Q(\(\alpha\)) Function and Squeezing Effect

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Abstract

The relation of squeezing and Q(\(\alpha\)) function is discussed in this paper. By means of Q function, the squeezing of field with gaussian Q(\(\alpha\)) function or negative P(\(\alpha\)) function is also discussed in detail.

1 Introduction

In quantum optics, P(\(\alpha\)), Q(\(\alpha\)) and W(\(\alpha\)) are common quasiprobability distribution functions [1], but only Q(\(\alpha\)) perseveres good function (positive and nonregular). Recently, by means of Fokker-Plank equation for Q function, M. S. Kim et al. discussed the fourth-order squeezing [2]. In this paper, we consider the relation between Q function and squeezing, and study the squeezing of field with gaussian Q function or negative P(\(\alpha\)) function.

For any field density operator \(\rho\), the Q function is defined as

\[
Q(\alpha) = \frac{1}{\pi} \langle \alpha | \rho | \alpha \rangle
\]

it satisfies the normalization condition

\[
\int d\alpha^2 Q(\alpha) = 1
\]

For antinormally ordered operator \(f(\alpha, \alpha^+) = f^{(a)}(\alpha, \alpha^+)\), one can get following equation

\[
\langle f(\alpha, \alpha^+) \rangle = \int d^2\alpha Q(\alpha) f^{(a)}(\alpha, \alpha^+) = 1
\]

where \(\alpha\) and \(\alpha^+\) are annihilation and creation operators respectively. Defining parameter

\[
S = \langle \alpha^2 + (\alpha + \alpha^+)^2 \rangle - \langle \alpha + \alpha^+ \rangle^2
\]

For squeezing, \(S\) should be negative.

Now, we suppose that Q function can be expanded as following form

\[
Q(\alpha) = \frac{1}{\pi} e^{-|\alpha|^2} \sum C_{m,n} a^m a^* n, \quad (C_{m,n} = C_{n,m}^*)
\]

Using mathematical identity [3]
\[
\int \frac{d^2 \alpha}{\pi} e^{-\beta |\alpha|^2 + \rho \alpha + \rho^*} = \frac{1}{\beta^{\rho^2/\beta}}, (\beta > 0)
\]  
(6)

one can have

\[
\int \frac{d^2 \alpha}{\pi} \alpha^m \alpha^* e^{-\beta |\alpha|^2} = \frac{n! \delta_{mn}}{\beta^{m+1}}
\]  
(7)

and the normalization condition is

\[
\sum_m C_{m,m} m! / \beta^{m+1} = 1
\]  
(8)

By means of equations (3) and (7), we have

\[
\langle a + a^+ \rangle = \sum_m \frac{2(m + 1)! Re C_{m,m+1}}{\beta^{m+2}}
\]  
(9)

\[
\langle a^2 + a^{+2} \rangle = \sum_m \frac{2(m + 2)! Re C_{m,m+2}}{\beta^{m+3}}
\]  
(10)

\[
\langle a^+ a \rangle = \sum_m \frac{(m + 1) - \beta}{\beta^{m+2}} m! C_{m,m}
\]  
(11)

and

\[
S = \sum_m \frac{2(m + 2)! Re C_{m,m+2}}{\beta^{m+3}} + 2 \sum_m \frac{m + 1 - \beta}{\beta^{m+2}} m! C_{m,m}
\]

\[- \left[ \sum_m \frac{2(m + 1)! Re C_{m,m+1}}{\beta^{m+2}} \right]^2
\]  
(12)

If the field exists squeezing, then

\[
\sum_m \left[ \frac{(m + 2)! Re C_{m,m+2}}{\beta^{m+3}} + \frac{(m + 1 - \beta)m! C_{m,m}}{\beta^{m+2}} \right]
\]

\[< \left[ \sum_m \frac{2(m + 1)! Re C_{m,m+1}}{\beta^{m+2}} \right]^2
\]  
(13)

2 Squddzing of field with gaussian Q function

We introduce the gaussian Q function as

\[
Q(\alpha) = \sqrt{t^2 - 4|A|^2} exp \left[ - t(\alpha^* - \omega^*) (\alpha - \omega) \right.
\]

\[+ A^* (\alpha^* - \omega^*)^2 + A(\alpha - \omega)^2 \]

\]  
(14)
where $t > 2|A|$. Using integration formula[3]

\[
\int \frac{d^2z}{\pi} e^{-|z|^2 + g z^2 + \kappa z^4 + t z^2 + s z^4} = \frac{1}{\sqrt{\mu^2 - 4fg}} e^{\frac{\mu^2 g + 4g^2}{2(\mu^2 - 4fg)}}
\]

and equation (3), one can show

\[
\langle \alpha + \alpha^+ \rangle = \omega + \omega^*
\]

(16)

\[
\langle \alpha^2 + \alpha^{+2} \rangle = \omega^2 + \omega^2 + \frac{2(A + A^*)}{t^2 - 4|A|^2}
\]

(17)

\[
\langle \alpha^+ \alpha \rangle = |\omega|^2 + \frac{t}{t^2 - 4|A|^2} - 1
\]

(18)

and easily obtain

\[
S = \frac{2(A + A^* + 4|A|^2 + t - t^2)}{t^2 - 4|A|^2}
\]

(19)

Thus the condition for the existence of squeezing is

\[
A + A^* + 4|A|^2 < t^2 - t
\]

(20)

If $A = 0$, squeezing means $t > 1$, if $t < 1$ and $A = 0$, no squeezing exists in the field. It is worth to point out that the field with $A = 0$ and $t > 1$ has not been found uptill now.

3 Squeezing of field with negative $P(\alpha)$ function

The relation of $P(\alpha)$ and $Q(\alpha)$ is

\[
Q(\alpha) = \int \frac{d^2\beta}{\pi} e^{-|\beta - \alpha|^2} P(\beta)
\]

(21)

for nonclassical field, its $P(\alpha)$ function has two situations [4]: i) $P(\alpha)$ is negative, ii) $P(\alpha)$ is more singular than $\delta -$ function. We consider the nonclassical field with negative $P(\alpha)$ function[5]

\[
\rho = \int d^2 \alpha P(\alpha) |\alpha\rangle \langle \alpha|
\]

(22)

Suppose $P(\alpha)$ as

\[
P(\alpha) = \frac{1}{\pi} e^{-|\alpha|^2} \sum_{i,j} P_{i,j} \alpha^i \alpha^* j
\]

(23)

Using equations (6) and (21), we obtain
\[
Q(\alpha) = \frac{1}{\pi} \sum_{i,j} P_{i,j} e^{\frac{-t}{1+t}|\alpha|^2} \sum_{l=0}^{\min(i,j)} \frac{i!j!a^{i-l}a^{*j-l}}{l!(i-l)!(j-l)!(l+t)^{i+j-l+1}}
\]  \tag{24}

Comparing with equation (\ref{eq:23}), one can have

\[
\beta = \frac{t}{1+t}
\]  \tag{25}

\[
C_{m,n} = \sum_{l} \frac{(m+l)!(n+l)!}{l!m!n!(1+t)^{m+n+l+1}}
\]  \tag{26}

Obviously, the field with negative $P$ function can exhibit squeezing for some situation, but, if $P(\alpha)$ is only the function of $|\alpha|$, i.e., $P(\alpha)$ is sphere symmetry in phase space, then

\[
P_{i,j} = 0 \quad (i \neq j)
\]  \tag{27}

\[
C_{m,n} = 0 \quad (m \neq n)
\]  \tag{28}

From equation (12), one can get

\[
S > 0
\]  \tag{29}

In conclusion, it is clearly that no squeezing exists in the field with negative $P(\alpha)$ function which is sphere symmetry in phase space.

\textbf{References}


