THE SQUEEZING OPERATOR AND THE SQUEEZING STATES OF "SUPERSPACE"

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Abstract

In this paper, the unitary squeezing operator of "superspace" is introduced and by making this operator act on the supercoherent state, the squeezing supercoherent states are obtained, then come out the four orthonormalization eigenstates of the square of annihilation operator A of the supersymmetry harmonic oscillator, and their squeezing character is also studied.

1 Introduction

Early in the 1970's, D. Stoler[1] put forward the concept of the squeezing state first. Following him, H. P. Yuen[2] made a detailed study of the quantum characteristic of the squeezing state which was obtained from the squeezing operator acting on the coherent state. This kind of squeezing state they studied is the squeezing coherent state. Having less noise than the coherent state, the squeezing state would be a vast applied vistas in the optical communication and the gravitational force wave probing, etc. The squeezing state has become an attentive problem.

In recent years, a lot of studies about the supersymmetry have been done, P. Salomonson[3] and other persons put forward the supersymmetry harmonic oscillator, and C. Aragone[4], along with other, introduced the supercoherent state. People found that the inner link of different atoms and ions are related to the abstract supersymmetry[5]. Chen Cheng—ming and Xu Donghui[6] acted the displacement operator on one supersymmetry Hamiltonian, and also drew the supercoherent state, moreover, made the discussion on the squeezing state extend into the supercoherent state. The eigenstate of the annihilation operator A of the supersymmetry harmonic oscillator which they introduced — —the supercoherent state can not be introduced by using the displacement operator to affect the supersymmetry harmonic oscillator Hamiltonian. Acting the squeezing operator on the Hamiltonian of the displacement harmonic oscillator, the eigenstate of the new constructed Hamiltonian is the squeezing state[7]. According to this theory, to discuss the problem about supersymmetry requires not only constructing proper annihilation operator of the Hamiltaonian of the supersymmetry harmonic oscillator, but also introducing the displacement operator and squeezing operator of "superspace".

This paper introduces the squeezing operator of "superspace", and acts it on the supercoherent state, so as to get the squeezing supercoherent state. This method is equivalent to acting the squeezing operator of "superspace" on the displacement supersymmetry harmonic oscillator, and then, to get the eigenstate of the new constructed Hamiltonian. In this paper, the annihilation operator A of the supersymmetry harmonic oscillaor has such characters: $[A, H] = \omega A$, $[A, A^+] = 1$ and $H = \omega A^+ A$, As a result, the obtained squeezing supercoherent state is different from the squeezing state in literature[6]. In this paper, the squeezing character of the eigenstate of A is also discussed.

2 Supercoherent State

The Hamiltonian of the supersymmetry harmonic oscillator is[3]

$$H = \frac{1}{2}P^2 + \frac{1}{2}\omega^2 X - \frac{1}{2}\omega\sigma_3 = \omega\begin{pmatrix} a^+ a & O \\ 0 & aa^+ \end{pmatrix} \tag{1}$$

Where, x and p are the coordinate operator and momentum operator in the general space, σ_3 is the third component in Parli matrix; a and a^+ are the annihilation and creation operators of the ordinary harmonic oscillator.

The Harmiltonian of the supersymmetry harmonic oscillator is also written as:

$$H = \omega A^+ A \tag{2a}$$

$$= \frac{1}{2}P^2 + \frac{1}{2}\omega^2 Q^2 - \frac{1}{2}\omega \tag{2b}$$

in it,

$$A = \begin{pmatrix} 0 & \sqrt{aa^+} \\ \frac{1}{\sqrt{aa^+}}a^2 & 0 \end{pmatrix} \tag{3}$$

$$Q = \sqrt{\frac{1}{2\omega}}(A^+ + A), P = i\sqrt{\frac{\omega}{2}}(A^+ - A) \tag{4a}$$

$$A = \sqrt{\frac{\omega}{2}}Q + i\sqrt{\frac{1}{2\omega}}P, A^+ = \sqrt{\frac{\omega}{2}}Q - i\sqrt{\frac{\omega}{2}}P \tag{4b}$$

test and verify easily,

$$[A, H] = \omega A \tag{5}$$

$$[A, A^+] = I, [Q, P] \doteq i \tag{6}$$

Because (5) is tenable, A is called the annihilation operator of the supersymmetry harmonic oscillator; A^+ the creation operator of the supersymmetry harmonic oscillator. (2b) is equal to the relevant expression form of the ordinary harmonic oscillator, and Q and P are called the generalized coordinate operator and the generalized momentum operator of "superspace" separately.

We can get the energy eigenvalue of H and the relevant eigenstate from literature[4], they are

$$E_0 = 0,\ \varphi_0 = \begin{pmatrix} |0\rangle \\ 0 \end{pmatrix} \tag{7a}$$

$$E_{n>0} = n\omega, \varphi_{n>0} = C_n^+\varphi_n^+ + C_n^-\varphi_n^- \tag{7b}$$

$$|C_n^+|^2 + |C_n^-|^2 = 1 \tag{7c}$$

where

$$\varphi_{n>0}^+ = \begin{pmatrix} |n\rangle \\ 0 \end{pmatrix}, \varphi_{n>0}^- = \begin{pmatrix} 0 \\ |n-1\rangle \end{pmatrix} \tag{7d}$$

The eigenvalue of σ_3 is $+1$ and -1.

It is easy to prove that

$$A\varphi_{n>1} = \sqrt{n}\,\varphi_{n-1}, A^+\varphi_n = \sqrt{n+1}\,\varphi_{n+1}, A^+A\varphi_n = n\varphi_n \tag{8}$$

the eigenquation of A is

$$A|\Psi(\alpha)\rangle = \alpha|\Psi(\alpha)\rangle \tag{9}$$

Where, α is the complex parameter, $\alpha = |\alpha|e^{i\theta}$.

The definition of the displacement operator of "superspace" is

$$D(\alpha) = \exp(\alpha A^+ - \alpha^* A) \tag{10}$$

It has the similar character of the ordinary displacement operator $D(\alpha)$:

$$D^+(\alpha) = D(-\alpha) = [D(\alpha)]^{-1} \tag{11a}$$

$$D^+(\alpha)AD(\alpha) = A + \alpha \tag{11b}$$

$$D^+(\alpha)A^+D(\alpha) = A^+ + \alpha^* \tag{11c}$$

The eigenstate of A (double degenerate) is obtained by solving the eigenequation of (9), or by using $D(\alpha)$

$$|\Psi_1(\alpha)\rangle = \exp(-\frac{|\alpha|^2}{2}(ch|\alpha|^2)^{\frac{1}{2}}\begin{pmatrix} |\alpha\rangle_e \\ \frac{\alpha}{\sqrt{aa^+}}|\alpha\rangle_e \end{pmatrix} \tag{12a}$$

$$|\Psi_2(\alpha) = \exp(-\frac{|\alpha|^2}{2})(sh|\alpha|^2)^{\frac{1}{2}}\begin{pmatrix} |\alpha\rangle_0 \\ \frac{\alpha}{\sqrt{aa^+}}|\alpha\rangle_0 \end{pmatrix} \tag{12b}$$

Where, $|\alpha\rangle_0$ and $|\alpha\rangle_e$ are odd coherent state and even coherent state respectively[8,9]. The two mathematical expression formulas produced by the translation of the orthonormalization eigenstate are

$$|\Psi_1(\alpha)\rangle = D(\alpha)\begin{pmatrix} |0\rangle \\ 0 \end{pmatrix} \tag{13a}$$

$$|\Psi_2(\alpha)\rangle = D(\alpha)\begin{pmatrix} |1\rangle \\ 0 \end{pmatrix} = D(\alpha)\begin{bmatrix} 0 \\ \frac{1}{\sqrt{aa^+}}a|0\rangle \end{bmatrix} \tag{13b}$$

For the eigenstate of A, it is easy to prove that

$$\langle \Delta Q\rangle\langle \Delta P\rangle = \frac{1}{2} \tag{14}$$

Namely, the eigenstate of A is the minimum uncertainty state of Q and P, they are the conjugate Hermitian operators. In this sense, the eigenstate of A is called the supercoherent state.

3 Squeeaing Supercoherent State

First, let us introduce the unitary evolutional operator of "superspace" generally:

$$S_k(Z) = \exp[Z_k(A^+)^k - Z_k^* A^k],\ Z_k = Z/K! \tag{15}$$

When $k = 1$, it is the displacement operator D (α) $(\alpha = z)$; When $k = 2$, $S_{K(Z)}$ is called the squeezing operator of complex parameter, written as $S(z)$.

$$S(z) = \exp\left[\frac{1}{2}z(A^+)^2 - \frac{1}{2}z^* A^2\right],\quad z = re^{i\theta} \tag{16}$$

Where r is the squeezing factor, θ the squeezing angle. Since the character of A is the same as a, and also

$$S^*(Z)AS(Z) = Achr + A^+ shre^{i\theta} \tag{17}$$

And

$$S(Z) = R(-\frac{\theta}{2})S(r)R(\frac{\theta}{2}) \tag{18a}$$

Where, $R(\theta)$ is the revolving operator of the phasespace, $S(r)$ the squeezing operator of the real parameter,

$$R(\theta) = \exp(-i\theta A^+ A) \tag{18b}$$

$$S(r) = \exp\left[\frac{r}{2}(A^{+2} - A^2)\right] \tag{18c}$$

To redefine the quadrature phase amplitude operator of "superspace"

$$X(\varphi) = \frac{1}{2}(Ae^{i\varphi} + A^+ e^{-i\varphi}) \tag{19}$$

Since the eigenstate of H and A all have the double degeneracy, the squeezing states are double. One of the squeezing state of "superspace" can be defined

$$|\alpha,z\rangle_1 = D(\alpha)S(z)\begin{pmatrix} |0\rangle \\ 0 \end{pmatrix} \tag{20}$$

Because of $D^+(\alpha)AD(\alpha) = A + \alpha$, and making use of (17) and (18), the expection value of $X(\varphi)$ in $|\alpha, Z\rangle_1$, can be calculated, that is

$$\langle X(\varphi)\rangle_1 = \frac{1}{2}(\alpha e^{I\varphi} + \alpha^* e^{-I\varphi}) \tag{21}$$

but the expection value of $X^2(\varphi)$ in $|\alpha, Z\rangle_1$ is

$$\langle X^2(\varphi)\rangle_1 = \frac{1}{4}\left[(\alpha e^{i\varphi} + \alpha^* e^{i\varphi})^2 + |chr + e^{i(2\varphi+\theta)}shr|^2\right] \tag{22}$$

thus,

$$\langle \Delta X^2(\varphi)\rangle_1 = \langle X^2(\varphi)\rangle_1 - \langle X(\varphi)\rangle_1^2 = \frac{1}{4}|chr + e^{i(2\varphi+\theta)}shr|^2 \tag{23}$$

When $r = 0$, the formula above is the fluctuation of the supercoherent state. , $\langle \Delta X^2\rangle_1$ is irrelevant to φ. When $r \neq 0$ (supposing $r > 0$), if φ satisfies the inequality.

$$\cos(2\varphi + \theta) < -thr \tag{24}$$

then

$$\langle \Delta X^2(\varphi)\rangle_1 < \frac{1}{4} \tag{25}$$

Namely, (24) is the condition that the squeezing of $X(\varphi)$ exists in $|\alpha,z\rangle_1$. $X(\varphi+\pi/2)$ is the phase amplitude operator which is quadrature with $X(\varphi)$. Its squeezing condition is

$$\cos(2\varphi+\theta) > thr \tag{26}$$

Obviously (24) and (26) can be tenable at the same time. That is, $|\alpha,z\rangle_1$ can not exist the squeezing of $X(\varphi)$ and $X(\varphi+\pi/2)$.

Especially, if

$$thr \geqslant \cos(2\varphi+\theta) \geqslant - thr \tag{27}$$

neither of the quadrature phase components has the squeezing.

From (23) we get

$$\langle\Delta X^2(\varphi)\rangle_1\langle\Delta X^2(\varphi+\frac{\pi}{2})\rangle_1 = \frac{1}{16}[1 + sh^2 2r sin^2(2\varphi+\theta)] \tag{28}$$

When $2\varphi+\theta=0$ or π, the formula above takes the minimum value,

$$\langle\Delta X^2(\varphi)\rangle_1\langle\Delta X^2(\varphi+\frac{\pi}{2})\rangle_1 = \frac{1}{16} \tag{29}$$

the relation of the minimum uncertainty is tenable.

When $2\varphi+\theta=0$, (29) and (26) are satisfied at the same time; when $2\varphi+\theta=\pi$, (29) and (24) too. Similar to the definition of the squeezing coherent state, the squeezing supercoherent state $|\alpha,z\rangle_1$ is named.

Using (17)

$$S^+(z)\,D(\alpha)S(z) = D(\beta) \tag{30a}$$

is solved, that is

$$D(\alpha)S(z) = S(z)\,D(\beta) \tag{30b}$$

Where,

$$\beta = \alpha chr - \alpha^* shre^{i\theta} \tag{30c}$$

Now make

$$|z,\beta\rangle_1 = S(z)D(\beta)\begin{pmatrix}|0\rangle\\0\end{pmatrix} = S(z)|\Psi_1(\beta)\rangle \tag{31}$$

from (30b), here is

$$|z,\beta\rangle_1 = |\alpha,z\rangle_1 \tag{32}$$

Next another squeezing state of "superspace" will be discussed. Let

$$|z,\beta_2\rangle = S(z)|\Psi_2(\beta)\rangle \tag{33}$$

Using (17), here is

$$\begin{aligned}\langle X(\varphi)\rangle_2 &= \frac{1}{2}\langle\Psi_2(\beta)|[chre^{i\varphi} + shre^{-i(\varphi+\theta)}]A + [chre^{-i\varphi} + shre^{i(\varphi+\theta)}]A^+|\Psi_2(\beta)\rangle \\ &= chrRe(\beta e^{i\varphi}) + shrRe[\beta e^{-i(\varphi+\theta)}]\end{aligned} \tag{34}$$

$$\begin{aligned}\langle X^2(\varphi)\rangle_2 &= \frac{1}{4}\langle A^2e^{2i\varphi}+2A^+A+A^{+2}e^{-2i\varphi}\rangle_2+\frac{1}{4} \\ &=[chrRe(\beta e^{i\varphi})+shrRe(\beta e^{-i(\varphi+\theta)})]^2+\frac{1}{4}[ch^2r+sh^2r+sh2rcos(2\varphi+\theta)]\end{aligned} \tag{35}$$

So,

$$\begin{aligned}\langle\Delta X^2(\varphi)\rangle_2 &= \langle X^2(\varphi)\rangle_2 - \langle X(\varphi)\rangle_2^2 \\ &= \frac{1}{4}[ch^2r + sh^2r + sh2r\cos(2\varphi+\theta)] \\ &= \frac{1}{4}|chr + shre^{i(2\varphi+\theta)}|^2\end{aligned} \tag{36}$$

It is clear that $|z,\beta\rangle_2$ and $|z,\beta\rangle_1 = |\alpha,z\rangle_1$ have the same squeezing character, and both are the squeezing supercoherent states.

The eigenstate and of A can be generally written as

$$|\Psi(\alpha)\rangle = C_1|\Psi_1(\alpha)\rangle + C_2|\Psi_2(\alpha)\rangle \tag{37a}$$

$$|C_1|^2 + |C_2|^2 = 1 \tag{37b}$$

To make

$$|z,\beta\rangle = S(z)|\Psi(\beta)\rangle \tag{38}$$

Similarly, $|z,\beta\rangle$ is the squeezing supercoherent state. It includes $|z,\beta\rangle_1$ and $|z,\beta\rangle_1$.

Since

$$S(z)AS^{+}(z)S(z)|\Psi(\beta)\rangle = \beta S(z)|\Psi(\beta)\rangle|$$

that is, $|z,\beta\rangle$ is the eigenstate of the unitary transformation operator $S^{+}(z)AS(z)$ of A. The unitary transformation does not change the eigenvalue of operator. It is still β.

$$S(z)AS^{+}(z)|z,\beta\rangle = \beta|z,\beta\rangle \tag{39}$$

The eigenstate of equation (39) is double degenerate, with the same characher.

4 The Squeezing Characher of The Eigenstate of A^2

As an example, the squeezing character of the eigenstate of A^2 will be discussed. The orthonormalization eigenstates (quartet degenerate state) of A^2 can be obtained easily. They are

$$|\Phi_1(\alpha)\rangle = \begin{pmatrix} |\alpha\rangle_e \\ 0 \end{pmatrix} = |\alpha\rangle_{ee} \tag{40a}$$

$$\Phi_2(\alpha)\rangle = (cth|\alpha|^2)^{\frac{1}{2}} \begin{bmatrix} 0 \\ \frac{a}{\sqrt{aa^{+}}}|\alpha\rangle_e \end{bmatrix} = |\alpha\rangle_{0e} \tag{40b}$$

$$|\Phi_3(\alpha)\rangle = \begin{pmatrix} |\alpha\rangle_0 \\ 0 \end{pmatrix} = |\alpha\rangle_{e0} \tag{40c}$$

$$\Phi_4(\alpha)\rangle = (th|\alpha|^2)^{\frac{1}{2}} \begin{bmatrix} 0 \\ \frac{a}{\sqrt{aa^{+}}}|\alpha\rangle_0 \end{bmatrix} = |\alpha\rangle_{00} \tag{40d}$$

$$\langle\Phi_i(\alpha)|\Phi_j(\alpha)\rangle = \delta_{ij} \tag{41}$$

$$A^2|\Phi_i(\alpha)\rangle = \alpha^2|\Phi_i(\alpha)\rangle, \quad (i = 1,2,3,4) \tag{42}$$

The eigenstate of A^2 has the character that can be converted by A acting on.

$$\begin{aligned} A|\Phi_1(\alpha)\rangle &= \alpha(th|\alpha|^2)^{\frac{1}{2}}|\Phi_2(\alpha)\rangle \\ A|\Phi_2(\alpha)\rangle &= \alpha(cth|\alpha|^2)^{\frac{1}{2}}|\Phi_1(\alpha)\rangle \\ A|\Phi_3(\alpha)\rangle &= \alpha(cth|\alpha|^2)^{\frac{1}{2}}|\Phi_4(\alpha)\rangle \\ A|\Phi_4(\alpha)\rangle &= \alpha(th|\alpha|^2)^{\frac{1}{2}}|\Phi_3(\alpha)\rangle \end{aligned} \tag{43}$$

According to (41), (42) and (43), the following can be got easily,

$$\langle X(\varphi)\rangle_{ee} = \langle X(\varphi)\rangle_{0e} = \langle X(\varphi)\rangle_{e0} = \langle X(\varphi)\rangle_{00} = 0 \tag{44}$$

thereby

$$\begin{aligned} \langle\Delta X^2(\varphi)\rangle_{ee} &= \langle X^2(\varphi)\rangle_{ee} \\ &= \frac{1}{2}|\alpha|^2[\cos2(\varphi+\xi) + th|\alpha|^2] + \frac{1}{4} \end{aligned} \tag{45a}$$

$$\begin{aligned} \langle\Delta X^2(\varphi)\rangle_{0e} &= \langle X^2(\varphi)\rangle_{0e} \\ &= \frac{1}{2}|\alpha|^2[\cos2(\varphi+\xi) + cth|\alpha|^2] + \frac{1}{4} \end{aligned} \tag{45b}$$

$$\begin{aligned} \langle\Delta X^2(\varphi)\rangle_{e0} &= \langle X^2(\varphi)\rangle_{e0} \\ &= \frac{1}{2}|\alpha|^2[\cos2(\varphi+\xi) + cth|\alpha|^2] + \frac{1}{4} \end{aligned} \tag{45c}$$

$$\begin{aligned} \langle\Delta X^2(\varphi)\rangle_{00} &= \langle X^2(\varphi)\rangle_{00} \\ &= \frac{1}{2}|\alpha|^2[\cos2(\varphi+\xi) + th|\alpha|^2] + \frac{1}{4} \end{aligned} \tag{45d}$$

Because the minimum value of $cth|\alpha|^2$ is 1, the squeeze can not exist in $|\Phi_2(\alpha)\rangle$ and $|\Phi_3(\alpha)\rangle$. But the maximum of the $|\alpha|^2$ is 1 and not negative, so if the value of φ can be chosen properly, it can make

$$\cos2(\varphi+\xi) < -th|\alpha|^2 \tag{46}$$

thus,

$$\langle\Delta X^2(\varphi)\rangle_{ee} = \langle\Delta X^2(\varphi)\rangle_{00} < \frac{1}{4} \tag{47}$$

That is to say, $|\Phi_1(\alpha)\rangle$ and $|\Phi_4(\alpha)\rangle$ both have the squeeze. But

$$\langle\Delta X^2(\varphi)\rangle_{ee}\langle\Delta X^2(\varphi+\frac{\pi}{2})\rangle_{ee} = \langle\Delta X^2(\varphi)\rangle_{00}\langle\Delta X^2(\varphi+\frac{\pi}{2})\rangle_{00} > \frac{1}{16}, \quad (|\alpha|^2 \neq 0) \tag{48}$$

so $|\Phi_1(\alpha)\rangle$ and $|\Phi_4(\alpha)\rangle$ are the generalized squeezing states.

5 Conclusion

As far as $[A, H]=\omega A$, the common expression of the annihilation operator of the supersymmetry harmonic oscillator is

$$A=\begin{pmatrix}\delta a & r\\ \lambda a^2 & \rho a\end{pmatrix} \tag{49}$$

in it, δ, r, λ and ρ can be either figure C, or the operator function of a^+a. If A can still satisfy the commutation relation $[A, A^+]=I$, then the eigenstate of A can be produced by the displacement operator of "superspace" acting on the two minimum energy state of H.

The annihilation operator of the supersymmetry harmonic oscillator, being discussed in this paper, has the special significance. Besides it satisfies the commutation relation $[A, H]=\omega A$ and $[A, A^+]=I$, there is $H=\omega A^+A$ also, which is like the general annihilation operator a. So the study in this paper is very resemble in forms to the similar discussion about the ordianry space. But on the other hand, it can make our study in this paper have many particularities because of the double degeneracies of H, A and $S(z)AS^+(z)$.

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