Q(α) Function and Squeezing Effect

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Abstract

The relation of squeezing and Q(α) function is discussed in this paper. By means of Q function, the squeezing of field with gaussian Q(α) function or negative P(α) function is also discussed in detail.

1 Introduction

In quantum optics, P(α), Q(α) and W(α) are common quasiprobability distribution functions [1], but only Q(α) preserves good function (positive and nonregular). Recently, by means of Fokker–Plank equation for Q function, M. S. Kim et al. discussed the fourth-order squeezing [2]. In this paper, we consider the relation between Q function and squeezing, and study the squeezing of field with gaussian Q function or negative P(α) function.

For any field density operator ρ, the Q function is defined as

\[ Q(α) = \frac{1}{\pi} \langle α | ρ | α \rangle \]  

(1)

it satisfies the normalization condition

\[ \int dα^2 Q(α) = 1 \]  

(2)

For antinormally ordered operator \( f(α, α^+) \), one can get the following equation

\[ \langle f(α, α^+) \rangle = \int d^2α Q(α) f^{(α)}(α, α^*) = 1 \]  

(3)

where \( α \) and \( α^+ \) are annihilation and creation operators respectively. Defining parameter

\[ S = \langle (α + α^+)^2 \rangle - \langle α + α^+ \rangle^2 \]  

(4)

For squeezing, \( S \) should be negative.

Now, we suppose that Q function can be expanded as following form

\[ Q(α) = \frac{1}{\pi} e^{-β|α|^2} \sum_{m,n} C_{m,n} α^m α^{*n}, (C_{m,n} = C_{n,m}^{*}) \]  

(5)

Using mathematical identity [3]
\[
\int \frac{d^2 \alpha}{\pi} e^{-\beta |\alpha|^2 + \rho \alpha^* + \rho^* \alpha} = \frac{1}{\beta} e^{\rho \beta}, (\beta > 0)
\]

one can have

\[
\int \frac{d^2 \alpha}{\pi} \alpha^m \alpha^* n e^{-\beta |\alpha|^2} = \frac{n! \delta_{mn}}{\beta^{m+1}}
\]

and the normalization condition is

\[
\sum_m C_{m,m} m! / \beta^{m+1} = 1
\]

By means of equations (3) and (7), we have

\[
\langle a + a^+ \rangle = \sum_m \frac{2(m + 1)! \text{Re} C_{m,m+1}}{\beta^{m+2}}
\]

\[
\langle a^2 + a^{+2} \rangle = \sum_m \frac{2(m + 2)! \text{Re} C_{m,m+2}}{\beta^{m+3}}
\]

\[
\langle a^+ a \rangle = \sum_m \frac{(m + 1) - \beta}{\beta^{m+2}} m! C_{m,m}
\]

and

\[
S = \sum_m \frac{2(m + 2)! \text{Re} C_{m,m+2}}{\beta^{m+3}} + 2 \sum_m \frac{m + 1 - \beta}{\beta^{m+2}} m! C_{m,m}
\]

\[= \left[ \sum_m \frac{2(m + 1)! \text{Re} C_{m,m+1}}{\beta^{m+2}} \right]^2 \]

If the field exists squeezing, then

\[
\sum_m \left[ \frac{(m + 2)! \text{Re} C_{m,m+2}}{\beta^{m+3}} + \frac{(m + 1 - \beta)m! C_{m,m}}{\beta^{m+2}} \right]
\]

\[< \left[ \sum_m \frac{2(m + 1)! \text{Re} C_{m,m+1}}{\beta^{m+2}} \right]^2 \]

2 Squddzing of field with gaussian Q function

We introduce the gaussian Q function as

\[
Q(\alpha) = \sqrt{\beta} - 4 |A|^2 \exp \left[ - t(\alpha^* - \omega^*)(\alpha - \omega) + A^*(\alpha^* - \omega^*)^2 + A(\alpha - \omega)^2 \right]
\]

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where \( t > 2|A| \). Using integration formula\([3]\)

\[
\int \frac{d^2z}{\pi} e^{-\mu|z|^2 + \sigma z^* z + \tau z + \rho z^*} = \frac{1}{\sqrt{\mu^2 - 4\phi g}} e^{\frac{\mu z^* - z + \phi}{\phi}}
\]  

(15)

and equation (3), one can show

\[
\langle \alpha + \alpha^+ \rangle = \omega + \omega^*
\]  

(16)

\[
\langle \alpha^2 + \alpha^{+2} \rangle = \omega^* 2 + \omega^2 + \frac{2(A + A^*)}{t^2 - 4|A|^2}
\]  

(17)

\[
\langle \alpha^+ \alpha \rangle = |\omega|^2 + \frac{t}{t^2 - 4|A|^2} - 1
\]  

(18)

and easily obtain

\[
S = \frac{2(A + A^* + 4|A|^2 + t - t^2)}{t^2 - 4|A|^2}
\]  

(19)

Thus the condition for the existence of squeezing is

\[
A + A^* + 4|A|^2 < t^2 - t
\]  

(20)

If \( A = 0 \), squeezing means \( t > 1 \), if \( t < 1 \) and \( A = 0 \), no squeezing exists in the field. It is worth to point out that the field with \( A = 0 \) and \( t > 1 \) has not been found until now.

### 3 Squeezing of field with negative \( P(\alpha) \) function

The relation of \( P(\alpha) \) and \( Q(\alpha) \) is

\[
Q(\alpha) = \int \frac{d^2\beta}{\pi} e^{-|\beta|^2 - \alpha^* \beta} P(\beta)
\]  

(21)

for nonclassical field, its \( P(\alpha) \) function has two situations \([4]\): i) \( P(\alpha) \) is negative, ii) \( P(\alpha) \) is more singular than \( \delta - \) function. We consider the nonclassical field with negative \( P(\alpha) \) function\([5]\)

\[
\rho = \int d^2\alpha P(\alpha) \langle \alpha | \langle \alpha
\]  

(22)

Suppose \( P(\alpha) \) as

\[
P(\alpha) = \frac{1}{\pi} e^{-|\alpha|^2} \sum_{i,j} P_{ij} \alpha^i \alpha^{*j}
\]  

(23)

Using equations (6) and (21), we obtain
\[
Q(\alpha) = \frac{1}{\pi} \sum_{i,j} P_{i,j} e^{\frac{-i^{\min(i,j)}}{1 + t}} \sum_{l=0}^{\min(i,j)} \frac{i!j!a^{i-l}a^{*j-l}}{l!(i-l)!(j-l)!(l+t)^{i+j-l+1}}
\]  
(24)

comparing with equation (5), one can have

\[
\beta = \frac{t}{1 + t}
\]
(25)

\[
C_{m,n} = \sum_{l} P_{m+l,n+l} \frac{(m + l)! (n + l)!}{l! m! n! (1 + t)^{m+n+l+1}}
\]
(26)

Obviously, the field with negative P function can exhibits squeezing for some situation, but, if \( P(\alpha) \) is only the function of \( |\alpha| \), i.e., \( P(\alpha) \) is sphere symmetry in phase space, then

\[
P_{i,j} = 0 \quad (i \neq j)
\]
(27)

\[
C_{m,n} = 0 \quad (m \neq n)
\]
(28)

Form equation (12), one can get

\[
S > 0
\]
(29)

In conclusion, it is clearly that no suqueezing exists in the field with negative P(\( \alpha \)) function which is sphere symmetry in phase space.

References


