S7-73

SQUEEZED STATES AND PARTICLE PRODUCTION IN HIGH ENERGY COLLISIONS

Bindu A. Bambah
School of Physics
Univ. Of Hyderabad Hyderabad-500134,India

Abstract

Using the 'quantum optical approach' we propose a model of multiplicity distributions in high energy collisions based on squeezed coherent states. We show that the k-mode squeezed coherent state is the most general one in describing hadronic mulitiplicity distributions in particle collision processes, describing not only $p\bar{p}$ collisions but e^+e^-, νp and diffractive collisions as well. The reason for this phenomenological fit has been gained by working out a microscopic theory in which the squeezed coherent sources arise naturally if one considers the Lorentz squeezing of hadrons and works in the covariant phase space formalism .

1 INTRODUCTION

Although Quantum Chromodynamics is widely believed to be the theory of Strong Interactions, very few experimental results support this claim. In particular the behaviour of QCD at small momentum transfer i.e low energies is not understood. This lack of understanding reflects itself in the fact that particle production in high energy collisions cannot be explained within QCD. Given the absence of a detailed dynamical theory of strong interactions , one can adopt a statistical outlook and try to forecast macroscopic behaviour of a strongly interacting system given only partial information about their internal states.Experimental information about hadronisation in high energy collisions comes from the observation of jets of hadrons and the distributions of the final state particles. By using analogies with quantum optical systems one can get information about the types of sources(Chaotic, coherent, etc) that are responsible for hadronic emission. Also, by using adapting another quantum optical effect such as the Hanbury-Brown Twiss effect one can study the size and lifetime of the emitting region. This information can then be used to put restrictions on the microscopic theory pursued from the quark-parton end [1].

The experimental quantities amenable to the quantum statistical approach are: the multiplicity Distribution of final state particles (PIONS)given by

$$P_n = \frac{\sigma_n}{\sigma_{inel}} \tag{1}$$

where σ_n n-pion cross-section, the number of particles produced per unit rapidity dN/dy , where $y = ln(\frac{E+p_L}{E-p_L})$ is the rapidity which plays the role of time in pion counting experiments, the moments of P_n and the two pion correlations which are analogous to Hanbury Brown Twiss effect for pions in rapidity space.

In particular, the quantum optical models are based on the assumption that multiparticle production takes place in two stages. In the initial stage formation of an excited system (fireball) which consists of a number of well defined phase space cells or 'sources' which then hadronize independently. In these models an ansatz is made about the statistical nature of these sources and the resulting multiplicity distributions are compared with data [2], [3]. Table 1. gives the comparison of various quantum optical models.

Table 1: Comparison of Quantum Optical Models of Multiplicity Distributions

Nature Of Source	Density matrix	Probability	Two pion
One Source	(Coherent State Rep)	Distribution	Correlations
$P(\alpha) = \frac{1}{\pi n} exp^{(-\|\alpha\|^2/\overline{n})}$	$\rho_{nm} = \frac{\overline{n}^n}{(1+\overline{n})^{n+1}} \delta_{nm}$	Geometrical	$g^2(0) = 2$
$P(\alpha) = \delta^2(\alpha - \alpha')$	$\rho_{nn} = \frac{\|\alpha'\|^{2n} e^{-\|\alpha'\|^2}}{\Gamma(n+1)}$	Poissonian	$g^2(0) = 1$
$P(\alpha) = \frac{e^{\|\alpha-\alpha'\|^2/\overline{n}}}{\pi \overline{n}}$	$\rho_{nn} = \frac{\overline{n}^n}{(1+\overline{n})^{n+1}} e^{-\|\alpha'\|^2/(1+\overline{n})}$ $\times L_n(\frac{-\|\alpha'\|^2}{\overline{n}(1+\overline{n})})$	Glauber-Lachs	$1 \leq g^2(0) \leq 2.$
K sources			
Gaussian (Chaotic)	$\rho_{nn} = \frac{n+k-1)!}{n!(k+1)!} (\frac{\overline{n}/k}{1+\overline{n}/k})^n \frac{1}{(1+\overline{n}/k)^k}$	Negative Binomial	” ”
Coherent +Chaotic	$\rho_{nn} = \frac{(p\overline{n}/k)^n}{(1+p\overline{n}/k)^{n+k}}$ $\times e^{\left[\frac{-\gamma p\overline{n}}{1+p\overline{n}/k}\right]} L_n^{k-1}(\frac{-\gamma k}{(1+p\overline{n}/k)})$	Perina-McGill	” ” ” ”

2 The Phenomenological model

Experimentally there exists a large class of data (νp) and low mass diffractive data that have mulplicity distributions with sub-Poissonian Statistics. Thus we seek a more general distribution than the ones given in table 1. A clue as to the appropriate distribution is that charged pions occur in pairs Furthermore the most general Gaussian source characterised by Gaussian Wigner Function . These facts point to the use of Squeezed Coherent states.
We find that the k-mode squeezed state $|\alpha, r >= |\alpha_1, r_1 > |\alpha_2, r_2 > \cdots |\alpha_k, r_k >$ characterised by the multiplicity distribution:

$$P_n{}^k = \Pi_i^k P_{n_i} \cdots \sum_i n_i = n \tag{2}$$

$$\begin{aligned} P_n{}^k &= e^{\left[-k\alpha^2(1+x)\right]}(1-x^2)^{k/2}(\frac{x}{2})^n \\ &\times \sum_{m=0}^{\left[\frac{n}{2}\right]} \frac{\gamma_m H_{n-2m}{}^2(\sqrt{k}y)2^{2m}}{m!(n-2m)!} \end{aligned}$$

$y = (\frac{\alpha^2(1+x)^2}{2x})^{\frac{1}{2}}$
$\gamma = \frac{k-1}{2}$; $\gamma_m = (\gamma+1)\cdots(\gamma+(m-1))$; $\gamma_0 = 1$

and the second order correlation function:

$$g_k^2(0) = 1 + \frac{2sinh^4(r) + (2\alpha^2+1)sinh^2(r) - sinh(2r)}{k(\alpha^2 + sinh^2(r))^2} \tag{3}$$

is the most general distribution that fits a wide range of data [4]. If $r > 0$ there are regions where $g_k^2(0) < 1$ and the distribution is narrower than Poissonian. If $r < 0$; $g_k^2(0)$ is always greater then 1 showing distributions which are broader than Poissonian.

Hadronic distributions in $p\overline{p}$ collisions show broader than Poisonnian multiplicity distributions with a long multiplicity tail, which gets broader and broader with the increase of energy. The $k = 3$ mode distribution for $\overline{n} = 13.6$, $x = -0.20$ and $\overline{n} = 26.1$, $x = -0.35$ respectively fit corresponding ISR ($62.2Gev$) and UA5 ($540Gev$) data, α for each of these is thus fixed.
To fit neutrino induced collisions in which the distribution is super-Poissonian $((\frac{\Delta n}{\overline{n}}) < 1)$, $k = 3, x = 0.5$ fit data well. e^+e^- collisions are fit by the k=2 squeezed coherent distribution with r close to zero . (nearly Poissonian.)

3 The Statistics confronts the Dynamics

We would now like to conjecture on the reason for this success and find an overlap with dyamical models. We search for incoming states of the hadronic fireball which will give rise to SQUEEZED COHERENT DISTRIBUTION. The candidate dynamical model of hadrons, which we find is appropriate is the covariant phase space model for hadrons which is a revival of Feynman et. al's relativistic harmonic oscillator model[5] Kim and Wigner pointed out that the covariant harmonic oscillator model is the natural language for a covariant description of phase-space [6], [7]. In this paper, we use the covariant phase space distribution description of relativistic extended particles to give a phenomenological description of multiplicity distributions in the high energy collisions of hadrons

Wave functions without time-like oscillations can be constructed by using the unitary representations of the Poincare group and imposing a covariant condition[8]. In this model two quarks bound together by a relativistic harmonic oscillator potential mapped onto O(3,1) invariant harmonic oscillator equation. The ground state wave function ψ_β^0 in the Lorentz boosted (primed) frame is

$$\psi_0^\beta(x') = [e^{-i\eta K_3}]\psi_0(x) \tag{4}$$

where K_3 boost generator along the z axis,

$$K_3 = i(z\frac{\partial}{\partial t} + t\frac{\partial}{\partial z}) \tag{5}$$

and $\eta = Tanh^{-1}(\beta)$

In 'Quantum Optical' language, using light- cone variables we have:

$$u = (t+z)/\sqrt{2} \quad v = (t-z)/\sqrt{2} \tag{6}$$

Then in the Lorentz-transformed frame:

$$\begin{gathered} q'_u = e^\eta q_u \quad q'_v = e^{-\eta} q_v \\ u' = e^{-\eta} u \quad v' = e^\eta v \end{gathered} \tag{7}$$

Introducing creation and annihilation operators

$$a_u = u + \frac{\partial}{\partial u} \ldots. a_v = v + \frac{\partial}{\partial v}$$
$$a_u^\dagger = u - \frac{\partial}{\partial u} \ldots\ldots a_v^\dagger = v - \frac{\partial}{\partial v}$$

we find that the wave function $\psi_n^\beta = \psi(u', v')$ is a two mode squeezed state.

$$\psi_0^\beta(u', v') = |0, \beta >= |0, \eta >_{u'} |0, -\eta >_{v'} \tag{8}$$

The excited state is given by:

$$|n, \beta >= (a_{u'}^\dagger)^n |0, \eta > (a_{v'}^\dagger)^n |0, -\eta > \tag{9}$$

The condition for absence of time-like oscillations in the hadronic rest-frame

$$(a_u' - a_v')|n, \beta >= 0. \tag{10}$$

The physical wave functions are

$$\psi_n^\beta(u', v') = |n, \beta >= \sum_{m=0}^{n} \begin{pmatrix} n \\ m \end{pmatrix} |n - m, \eta >_{u'} |m, -\eta >_{v'} \tag{11}$$

in the Fock-space representation

$$\psi_n^\beta(u', v') = \sum_{n_1, n_2} \sum_{m=0}^{n} \begin{pmatrix} n \\ m \end{pmatrix} G_{n_1, n-m}(\eta) G_{n_2, m}(-\eta) \psi_{n_1}^0(u) \psi_{n2}^0(v) \tag{12}$$

Where [9]:

$$G_{n,m} = (-1)^{\frac{m+n}{2}} (\frac{m!n!}{cosh(\eta)})(\frac{tanh(\eta)}{2})^{\frac{m+n}{2}} \times \sum_{\lambda}^{min[\frac{n}{2} \frac{m}{2}]} \frac{(\frac{-4}{sinh^2(\eta)})^\lambda}{(2\lambda)!(m/2 - \lambda)!(n/2 - \lambda)!} \tag{13}$$

for n,m even and

$$G_{n,m} = (-1)^{\frac{m+n}{2} - 3/2} (\frac{m!n!}{cosh(\eta)})(\frac{tanh(\eta)}{2})^{\frac{m+n}{2} - 1} \times \sum_{\lambda}^{min[\frac{n-1}{2}, \frac{m-1}{2}]} \frac{(\frac{-4}{sinh^2(\eta)})^\lambda}{(2\lambda + 1)!(m - 1/2 - \lambda)!(n - 1/2 - \lambda)!} \tag{14}$$

for n,m odd. $G_{n,m}$ is non zero for both n,m even or both n,m odd , thus excitations of quarks occur in pairs. and the Lorentz squeezed vacuum is a many particle state . The above suggests

the identification of Hadronic sources in terms of squeezed states.
In the 'fireball picture ' the Wigner function of the source is , [10]

$$\begin{aligned} W^n(u,v,q_u,q_v) &= (\frac{2}{\pi})^2 e^{-\frac{1}{2}(e^\eta u^2+e^{-\eta}q_u^2+e^{-\eta}v^2+e^\eta q_v^2)} \\ &\times \sum_{m=0}^{n} \left(\begin{array}{c} n \\ m \end{array} \right) (-1)^n L_{n-m}[e^\eta u^2 - e^{-\eta}q_u^2] L_m[e^{-\eta}v^2 - e^\eta q_v^2] \end{aligned} \tag{15}$$

The number distribution for l particles in the n^{th} excited state.

$$P_l = \sum_{l_1+l_2=l} \sum_{m=0}^{n} \left(\begin{array}{c} n \\ m \end{array} \right) P_{l_1}^{n-m,sq}(\eta) P_{l_2}^{m,sq}(-\eta) \tag{16}$$

$$\begin{aligned} P_{l_2}^{m,sq} &= \frac{(m)!l_2!}{(cosh(-\eta))^{2l_2+1}} (\frac{tanh(-\eta)}{2})^{m-l_2} \\ &\times F(-\eta,l_2,m)\cos^2(\frac{(m-l_2)\pi}{2}) \end{aligned} \tag{17}$$

where:

$$F(-\eta,l_2,m) = \sum_{\lambda=0}^{min(l_2/2,(m-l_2)/2} \frac{(\frac{-4}{sinh^2(-\eta)})^\lambda}{(\lambda)!(l_2-2\lambda)!(m-l_2-2\lambda)!} \tag{18}$$

The cosine terms imply P_l vanishes when $|m-l_2|$ or $|n-m-l_1|$ is odd. so that the excited each of oscillator modes is excited in pairs.
If each pair is associated with a two quark bound state(pion), the excited state contains pair correlated pions!!!

4 Results and Conclusion

The picture emerging is as follows the distribution of the fireball results from the excitation of oscillator modes of the colliding hadrons. This excitation takes place in pairs. Modes de-excite statistically emitting 2 pairs of quarks which we identify as two pions The phase space distribution of the fireball:

$$| < n,\beta|n,-\beta' > |^2 = (2\pi) \int dudv W_\beta^n(u,v,q_u,q_v) W_{\beta'}^n(u,v,q_u,q_v) \tag{19}$$

Probability of emission of m particles from two independent populations 1 and 2 corresponding to each of the incident hadrons. forming an overlapping distributions is given as:

$$P_m = \sum_{m'=0}^{m} P_{m-m'}^1 P_{m'}^2 \tag{20}$$

Total probability distribution thus becomes a product of the probability distribution of four squeezed sources:

$$P_m = \sum_{m_1+-m_2+m_3+m_4=m} P_{m_1}^{sq}(\eta) P_{m_2}^{sq}(-\eta) P_{m_3}(\eta') P_{m_4}(-\eta')$$

For target Projectile collisions $\beta' = 0$ thus the probability of emitting n' particles is:

$$
\begin{aligned}
P_{n'} &= \sum_{p=0}^{n'} \sum_{m=0}^{n} \binom{n'}{p} \binom{n}{m} \\
&\times (-1)^{\frac{n'+n}{2}} (p!(n-m)!m!(n'-p)!)^{1/2} \frac{1}{cosh(\eta)} (\frac{tanh(\eta)}{2})^{\frac{n'+n}{2}} (-1)^{\frac{m+n'-p}{2}} \\
&\times \sum_{\mu\lambda} \frac{(\frac{-4}{sinh^2(\eta)})^{\lambda+\mu}}{(2\mu)!(p/2-\mu)!(\frac{n'-p}{2}-\mu)!(m/2-\lambda)!(\frac{n-m}{2}-\lambda)!}
\end{aligned}
$$

As β increases the distribution gets broader .

For Central Collisions $\beta = \beta'$ and by plotting mP_m vs. $\frac{m}{<m>}$ for different values of β we see that the distributions become wider and skew symmetric as the value of β becomes larger. This is consistent with the variation seen in experimental data.

The total probability distribution for the two nucleon system for n pions is:

$$P_n^k = \sum_{\sum n_i = n} \prod_i^{k/2} P_{n_i}^{sq}(\eta) \sum_{\sum n_i = n} \prod_i^{k/2} P_{n_i}^{sq}(-\eta) \quad (21)$$

where k=6 for nucleon-nucleon collisions , k=4 for $\pi\pi$ collisions and k=3 for νp collisions (with η positive).

We include final state interactions in a simple fashion by assuming that the effect of interaction is to add coherence into the final state. This is consistent with the fact that in particle collisions experimental data shows some amount of coherence, especially in the low energy region , among the emitted particles. With the resulting density matrix we obtain the mutiplicity distribution for a variety of collisions and compare to data. The distribution we get is:

$$P_n^k = \sum_{\sum n_i = n} \prod_i^{k/2} P_{n_i}^{sq,coherent}(\alpha, \eta) \sum_{\sum n_i = n} \prod_i^{k/2} P_{n_i}^{sq,coherent}(\alpha, -\eta) \quad (22)$$

Where the average number of particles emitted by each mode is given by:$\overline{n}_i = \alpha^2 + Sinh^2(\eta)$ Above distribution fits the CERN ISR 62.2 GeV and UA5 540 GeV data. The k=3 distribution is compared with νp data. The data is well reproduced by the distribution. For e^+e^- collisions we take k=2 because the intermedeate state is the virtual $\overline{q}q$ state formed by the colliding electron and positron.

In terms of hadronic final states the LEP energy ($\sqrt{s}$ = 100 GeV) is equivalent to the SPS energy ($\sqrt{s}$ = 546 GeV) as far as total mutiplicities are concerned, in so far as $\overline{n}^{e^+e^-}$(LEP) $\approx \overline{n}^{\overline{p}p}$(SPS) $\approx$26 .

For the same value of $\overline{n}$ much narrower distribution for e^+e^- distributions than the $\overline{p}p$ distributions. This is consistent with recent LEP data [11].

We can make some predictions for higher energies such as those observed at the LHC and SSC. Since widening of the distributions is related to the squeezing parameter η the lorentz boost of the hadronic fireball, at C.M.S. energies of 20 TeV and above we have a large β value and higher modes will be excited. The multiplicity distribution for ultra-high energies is very broad and skew-symmetric. plot $\overline{n}P_n vs.\frac{n}{\overline{n}}$ for $\overline{p}p$ collisions for $\overline{n} = 50$

We can also calculate the Bose-Einstein Correlations of pions in this model by using the two mode state. Ongoing work is in progress to establish the connection of this model with QCD using the light cone formalism [12]. In this formalism it is also easy to incorporate temperature dependence by using Thermal Squeezed Coherent states. These would be of interest in heavy ion collisions.

5 Acknowledgements

I would like to thank Prof. Y.S. Kim and all the other organizers of ICSSUR '95 for their kind hospitality and support. I also acknowledge the support of the University Grants Commision,National Board of Higher Mathematics, INSA , DST and CSIR for their support.

References

[1] P.Carruthers and C.C.Shih, International Journal of Mod. Phys.**A2**(1987)1447.

[2] G.N. Fowler and R.M. Weiner in **'Hadronic Multipaticle Production'** Ed. P.Carruthers, (1988) 481.

[3] T.C. Meng in **Hadronic Multiparticle Production** Ed. P. Carruthers (1988), and references therein.

[4] B.Bambah and M.V.Satyanarayana Phys. Rev. D **38**(1988) 2202.

[5] R.P.Feynman,M.Kislinger and F.Ravndal Phys. Rev. D **3**(1971)2706.

[6] Y.S.Kim and E.P.Wigner, Phys. Rev. A **39**(1989)2829.

[7] Y.S.Kim and E.P.Wigner, Phys. Rev. D **49**(1990).

[8] Y.S. Kim and M.E.Noz.**'Theory and Applications of the Poincare Group'** (Reidel and Dortrecht,1986).

[9] M.V.Satyanarayana Phys. Rev. D **32** (1985) 400.

[10] S. Chaturvedi and V. Srinivasan Phys. Rev. A **40** (1989)6095.
; M.S.Kim, F.A.M. de Oliviera and P.L.Knight"**The Squeezing of Fock and Thermal States"** Imperial College Preprint,1991

[11] D. Decamp et. al. ALEPH COLLAB. , Phys. Lett. B**273** (1991)180.

[12] S.J.Brodsky in **Nuclear Chromodynamics** S.J. Brodsky and E.J. Moniz .eds. (World Scientific, 1986) and F.Lenz. ibid.