Q(a) Function and Squeezing Effect

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Abstract

The relation of squeezing and Q(a) function is discussed in this paper. By means of Q function, the squeezing of field with gaussian Q(a) function or negative P(a) function is also discussed in detail.

1 Introduction

In quantum optics, P(a), Q(a) and W(a) are common quasiprobability distribution functions [1], but only Q(a) persists a good function (positive and nonregular). Recently, by means of Fokker–Plank equation for Q function, M. S. Kim et al. discussed the fourth-order squeezing [2]. In this paper, we consider the relation between Q function and squeezing, and study the squeezing of field with gaussian Q function or negative P(a) function.

For any field density operator \( \rho \), the Q function is defined as

\[
Q(a) = \frac{1}{\pi} \langle a | \rho | a \rangle
\]

(1)

It satisfies the normalization condition

\[
\int da^2 Q(a) = 1
\]

(2)

For antinormally ordered operator \( f(a, a^+) = f^{(a)}(a, a^+) \), one can get following equation

\[
\langle f(a, a^+) \rangle = \int d^2aQ(a)f^{(a)}(a, a^+) = 1
\]

(3)

where \( a \) and \( a^+ \) are annihilation and creation operators respectively. Defining parameter

\[
S = \langle (a + a^*)^2 \rangle - \langle a + a^* \rangle^2
\]

(4)

For squeezing, \( S \) should be negative.

Now, we suppose that Q function can be expanded as following form

\[
Q(a) = \frac{1}{\pi} e^{-|a|^2} \sum C_{m,n} a^m a'^n, (C_{m,n} = C_{n,m})
\]

(5)

Using mathematical identity [3]
\[ \int \frac{d^2 \alpha}{\pi} e^{-\beta|\alpha|^2 + i\alpha \cdot \xi} = \frac{1}{\beta} e^{\nu/\beta}, (\beta > 0) \] (6)

one can have

\[ \int \frac{d^2 \alpha}{\pi} a^m a^n e^{-\beta|\alpha|^2} = \frac{n! \delta_{mn}}{\beta^{m+1}} \] (7)

and the normalization condition is

\[ \sum_m C_{m,m} m! / \beta^{m+1} = 1 \] (8)

By means of equations (3) and (7), we have

\[ \langle a + a^+ \rangle = \sum_m \frac{2(m + 1)! \text{Re} C_{m,m+1}}{\beta^{m+2}} \] (9)

\[ \langle a^2 + a^{+2} \rangle = \sum_m \frac{2(m + 2)! \text{Re} C_{m,m+2}}{\beta^{m+3}} \] (10)

\[ \langle a^+ a \rangle = \sum_m \frac{(m + 1) - \beta}{\beta^{m+2}} m! C_{m,m} \] (11)

and

\[ S = \sum_m \frac{2(m + 2)! \text{Re} C_{m,m+2}}{\beta^{m+3}} + 2\sum_m \frac{m + 1 - \beta}{\beta^{m+2}} m! C_{m,m} \]
\[ - \left( \sum_m \frac{2(m + 1)! \text{Re} C_{m,m+1}}{\beta^{m+2}} \right)^2 \] (12)

If the field exists squeezing, then

\[ \sum_m \left[ \frac{(m + 2)! \text{Re} C_{m,m+2}}{\beta^{m+3}} + \frac{(m + 1 - \beta)m! C_{m,m}}{\beta^{m+2}} \right] \]
\[ < \left[ \sum_m \frac{2(m + 1)! \text{Re} C_{m,m+1}}{\beta^{m+2}} \right]^2 \] (13)

2 Squddzing of field with gaussian Q function

We introduce the gaussian Q function as

\[ Q(\alpha) = \sqrt{t^2 - 4|A|^2} \exp[-t(\alpha^* - \omega^*)(\alpha - \omega)] \]
\[ + A^*(\alpha^* - \omega^*)^2 + A(\alpha - \omega)^2 \] (14)
where $t > 2|A|$. Using integration formula[3]

$$\frac{d^2z}{\pi} e^{-\mu|z|^{2} + f_{2}z^{2} + \kappa z^{*} + rz + oz} = \frac{1}{\sqrt{\mu^2 - 4fg}} e^{\frac{\mu r^2 + rz + oz}{\mu^2 - 4fg}}$$  \hspace{1cm} (15)$$

and equation (3), one can show

$$\langle \alpha + \alpha^{+} \rangle = \omega + \omega^{*}$$  \hspace{1cm} (16)$$

$$\langle \alpha^{2} + \alpha^{+2} \rangle = \omega^{*2} + \omega^{2} + \frac{2(A + A^{*})}{t^2 - 4|A|^2}$$  \hspace{1cm} (17)$$

$$\langle \alpha^{+} \alpha \rangle = |\omega|^2 + \frac{t}{t^2 - 4|A|^2} - 1$$  \hspace{1cm} (18)$$

and easily obtain

$$S = \frac{2(A + A^{*} + 4|A|^2 + t - t^2)}{t^2 - 4|A|^2}$$  \hspace{1cm} (19)$$

Thus the condition for the existence of squeezing is

$$A + A^{*} + 4|A|^2 < t^2 - t$$  \hspace{1cm} (20)$$

If $A = 0$, squeezing means $t > 1$, if $t < 1$ and $A = 0$, no squeezing exists in the field. It is worth to point out that the field with $A = 0$ and $t > 1$ has not been found up till now.

3 Squeezing of field with negative $P(\alpha)$ function

The relation of $P(\alpha)$ and $Q(\alpha)$ is

$$Q(\alpha) = \int \frac{d^2\beta}{\pi} e^{-|\beta - \alpha|^2} P(\beta)$$  \hspace{1cm} (21)$$

for nonclassical field, its $P(\alpha)$ function has two situations [4]: i) $P(\alpha)$ is negative, ii) $P(\alpha)$ is more singular than $\delta$-function. We consider the nonclassical field with negative $P(\alpha)$ function[5]

$$\rho = \int d^2a P(\alpha) |\alpha\rangle \langle \alpha|$$  \hspace{1cm} (22)$$

Suppose $P(\alpha)$ as

$$P(\alpha) = \frac{1}{\pi} e^{t|\alpha|^2} \sum_{i,j} P_{i,j} \alpha^i \alpha^{*j}$$  \hspace{1cm} (23)$$

Using equations (6) and (21), we obtain
\[ Q(\alpha) = \frac{1}{\pi} \sum_{i,j} P_{i,j} e^{\frac{-z}{\pi |\alpha|^2}} \sum_{l=0}^{\min(i,j)} \frac{i! j! i^j a^i \alpha^j}{l!(i-l)!(j-l)!(l+t)!^{i+j-l+1}} \]  

(24)

Comparing with equation (5), one can have:

\[ \beta = \frac{t}{1+t} \]  

(25)

\[ C_{m,n} = \sum_{l} P_{m+l,n+l} \frac{(m+l)! (n+l)!}{l! m! n! (1+t)^{m+n+l+1}} \]  

(26)

Obviously, the field with negative P function can exhibit squeezing for some situations, but if \( P(\alpha) \) is only the function of \(|\alpha|\), i.e., \( P(\alpha) \) is sphere symmetry in phase space, then:

\[ P_{i,j} = 0 \quad (i \neq j) \]  

(27)

\[ C_{m,n} = 0 \quad (m \neq n) \]  

(28)

Form equation (12), one can get:

\[ S > 0 \]  

(29)

In conclusion, it is clearly that no squeezing exists in the field with negative \( P(\alpha) \) function which is sphere symmetry in phase space.

References


