Experiments with explicit filtering for LES using a finite-difference method

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1. Motivation and objectives

The equations for large-eddy simulation (LES) are derived formally by applying a spatial filter to the Navier-Stokes equations. The filter width as well as the details of the filter shape are free parameters in LES, and these can be used both to control the effective resolution of the simulation and to establish the relative importance of different portions of the resolved spectrum. In spectral simulations, the natural choice for the LES filter is the truncation associated with the use of a finite number of modes. This choice is "automatic" in the sense that no explicit filtering operations need to be performed during the course of the simulation. In other words, selection of the number of modes dictates the range of scales that can be resolved, and the usual numerical procedures ensure that the higher frequencies generated by nonlinear interactions are excluded from the simulation.

An analogous, but less well justified, approach to filtering is more or less universally used in conjunction with LES using finite-difference methods. In this approach, the finite support provided by the computational mesh as well as the wavenumber-dependent truncation errors associated with the finite-difference operators are assumed to define the filter operation. This approach has the advantage that it is also "automatic" in the sense that no explicit filtering operations need to be performed.

While it is certainly convenient to avoid the explicit filtering operation, there are some practical considerations associated with finite-difference methods that favor the use of an explicit filter. Foremost among these considerations is the issue of truncation error. All finite-difference approximations have an associated truncation error that increases with increasing wavenumber. These errors can be quite severe for the smallest resolved scales, and these errors will interfere with the dynamics of the small eddies if no corrective action is taken. Years of experience at CTR with a second-order finite-difference scheme for high Reynolds number LES has repeatedly indicated that truncation errors must be minimized in order to obtain acceptable simulation results.

Explicit filtering can be used as a means of controlling truncation error by simply removing from the simulation the smallest motions that would otherwise be affected by the error. To implement this approach, an LES filter with a characteristic width greater than the mesh spacing is applied explicitly at the conclusion of each time step during the course of the simulation. The filter operation insures that the error-prone high-frequency solution components are either removed entirely or diminished in amplitude. The ratio of the filter width to the mesh spacing provides a useful measure of the degree to which the truncation error is reduced. As the filter width ratio becomes large the finite-difference approximations can be reasonably accurate over the entire range of scales passed by the filter.

Explicit filtering can also be used to control aliasing, interpolation, and subgrid-scale modeling errors. Aliasing errors arise from the nonlinear generation of frequencies higher than the maximum which the mesh can support. These unresolvable high frequencies "alias" to lower, resolved frequencies. It turns out that for bilinear products, one of the interaction partners must be in the upper third of the wavenumber range in order for the product to alias. Thus aliasing error can be reduced or eliminated by reducing the energy in the upper wavenumber portion of the spectrum. In particular, aliasing error will be eliminated entirely for a filter width ratio greater than or equal to 1.5 when a sharp cutoff filter is used (i.e., the usual 3/2 rule). Interpolation errors are analogous to finite-difference truncation errors in that their magnitude increases with increasing wavenumber. These errors will be reduced in much the same way as the finite-difference truncation error when the solution is filtered. Finally, explicit filtering can be used to control subgrid-scale modeling errors that arise in the implementation of the dynamic subgrid-scale model (Germano *et al.* 1991). In order to compute the subgrid-scale model coefficient, the dynamic model samples turbulent stresses generated by a band of the smallest motions resolved in the simulation. This is also the region of the spectrum where the truncation, interpolation, and aliasing errors are the most severe. If no explicit filtering is performed, the stresses sampled in the dynamic model will be contaminated with the various sources of numerical error, which could lead to erroneous estimates for the subgrid-scale model coefficient.

While the potential advantages of explicit filtering are rather clear, there is a significant cost associated with its implementation. In particular, explicit filtering reduces the effective resolution of the simulation compared with that afforded by the mesh. The resolution requirements for LES are usually set by the need to capture most of the energy-containing eddies, and if explicit filtering is used, the mesh must be enlarged so that these motions are passed by the filter. In simpler terms, the mesh must be expanded in each direction by a factor equal to the filter width ratio in order to retain the effective resolution of an unfiltered simulation. This is a significant overhead for a three-dimensional simulation; a filter width ratio of 2 increases the cost of the simulation by a factor of 8, whereas a filter ration of 3 increases the cost by a factor of 27!

Given the high cost of explicit filtering, the following interesting question arises. Since the mesh must be expanded in order to perform the explicit filter, might it be better to take advantage of the increased resolution and simply perform an unfiltered simulation on the larger mesh? The cost of the two approaches is roughly the same, but the philosophy is rather different. In the filtered simulation, resolution is sacrificed in order to minimize the various forms of numerical error. In the unfiltered simulation, the errors are left intact, but they are concentrated at very small scales that could be dynamically unimportant from a LES perspective. Very little is known about this tradeoff and the objective of this work is to study this relationship in high Reynolds number channel flow simulations using a second-order finite-difference method.

2. Accomplishments

2.1 Numerical method

The second-order staggered mesh scheme of Harlow and Welch (1965) was chosen for this work due to its popularity for contemporary LES. This scheme has a number of practical advantages including mass, momentum, and kinetic energy conservation, coupled pressure and velocity fields, ease of implementation, and straightforward extension to generalized coordinate systems. On the down side, the scheme is of low accuracy and is susceptible to point-to-point oscillations. In an attempt to assess the role of truncation error, the scheme was tested for direct numerical simulation of low Reynolds number turbulent channel flow by Choi *et al.* (1992) and Choi and Moin (1994). They found good agreement in mean and rms velocity profiles when compared with pseudo spectral simulation results on the same mesh. However, they needed to double the mesh in all three directions in order to obtain a good comparison of the vorticity fluctuation profiles. Rai and Moin (1991) performed similar tests but used a much coarser grid for the finite-difference calculation (factor of 14 fewer points than the spectral simulation). They were primarily interested in testing higher-order upwind schemes and found these to be superior to the second-order scheme on the coarse mesh.

Recent experience with the second-order scheme at CTR for high Reynolds number LES has lead to a different conclusion. The scheme has been found to produce acceptable results, but only when rather fine meshes are used (Akselvoll and Moin, 1995; Kaltenbach, 1994, Lund and Moin, 1995). The difference in behavior for LES is probably due to the increased energy level in the smallest resolved scales. These scales make a non-negligible contribution to the low-order statistics in LES, and thus the effects of numerical error are more apparent in this case.

It is hypothesized that explicit filtering should improve the second-order simulation results by removing a portion of the numerical error. It is already known that the simulation results improve as the mesh is refined, and thus the relevant question is whether a greater benefit can be realized through explicit filtering.

2.2 High Reynolds number channel flow test case

The test case for this study is turbulent channel flow at a Reynolds number of 47100 based on centerline velocity and channel half-width (a friction velocity Reynolds number of 2000). This particular Reynolds number was chosen due to the availability of pseudo spectral results (Piomelli, 1993) that are used as a basis for comparison. Piomelli used a computational domain of height 2δ, length $(5\pi/2)\delta$, and width $(\pi/2)\delta$. Fourier expansions were used in the homogeneous direction, whereas a Chebychev expansion was used in the normal direction. The advective terms were cast in skew-symmetric form and no explicit de-aliasing was performed. 64 Fourier modes were used in the streamwise direction, 80 were used in the spanwise direction, and 80 Chebychev modes were used in the normal direction.

The finite-difference mesh is identical to that used in the pseudo spectral simulation with the exception of the distribution of points in the normal direction. The pseudo spectral simulation uses a cosine mapping function to distribute the collocation points in the normal direction. While this distribution is necessary in order to make use of the fast Fourier transform, it leads to a mesh that is strongly stretched in the near-wall region. Experience with this type of mesh for finite-difference calculations indicates that the grid spacing becomes too coarse within a short distance from the wall. In order to avoid this problem, the standard hyperbolic tangent mapping is used. The hyperbolic tangent mesh is designed so that the spacing of the first mesh cell away from the wall as well as the spacing at the channel centerline are very close to those of the cosine mesh. It turns out that these constraints can be met only by increasing the number of points in the normal direction from 81 to 141.

The mesh spacings in wall units are $\Delta x^+ = 250$, $\Delta y^+_{\rm min} = 1.6$, $\Delta y^+_{\rm max} = 150$, and $\Delta z^+ = 40$. In terms of channel half-heights the mesh spacings are $\Delta x/\delta = 0.12$, $\Delta y_{\rm min}/\delta = 8.0 \times 10^{-4}$, $\Delta y_{\rm max}/\delta = 0.075$, and $\Delta z/\delta = 0.02$.

Both the spectral and finite-difference simulations make use of the dynamic subgrid-scale model (Germano *et al.* 1991) with both test filtering and averaging of the equations for the model coefficient performed in planes parallel to the wall. The ratio of the test filter to LES filter is fixed at 2 in all simulations. In cases where an explicit LES filter is used, the test filter is simply adjusted to be twice as wide as the LES filter. The test filter operation is applied in physical space and the stencil width is varied to accommodate filters of various widths.

The simulations are performed with a fixed mean pressure gradient. The mass flow is not constrained and, therefore, will differ from simulation to simulation.

2.3 Explicit filtering strategy

Explicit filtering is restricted to the streamwise and spanwise directions. Several factors dictate this choice. Foremost of these is that the mesh in the wall-normal direction is non-uniform and, therefore, the filtering and derivative operation do not commute. Corrections can be applied in this case (Ghosal and Moin, 1994), but the effectiveness of these has not yet been established. Second, the cost of performing simulations with large filter width ratios is not as severe if the mesh is only expanded in two directions. Finally, except for the core region, the wall-normal mesh is substantially finer than the other two directions. It is therefore plausible that the dominant sources of error arise from the streamwise and spanwise directions and not the normal direction. Indeed in a related study (Lund *et al.* 1995), it was found that refining the wall-normal mesh while leaving the other two directions unchanged resulted in very little improvement in the computed statistics. The same experiment applied to the other two directions, however, lead to a marked improvement in the results.

A sharp spectral cutoff is used for the explicit filter. This choice is dictated primarily by the desire to maintain kinetic energy conservation. The sharp cutoff filter does not alter the non-linear energy transfer since this term is the convolution of the velocity with the advective terms. If the velocity has no energy beyond the cutoff wavenumber, then the energy transfer is the same whether or not the advective terms are filtered with the sharp cutoff. Smooth filters do not share

Case	Mesh			Filter	Effective resolution		
	N_x	N_y	N_z	ratio	N_x	N_y	N_z
A	64	141	80	1.0	64	141	80
B	96	141	120	1.5	64	141	80
C	192	141	240	3.0	64	141	80
D	96	141	120	1.0	96	141	120
E	192	141	240	1.0	192	141	240

Table 1. Mesh and effective resolution for the various simulations.

this property and a non-physical energy drain will result if they are used. Energy conservation for simulations filtered with the sharp cutoff was verified in filtered simulations of isotropic turbulence.

Cutoff filtering is performed with fast Fourier transforms. The current flow solver uses a third-order Runge-Kutta time stepping algorithm and the velocity field is explicitly filtered at the conclusion of each of the three substeps. The computational overhead for the filtering operation is roughly 30%.

2.4 Results from the explicitly filtered simulations

Simulations were run with filter width ratios of 1.0, 1.5, and 3.0 (refer to Table 1.) The mesh was enlarged in the streamwise and spanwise directions by a factor equal to the filter width ratio in each case so that the effective resolution was constant. The modified wavenumber diagram for these simulations are shown in Fig. 1. The chain-dashed vertical line denotes the fixed effective resolution, while the solid curves to the left of this line show the modified wavenumber distributions for the various levels of filtering. When no filter is applied (lowest solid curve in Fig. 1) considerable truncation error is evident for the upper half of the wavenumber range. As the filter width ratio is increased, the situation improves. The error might seem to be acceptable for a filter width ratio of 3.

Figure 2 shows a comparison of the mean velocity profiles from the explicitly filtered simulations, plotted in wall coordinates. The pseudo spectral results of Piomelli (1993) are also included for reference. Starting with the unfiltered simulation, it is seen that the velocity profile deviates strongly from the accepted log-law. Although a logarithmic region is present, the slope is too low and intercept is overpredicted by more than 100%. The mass flow is also overpredicted by 6.3% compared with the correlations of Dean (1978).

A comparison of the unfiltered case with the pseudo spectral simulation provides some insight regarding the role of truncation errors when the second-order scheme is used for high Reynolds number LES. From Fig. 2 it is clear that the second-order scheme is not able to reproduce even the lowest order statistics when compared with a pseudo spectral simulation at the same resolution. Although this might be

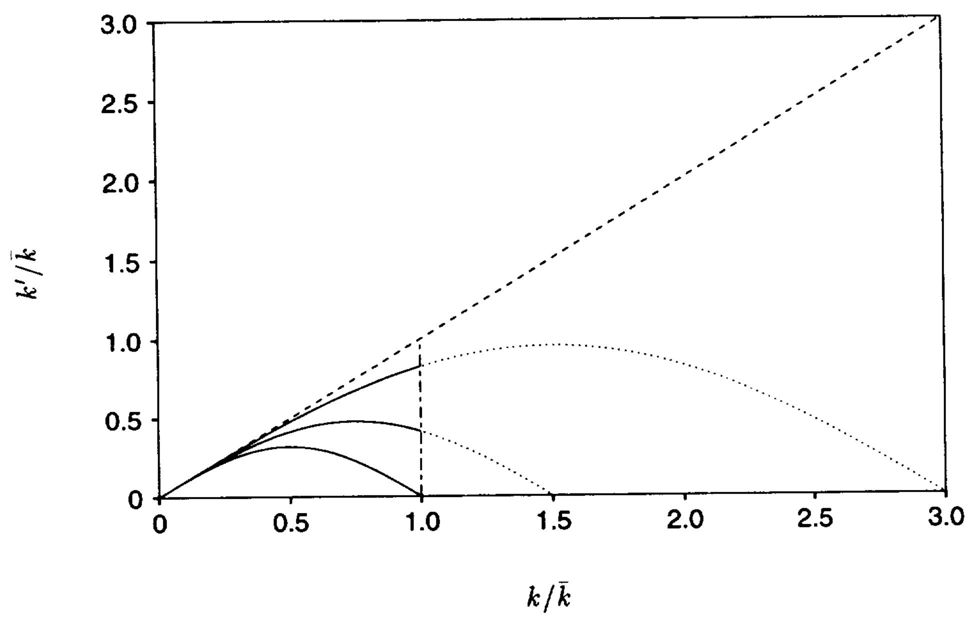

FIGURE 1. Modified wavenumber diagram for the various simulations. From bottom to top the solid lines are for filtered simulations using filter width ratios of 1.0, 1.5, and 3.0, respectively. The vertical chain-dashed line represents the effective resolution of the filtered simulations which was held fixed by expanding the mesh by factor equal to the filter width ratio in each case. From bottom to top, the dotted curves show the modified wavenumber distributions for the unfiltered simulations performed on the meshes expanded by factors of 1.5 and 3.0, respectively. The dashed line is the exact distribution that is achieved with a pseudo spectral method.

expected, it is in contrast to the findings of Choi *et al.* (1992) who obtained a good match with pseudo spectral results for low Reynolds number direct numerical simulations (DNS) of channel flow. As discussed in the introduction, the shift in behavior is suspected to result from a relative increase in numerical error in the LES resulting from the substantial increase in energy in the smallest resolved length scales. The relatively good performance of the second-order scheme in the DNS of Choi *et al.* (1992) was probably aided further by the fact that the DNS was very well resolved. Kim, Moin and Moser (1987) reported no significant change of their spectral DNS results when they coarsened the resolution in the streamwise and spanwise directions by approximately 30%.

Returning to the curves in Fig. 2, it is clear that filtering improves the mean velocity profile. In particular, the log-law intercept decreases toward the usual value and the slope improves. A noticeable wake develops in the outer region of the velocity profile for the case with a filter width ratio of 3. This wake is somewhat larger than the one observed in the pseudo spectral results, and it could be a spurious effect resulting from truncation errors associated with differentiation in

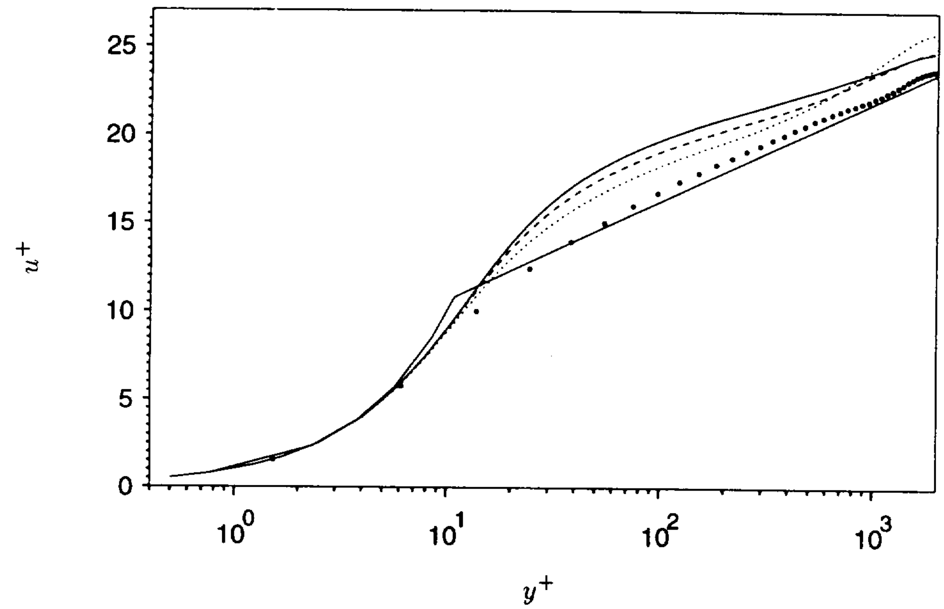

FIGURE 2. Mean velocity profiles from the explicitly filtered simulations. ——— : filter width ratio 1.0; ---- : 1.5; ········ : 3.0; • : pseudo spectral results of Piomelli (1993). The viscous sublayer ($u^+ = y^+$) and log-law ($u^+ = 2.44\ln(y^+) + 5.0$) solutions are also shown for reference.

the wall-normal direction. Explicit filtering is not performed in this direction, and the wall-normal mesh is somewhat coarse in the vicinity of the channel centerline. A simulation with a 25% refinement of the wall-normal mesh spacing was found to give a slightly smaller wake.

Although explicit filtering clearly improves the mean velocity profile, the rate of convergence to the pseudo spectral results appears to be rather slow. Significant errors still exist for a filter width ratio of 3, and a simple extrapolation of these results would seem to indicate that a filter width ratio as large as 6 would be required to recover the standard log-law.

Figure 3 shows the velocity fluctuation profiles plotted in wall coordinates. Starting with the unfiltered case, it is apparent that the second-order scheme is unable to reproduce the pseudo spectral results at high Reynolds number. The streamwise fluctuation is overpredicted, and the other two components are underpredicted. This exaggerated near-wall anisotropy is characteristic of the second-order scheme when the mesh is too coarse. When explicit filtering is used, the results are seen to improve. The streamwise velocity fluctuation is reduced and the anisotropy is improved. Once again, the rate of convergence to the pseudo spectral results is slow, and it appears that a filter width ratio in excess of 3 is required to recover spectral-like accuracy.

As discussed in the introduction, explicit filtering can improve the dynamic model

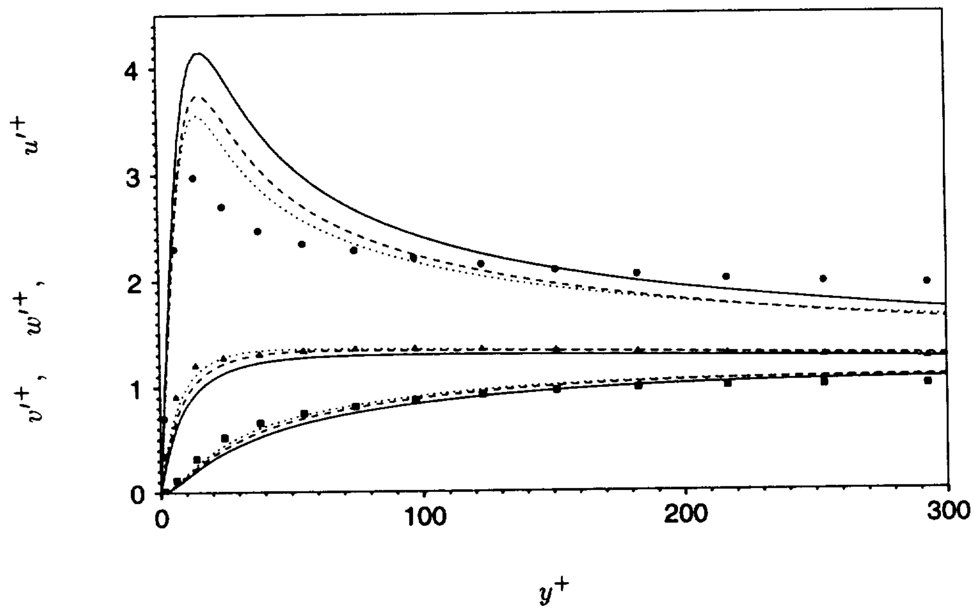

FIGURE 3. Velocity fluctuation profiles from the explicitly filtered simulations. ——— : filter width ratio 1.0; ---- : 1.5; ········ : 3.0; ● , ■ , and ▲ : u'^+, v'^+, and w'^+ from the pseudo spectral calculation of Piomelli (1993).

calculation of the subgrid-scale model constant since the scales that it samples will be better resolved. This effect is demonstrated in Fig. 4 (a) where the subgrid-scale shear stress is plotted in the near-wall region. When no explicit filter is used, the subgrid-scale shear stress is underpredicted by about a factor of 2 when compared with the value from the pseudo spectral simulation. Although it can not be seen from Fig. 4 (a), the stress is too low over the entire channel. Filtering improves this situation by increasing the stress level throughout the channel. When a filter width ratio of 3 is used, the stress is still about 20% low at the maximum but is very close to the pseudo spectral prediction over much of the rest of the channel.

One interesting feature of the subgrid-scale shear stress distributions is the discrepancy in the location of the maximum value between the finite-difference and pseudo spectral calculations. The peak value from the pseudo spectral simulation is at roughly 12 wall units, whereas a maximum does not occur until about 30 wall units in the finite-difference simulation. The position of the maximum in the finite-difference simulation is insensitive to filter width ratio, which seems to indicate that the discrepancy is not a result of truncation error from the streamwise or spanwise directions. The discrepancy could result from wall-normal truncation error in the finite-difference calculation although this would seem unlikely given the very fine mesh in the near-wall region. At the same time, the collocation points near the wall are much more coarsely spaced in the pseudo spectral simulation and this may affect the prediction of the stress maximum.

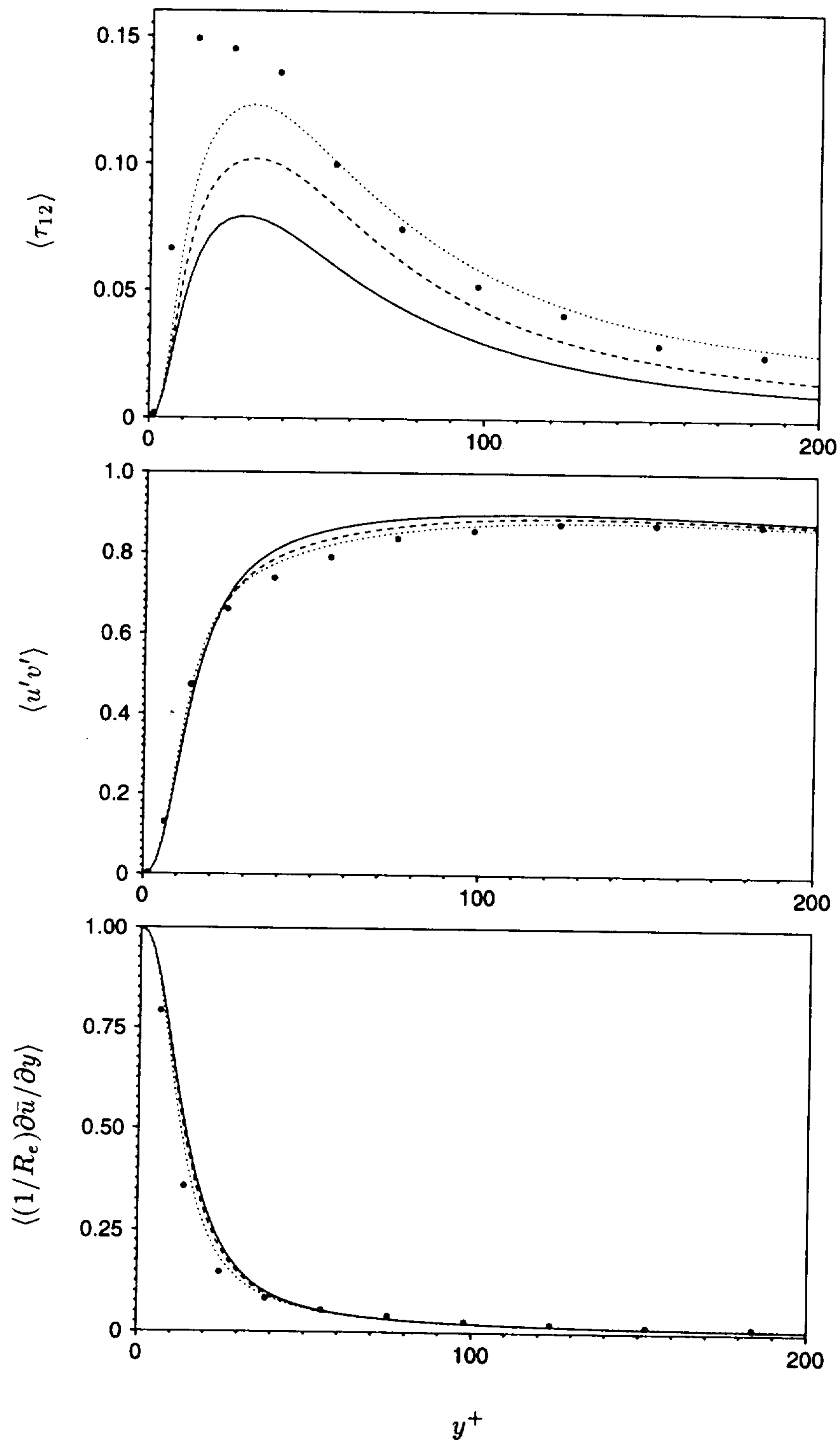

FIGURE 4. Shear stress profiles from the explicitly filtered simulations. (a): subgrid-scale shear stress; (b): resolved shear stress; (c): viscous shear stress. In each case ——— : filter width ratio 1.0; ---- : 1.5; ········ : 3.0; • : pseudo spectral results of Piomelli (1993).

The resolved and viscous shear stress profiles are shown in Figs. 4 (b) and (c). Both these stress components are generally over-predicted when no explicit filter is used. The results improve when the simulation is filtered, and the stresses from the case using a filter width ratio of 3 are in reasonable agreement with the pseudo spectral results.

2.5 Results from mesh refinement without explicit filtering

As discussed in the introduction, it is of interest to compare the effectiveness of explicit filtering against straightforward mesh refinement. The explicitly filtered simulations make use of a fine mesh but discard the high-frequency, error-prone scales. Simulations performed on the same fine mesh but without explicit filtering cost roughly the same but include a broader range of motions. The smallest of these are certainly polluted by numerical error, but they may be far enough removed from the energy-containing scales that the errors do not significantly effect the low-order statistics.

The tradeoff between explicit filtering and straightforward mesh refinement was studied by performing two additional simulations on the same meshes used in the explicit filter study, but without application of the filter. The parameters for these simulations are summarized in Table 1 and the corresponding modified wavenumber diagrams are shown in Fig. 1. Note that the modified wavenumber distributions for the refined simulations are identical to the filtered cases up to the cutoff wavenumber. Thus this portion of the spectrum is subject to the same numerical errors in both the filtered and refined cases. The difference between the two series is that the refined simulations include the motions intermediate between the LES filter and the mesh resolution limit. The additional scales are subject to considerable numerical error, but these errors are concentrated at increasing wavenumber as the level of refinement is increased. In particular, note that when the mesh is refined by a factor of 3, the modified wavenumber does not begin to decrease until 1.5 times the cutoff wavenumber (for the filtered simulations). The error increases appreciably only after this point and it is plausible that the useful resolution of this simulation is roughly 50% higher than in the corresponding filtered case.

Figure 5 shows a comparison of the mean velocity profile from the simulations with mesh refinement. The most noticeable change is a decrease in the mean velocity for the fixed wall shear as the mesh is refined. The quality of the logarithmic region is essentially unchanged, however, and its extent decreases with increasing resolution. If a straight line is fit through the "logarithmic" region, the log law intercept is found to improve as the resolution is increased and is roughly correct for a factor of 3 mesh refinement. The slope of the "logarithmic" region does not improve with mesh refinement, however, and the profile for the factor of 3 refinement displays an unusual oscillation about the expected logarithmic distribution. In comparing the profiles from the filtered and unfilter simulations performed on the same mesh (Figs. 2 and 5), it is clear that the log-law intercept is better predicted by the refined simulations without filtering, whereas the slope and extent of the log region is better predicted when the simulation is filtered. Thus it appears that a rough prediction of the correct profile shape can be achieved more efficiently via mesh refinement,

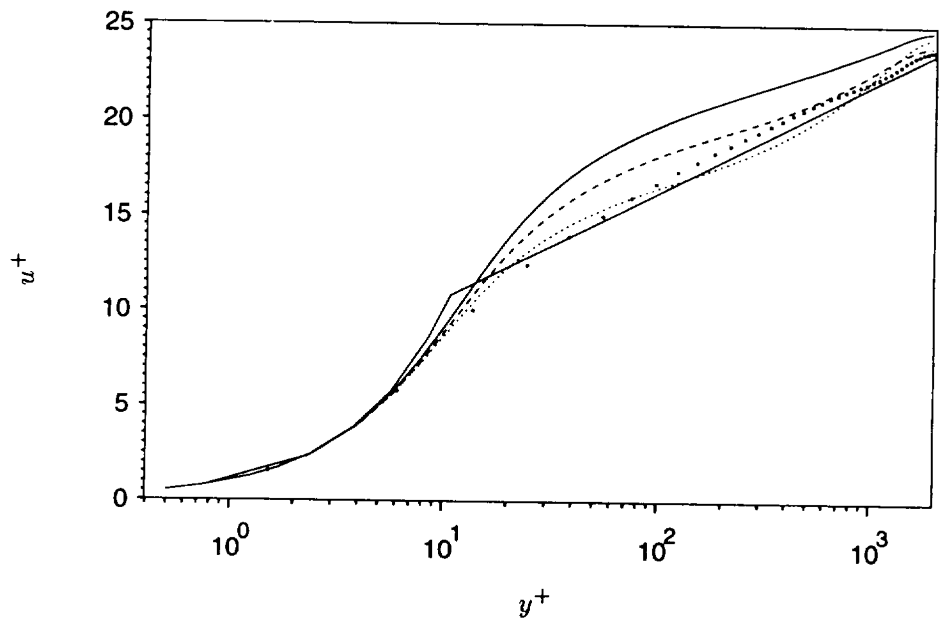

FIGURE 5. Mean velocity profiles from the refined simulations. —— : no refinement; ---- : 1.5 increase in resolution; ······· : 3.0 times increase in resolution; • : pseudo spectral results of Piomelli (1993). The viscous sublayer ($u^+ = y^+$) and log-law ($u^+ = 2.44 \ln(y^+) + 5.0$) solutions are also shown for reference.

whereas the finer details of the velocity distribution may require the removal of at least some of the numerical error. It is also interesting to note that the profiles from the filtered simulations (Fig. 2) have evidently not saturated due to numerical error arising from the wall-normal direction. Figure 5 for the unfiltered simulations shows that it is possible to achieve roughly the correct log-law intercept without improving the wall-normal resolution. Thus it might be expected that the filtered simulation profiles shown in Fig. 2 would continue to improve if the filter width ratio were increased further.

Velocity fluctuation profiles from the mesh refinement series are shown in Fig. 6. The velocity fluctuations are seen to respond strongly to increased resolution with the streamwise component showing the greatest improvement. For a factor of 3 increase in resolution, the streamwise velocity fluctuation agrees very well with the pseudo spectral results in the vicinity of the maximum but appears to be somewhat low as the distance from the wall is increased. Both the wall-normal and spanwise velocity fluctuations increase in the near-wall region as the mesh is refined and appear to exceed the values from the pseudo spectral simulation. Part of this effect is due to increased variance coming from the additional small-scale motions supported by the refined meshes in the finite-difference simulations. In order to make an exact comparison, the finite-difference data in Fig. 6 should really have been filtered back to the resolution of the pseudo spectral simulation as the statistics

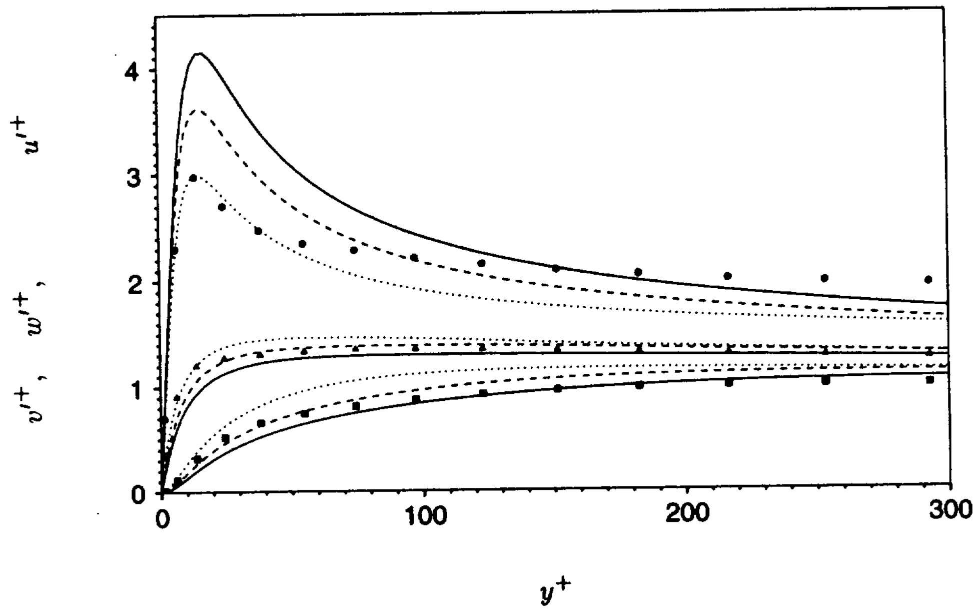

FIGURE 6. Velocity fluctuation profiles from the refined simulations. —— : no refinement; ---- : 1.5 times increase in resolution; ········ : 3.0 times increase in resolution; ● , ■ , and ▲ : u'^+, v'^+, and w'^+ from the pseudo spectral calculation of Piomelli (1993).

were accumulated. Such a filtering of the statistics might also lower the streamwise fluctuation and could affect the apparent agreement with the pseudo spectral results.

In comparing the filtered and unfiltered simulations run on the same mesh (Figs. 3 and 6), it is again apparent that the statistics improve faster when the mesh is simply refined. Unlike the mean velocity profile, however, there do not appear to be any anomalous features associated with the velocity fluctuations when the numerical error is not removed from the simulation.

2.6 Conclusions

The forgoing results have shown that explicit filtering can improve the accuracy of LES performed with a second-order accurate finite-difference scheme. In particular, the quality of the logarithmic region of the mean velocity profile for turbulent channel flow is improved as is the near-wall anisotropy of the velocity fluctuations. The dynamic subgrid-scale model estimation of the shear stress component is also improved. While the statistics clearly benefit from explicit filtering, the rate at which the solution improves is rather slow. Even a filter width ratio of 3 is evidently insufficient to produce results that compare well with a pseudo spectral simulation at the same effective resolution. Based on this result, it appears that a filter width ratio as great as 6 may be required to recover pseudo spectral accuracy if the same effective resolution is used. This is clearly impractical as the cost of performing such a simulation with filtering in all three directions would be 216 times greater than would be dictated by the basic resolution requirements.

Mesh refinement without explicit filtering was found to improve the statistics at a greater rate when compared with the filtered simulations. This result seems to indicate that there is some benefit from including additional smaller scales in the simulations even if they are contaminated by numerical error. This is probably due to the fact that the error is pushed out to higher wavenumber where it has a relatively weak impact on the low-order statistics. Signs of the residual error are evident in the mean velocity profile, however, and it may not be possible to obtain highly accurate statistics without at least some level of numerical error removal.

The basic message from both the explicit filtering and mesh refinement simulations is that, while the results are clearly improved when numerical error is reduced, the cost of doing so via either mechanism is considerable. Although a factor of 3 refinement of the mesh gives acceptable agreement with pseudo spectral simulation results, this represents a factor of 27 increase in cost for a simulation that is refined in all three directions. Even in the present case of two-dimensional refinement, the cost is increased by nearly an order of magnitude. It is possible that a slight gain may be realized by combining some level of mesh refinement and explicit filtering. For example, it is possible that even better results could be obtained using a mesh that is expanded by a factor of three and then filtered using a filter width ratio of 1.5 so that the effective resolution is doubled. It is doubtful that this strategy would lead to a significant reduction in cost, however.

The results of the present study also hint that a higher-order scheme may be a more cost-effective means at achieving acceptable accuracy. For example, the relative truncation error in a fourth-order scheme can be reduced by the same amount as in the second-order simulation using a mesh expanded by a factor of 1.7 as opposed to a factor of 3. By the same token, the use of an explicit filter may be more effective at moderate filter width ratios when applied to a fourth-order scheme.

Until very recently, there did not exist a fourth-order fully-conservative finite-difference scheme for the three-dimensional Navier-Stokes equations that was applicable in generalized coordinates. Such a scheme has been developed by Y. Morinishi during the past several months and the details are reported in this volume. This scheme has not yet been tested for high Reynolds number LES, but tests in coarse DNS show that it is considerably more accurate than the second-order scheme. The fourth-order scheme will be used to repeat some of the present high Reynolds number channel flow simulations in the coming months. Depending on the outcome of these tests, it may be useful to investigate the use of explicit filtering in conjunction with the fourth-order scheme.

3. Future plans

The main focus during the coming year will be to incorporate the results of the present numerical experiments (and the work of Y. Morinishi) into the CTR complex flow LES program. At this point it looks as if the most promising avenue will be to convert our existing second-order codes to Morinishi's fourth-order scheme. As more experience is gained with the fourth-order scheme, it will be determined whether or not explicit filtering is a cost-effective means of improving the simulation results. If so, methods will be perfected for explicit filtering in generalized coordinates. Filtering in such a situation is not straightforward since the filter must approximate a spectral cut-off in order to minimize errors in kinetic energy conservation. Filters based on Pade approximates have been suggested by Lele (1992) for this purpose. These ideas were used by Akselvoll (1995) to explicitly filter a LES simulation in a single coordinate direction. While the filter appeared to be effective, there were some ambiguities associated with the boundary conditions necessary to perform the operation. This issue will be addressed if explicit filtering is decided to be used in conjunction with the fourth-order scheme.

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