Abstract

The relation of squeezing and $Q(\alpha)$ function is discussed in this paper. By means of $Q$ function, the squeezing of field with gaussian $Q(\alpha)$ function or negative $P(\alpha)$ function is also discussed in detail.

1 Introduction

In quantum optics, $P(\alpha), Q(\alpha)$ and $W(\alpha)$ are common quasiprobability distribution functions [1], but only $Q(\alpha)$ perserve good function (positive and nonregular). Recently, by means of Fokker-Plank equation for $Q$ function, M. S. Kim et. al discussed the fourth-order squeezing [2]. In this paper, we consider the relation between $Q$ function and squeezing, and study the squeezing of field with gaussian $Q$ function or negative $P(\alpha)$ function.

For any field density operator $\rho$, the $Q$ function is defined as

$$Q(\alpha) = \frac{1}{\pi} \langle \alpha | \rho | \alpha \rangle$$

(1)

it satisfies the normalization condition

$$\int d\alpha^2 Q(\alpha) = 1$$

(2)

For antinormally ordered operator $f(a, a^+) = f^{(a)}(a, a^+)$, one can get following equation

$$\langle f(a, a^+) \rangle = \int d^2 a Q(\alpha) f^{(a)}(\alpha, a^*) = 1$$

(3)

where $a$ and $a^+$ are annihilation and creation operators respectively. Defining parameter

$$S = \langle (a + a^+)^2 \rangle - \langle a + a^+ \rangle^2$$

(4)

For squeezing, $S$ should be negative.

Now, we suppose that $Q$ function can be expanded as following form

$$Q(\alpha) = \frac{1}{\pi} e^{\beta |\alpha|^2} \sum C_{m,n} a^m a^{*-n}, (C_{m,n} = C_{n,m}^*)$$

(5)

Using mathematical identity [3]
\[
\int \frac{d^2\alpha}{\pi} e^{-\beta|\alpha|^2 + r\alpha + r^*\alpha^*} = \frac{1}{\beta} e^{r^2/\beta} , (\beta > 0)
\] (6)

one can have
\[
\int \frac{d^2\alpha}{\pi} e^{m^2\alpha^*\alpha} e^{-\beta|\alpha|^2} = \frac{n! \delta_{mn}}{\beta^{m+1}}
\] (7)

and the normalization condition is
\[
\sum_m C_{m,m} m!/\beta^{m+1} = 1
\] (8)

By means of equations (3) and (7), we have
\[
\langle a + a^+ \rangle = \sum_m \frac{2(m+1)! Re C_{m,m+1}}{\beta^{m+2}}
\] (9)

\[
\langle a^2 + a^{+2} \rangle = \sum_m \frac{2(m+2)! Re C_{m,m+2}}{\beta^{m+3}}
\] (10)

\[
\langle a^+ a \rangle = \sum_m \frac{(m+1) - \beta}{\beta^{m+2}} m! C_{m,m}
\] (11)

and
\[
S = \sum_m \frac{2(m+2)! Re C_{m,m+2}}{\beta^{m+3}} + 2 \sum_m \frac{m+1 - \beta}{\beta^{m+2}} m! C_{m,m}
\]
\[
- \left[ \sum_m \frac{2(m+1)! Re C_{m,m+1}}{\beta^{m+2}} \right]^2
\] (12)

If the field exists squeezing, then
\[
\sum_m \left[ \frac{(m+2)! Re C_{m,m+2}}{\beta^{m+3}} + \frac{(m+1) - \beta}{\beta^{m+2}} m! C_{m,m} \right]
\]
\[
< \left[ \sum_m \frac{2(m+1)! Re C_{m,m+1}}{\beta^{m+2}} \right]^2
\] (13)

2 Squddzing of field with gaussian Q function

We introduce the gaussian Q function as
\[
Q(a) = \sqrt{t^2 - 4|A|^2} exp \left[ - t(a^* - \omega^*)(a - \omega) + A^*(a^* - \omega^*)^2 + A(a - \omega)^2 \right]
\] (14)
where $t > 2|A|$. Using integration formula [3]

$$\int \frac{d^2z}{\pi} e^{-\mu |z|^2 + f^2 z + g z^2 + rz + sz} = \frac{1}{\sqrt{\mu^2 - 4fg}} e^{\frac{\mu r z + rz^2 + sz^2}{\mu^2 - 4fg}}$$ (15)

and equation (3), one can show

$$\langle \alpha + \alpha^+ \rangle = \omega + \omega^*$$ (16)

$$\langle \alpha^2 + \alpha^{+2} \rangle = \omega^2 + \omega^2 + \frac{2(A + A^*)}{t^2 - 4|A|^2}$$ (17)

$$\langle \alpha^+ \alpha \rangle = |\omega|^2 + \frac{t}{t^2 - 4|A|^2} - 1$$ (18)

and easily obtain

$$S = 2(A + A^* + 4|A|^2 + t - t^2)$$ (19)

Thus the condition for the existence of squeezing is

$$A + A^* + 4|A|^2 < t^2 - t$$ (20)

If $A = 0$, squeezing means $t > 1$, and if $t < 1$ and $A = 0$, no squeezing exists in the field. It is worth to point out that the field with $A = 0$ and $t > 1$ has not been found up till now.

3 Squeezing of field with negative $P(\alpha)$ function

The relation of $P(\alpha)$ and $Q(\alpha)$ is

$$Q(\alpha) = \int \frac{d^2\beta}{\pi} e^{-|\beta - a|^2} P(\beta)$$ (21)

for nonclassical field, its $P(\alpha)$ function has two situations [4]: i) $P(\alpha)$ is negative, ii) $P(\alpha)$ is more singular than $\delta-$ function. We consider the nonclassical field with negative $P(\alpha)$ function [5]$

$$\rho = \int d^2a P(\alpha) |\alpha\rangle \langle \alpha|$$ (22)

Suppose $P(\alpha)$ as

$$P(\alpha) = \frac{1}{\pi} e^{-|a|^2} \sum_{i,j} P_{i,j} \alpha^i \alpha^{*j}$$ (23)

Using equations (6) and (21), we obtain
$$Q(\alpha) = \frac{1}{\pi} \sum_{i,j} P_{i,j} e^{-\frac{1}{2} |\alpha|^2} \sum_{l=0}^{\min(i,j)} \frac{i! j! \alpha^{i-l} \alpha^{* j-l}}{l! (i-l)! (j-l)! (l+t)^{i+j-l+1}}$$  \hspace{1cm} (24)$$

Comparing with equation (5), one can have

$$\beta = \frac{t}{1 + t}$$  \hspace{1cm} (25)$$

$$C_{m,n} = \sum_{l} P_{m+i,n+i} \frac{(m + l)! (n + l)!}{l! m! n! (1 + t)^{m+n+l+1}}$$  \hspace{1cm} (26)$$

Obviously, the field with negative $P$ function can exibites squeezing for some situation, but, if $P(\alpha)$ is only the function of $|\alpha|$, i.e., $P(\alpha)$ is sphere symmetry in phase space, then

$$P_{i,j} = 0 \quad (i \neq j)$$  \hspace{1cm} (27)$$

$$C_{m,n} = 0 \quad (m \neq n)$$  \hspace{1cm} (28)$$

Form equation (12), one can get

$$S > 0$$  \hspace{1cm} (29)$$

In conclusion, it is clearly that no suqueezing exists in the field with negative $P(\alpha)$ function which is sphere symmetry in phase space.

References

    W. H. Louisell, Quantum statistical properties of radiation (Wiley, New York, 1973)