant regarding the reliability of the restoration.

The degree of crack growth is dependent on both the magnitude and the duration of the applied stress. Therefore, it is not only difficult to estimate the effective applied stress in the oral environment, but the life estimation may be further complicated by crack healing or blunting of the crack tip during prolonged non-stress periods. This analysis is based on conservative assumptions regarding the statistics and physics of crack growth. Thus, predicted lifetimes will represent lower bounds with respect to material integrity. Of course, unaccounted failure modes will change lifetime estimations and this analysis did not consider the integrity of the bonding. Keeping this caveat in mind, reliability results for the three loading cases can be discussed. Fractography of specimen surfaces were not segregated into flaw type, which would allow concurrent flaw populations to be considered in the reliability analysis. Reliability results are presented based either on volume flaw failures or surface flaw failures, but not both simultaneously. Figure 7 shows a Weibull plot of the overall results for the fast-fracture probability of failure versus the magnitude of applied loading for the molar crown for volume flaw analysis. The crown is much more sensitive to flaws distributed in the volume than flaws residing on the surface. This is a reasonable result, since the loading is much greater in the crown than the flexure specimen disks. In contrast to this, the area fraction of the crown that experiences tensile stress relative to that of the disk specimen is significantly smaller. This does not mean that the crown will fail from volume flaws and not surface flaws. Rather it points to the need to firmly establish the identity of flaws in the disk specimen experiments. Also, since flexure tests are especially sensitive to surface flaws, these results perhaps point to the need for further tensile specimen testing where more of the specimen volume can be exposed to high levels of stress. Table 3 is a breakdown of the failure probability for each material under the three load cases for surface and volume flaw analysis. The material numbers represent various slices from the whole model geometry. It is clear that the porcelain B is driving the failure response and not the porcelain O. This is understandable, since the porcelain B is a weaker material than the porcelain O (they both have similar Weibull moduli) and the highest stresses are experienced by the body porcelain, especially as a result of Hertzian contact stresses. Obviously, Hertzian stresses are strongly dependent on the nature of the load, placement of the load, and the topology of the crown surface. Consequently one must be careful when extrapolating these results in a general fashion to clinical experiences. On the other hand, stresses around the metal layer seem less sensitive to the loading. Therefore, meaningful design guidelines may be more feasible here. Obviously, a clinical and laboratory study further exploring these issues is needed here before firm conclusions can be reached.

The time-dependent probability of failure is shown in Figure 8. This was for surface flaws on the exposed areas of the porcelain B. It was assumed that the interior surfaces of the porcelains at the metallic layer and at the interface of the two porcelains were not exposed directly to the environment. Further it was assumed that the volume flaws were not exposed to the oral environment. Of course, depending on porosity or the ability of the saliva to diffuse into the material this may or may not be a correct assumption. However, without a firm basis to assume stress corrosion can occur inside the material, the issue is ignored here. Since dental porcelains are sensitive to mechanical fatigue effects, a volume and surface flaw analysis investigating ramifications of material integrity using the Walker law (detailed in equations (15) and (16)) is desirable. Unfortunately, the current lack of usable data regarding this phenomenon precludes presenting results here. Based on the stress corrosion data from Fairhurst et al., we are assuming a cyclically applied sawtooth loading waveform with a stress ratio of zero (R=0.0). Assuming a cyclic frequency of 1 Hertz allows reliability to be predicted as a function of the number of cycles. For the various constant peak applied loads indicated in the figure, the effect of stress cycles on reliability is revealed.

CONCLUSION

The use of ceramics for load-bearing applications depends on the strength, toughness, and reliability of these materials. Ceramic components can be designed for long service life if the factors that cause material failure are considered. This design methodology must combine the statistical nature of strength controlling flaws with fracture mechanics to allow for multiaxial stress states, concurrent flaw populations, and subcritical crack growth. This has been accomplished with the CARES/LIFE public domain computer program for predicting the time-dependent reliability of monolithic structural ceramic components. CARES/LIFE has been used in this study to examine the durability of a ceramic-metal molar dental restoration.

For all load cases, fast-fracture failure occurred in the porcelain B volume. One of the reasons for this was the stress distribution within that volume. Results of this analysis have shown that the nickel-chromium coping successfully relieves stress from the ceramic components of the PFM restoration by absorbing the high stresses. The maximum stress in the metal layer was 73 MPa, which is well under the 690 MPa yield strength of the metal. Since the stresses are relatively low in the region of porcelain O and they are relatively insensitive to surface topology, failure of the restoration is not likely to occur here. However, if debonding or metal fatigue was to occur because of the high stress in the metal, the chance for failure in the porcelain O would likely be much higher.

The combination of a concentrated load, stress distribution, and volume flaws control the probability of failure within the volume of the restoration. The concentrated load produced a Hertzian stress distribution. The load most likely to initiate failure was oriented 30 degrees from perpendicular, which produced very high tensile stresses on the tip of the loaded cusp. Porcelain B sustained the highest stresses and therefore the greatest probability of failure.
FUTURE WORK...

Future work should contrast the results in this paper for a PFM crown versus an all-ceramic crown luted with cement. The influence of reduced metal thickness on the reliability of PFM crowns is also being studied. Ultrathin metal thickness of 0.1 mm has been proposed for use with PFM restorations, but the long-term durability of these crowns has not been fully investigated.

REFERENCES


Table 1: Summary of statistical parameters for the biaxial flexure specimens.

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Table 2: Summary of statistical parameters for the biaxial flexure specimens

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<th>Porcelain C/C3</th>
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<td>$\mu$</td>
<td>15.7</td>
<td>15.7</td>
</tr>
<tr>
<td>$A_s$ (mm$^2$)</td>
<td>167</td>
<td>N/A</td>
</tr>
<tr>
<td>$V_s$ (mm$^2$)</td>
<td>133</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 3: Probability of failure and maximum stress for each group of elements within the porcelain. Element groups are listed by material type and (material number).

### 300 Newton Load Perpendicular to Surface

<table>
<thead>
<tr>
<th>Material Type(n)</th>
<th>Fracture Origin</th>
<th>$P_F$</th>
<th>$\sigma_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(1)</td>
<td>Volume</td>
<td>0.1146E-01</td>
<td>70.2</td>
</tr>
<tr>
<td>O(2)</td>
<td>Volume</td>
<td>0.6416E-06</td>
<td>50.2</td>
</tr>
<tr>
<td>O(6)</td>
<td>Surface</td>
<td>0.3232E-07</td>
<td>26.2</td>
</tr>
<tr>
<td>B(7)</td>
<td>Surface</td>
<td>0.4219E-14</td>
<td>9.8</td>
</tr>
<tr>
<td>O(9)</td>
<td>Surface</td>
<td>0.0000E+00</td>
<td>6.2</td>
</tr>
<tr>
<td>Overall Volume $P_F$</td>
<td>0.1146E-01</td>
<td>0.3232E-07</td>
<td>10.2</td>
</tr>
<tr>
<td>Overall Surface $P_F$</td>
<td>0.3232E-07</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 300 Newton Load 30° From Perpendicular Load Case

<table>
<thead>
<tr>
<th>Material Type(n)</th>
<th>Fracture Origin</th>
<th>$P_F$</th>
<th>$\sigma_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(1)</td>
<td>Volume</td>
<td>0.1194E+00</td>
<td>81.2</td>
</tr>
<tr>
<td>O(2)</td>
<td>Volume</td>
<td>0.4516E-06</td>
<td>50.2</td>
</tr>
<tr>
<td>O(6)</td>
<td>Surface</td>
<td>0.5865E-07</td>
<td>27.2</td>
</tr>
<tr>
<td>B(7)</td>
<td>Surface</td>
<td>0.0000E+00</td>
<td>9.2</td>
</tr>
<tr>
<td>O(8)</td>
<td>Surface</td>
<td>0.0000E+00</td>
<td>0.2</td>
</tr>
<tr>
<td>Overall Volume $P_F$</td>
<td>0.1194E+00</td>
<td>0.5865E-07</td>
<td>2.2</td>
</tr>
<tr>
<td>Overall Surface $P_F$</td>
<td>0.5865E-07</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 600 Newton Load Applied Vertical to Crown

<table>
<thead>
<tr>
<th>Material Type(n)</th>
<th>Fracture Origin</th>
<th>$P_F$</th>
<th>$\sigma_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(1)</td>
<td>Volume</td>
<td>0.3552E+00</td>
<td>91.2</td>
</tr>
<tr>
<td>O(2)</td>
<td>Volume</td>
<td>0.3897E-03</td>
<td>83.2</td>
</tr>
<tr>
<td>O(6)</td>
<td>Surface</td>
<td>0.8173E-11</td>
<td>15.2</td>
</tr>
<tr>
<td>B(7)</td>
<td>Surface</td>
<td>0.1060E+10</td>
<td>17.2</td>
</tr>
<tr>
<td>O(8)</td>
<td>Surface</td>
<td>0.0000E+00</td>
<td>7.2</td>
</tr>
<tr>
<td>Overall Volume $P_F$</td>
<td>0.3552E+00</td>
<td>0.1060E+10</td>
<td>17.2</td>
</tr>
<tr>
<td>Overall Surface $P_F$</td>
<td>0.1060E+10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1.—Fractographic results of an all Dicor® crown showing failure initiation along the internal surface of the crown. Figure taken from reference 36.

Figure 2.—Weibull plot of (a) body porcelain and (b) opaque porcelain fast-fracture data reproduced (39) and (40). Dotted lines represent Kanofsky-Srinivasan 90 percent confidence bands.
Figure 3.—ANSYS® Plot of the principal stress on a biaxial test specimen. This model was used to analyze data from reference 40.

Figure 4.—Plot of median data values at various stressing rates (dynamic fatigue) of (a) body porcelain and (b) C1/C3 model porcelain taken from reference 28. Dotted lines represent 90 percent scatter band of the individual rupture stresses.
Figure 5.—Cutting plane view of three dimensional PFM molar crown used for this analysis. (Top of model is titled in 30°).

Figure 6.—(a) First principle stress plot for load case perpendicular to surface. (b) First principle stress plot for load case 30° from perpendicular to surface. (c) First principle stress plot for vertical load case.
Figure 7.—Weibull plot of fast-fracture failure probability versus applied load for each load case for (a).

Figure 8.—Cycle dependent probability of failure for surface flaws on exposed surface of body 1 porcelain. Highest failure rates correspond to the perpendicular load case (Load case 1). Followed by the 30 degrees from perpendicular load case (Load case 2).
Stress and Reliability Analysis of a Metal-Ceramic Dental Crown

Todd M. Sokolowski, Barry Hojjatie, Noel N. Nemeth, and Kenneth J. Anusavice

Interaction of mechanical and thermal stresses with the flaws and microcracks within the ceramic region of metal-ceramic dental crowns can result in catastrophic or delayed failure of these restorations. The objective of this study was to determine the combined influence of induced functional stresses and pre-existing flaws and microcracks on the time-dependent probability of failure of a metal-ceramic molar crown. A three-dimensional finite element model of a porcelain-fused-to-metal (PFM) molar crown was developed using the ANSYS® finite element program. The crown consisted of a body porcelain, opaque porcelain, and a metal substrate. The model had a 300 Newton load applied perpendicular to one cusp, a load of 300N applied at 30 degrees from the perpendicular load case, directed toward the center, and a 600 Newton vertical load. Ceramic specimens were subjected to a biaxial flexure test and the load-to-failure of each specimen was measured. The results of the finite element stress analysis and the flexure tests were incorporated in the NASA developed CARES/LIFE program to determine the Weibull and fatigue parameters and time-dependent fracture reliability of the PFM crown. CARES/LIFE calculates the time-dependent reliability of monolithic ceramic components subjected to thermomechanical and/or proof test loading. This program is an extension of the CARES (Ceramics Analysis and Reliability Evaluation of Structures) computer program.