A PROPOSAL FOR TESTING LOCAL REALISM
WITHOUT USING ASSUMPTIONS RELATED
TO HIDDEN VARIABLE STATES

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Abstract

A feasible experiment is discussed which allows us to prove a Bell's theorem for two
particles without using an inequality. The experiment could be used to test local
realism against quantum mechanics without the introduction of additional assumptions
related to hidden variables states. Only assumptions based on direct experimental
observation are needed.

The experiment I wish to discuss is represented in Fig. 1. It is a variant of
Franson's two-photon correlation experiment [1]. However, variants of other experiments
could also be considered [2-4]. A source (S) emits pairs of photons (γ₁ and γ₂). The
photons are emitted simultaneously [5], but there is uncertainty about the time of
emission. H₁ and H₂ are 50%:50% beam splitters. As in an experiment recently discussed
[6], H₁', H₂', H₃, and H₄ are not 50%:50% beam splitters, and have real amplitude
transmissivities T₁, T₂, T₃, and T₄, and real amplitude reflectivities R₁, R₂, R₃, and
R₄. M₁, M₁', M₂, M₂' and M₃' are mirrors, and φ₁, φ₂, and φ₃ are phase shifters.
L₂-S₂=L₁-S₁=cΔT is much greater than the coherence lengths of the packets associated
with γ₁ and γ₂. This implies that Δω₁ΔT>1 and Δω₂ΔT>1, where Δω₁ and Δω₂ are the
uncertainties in the angular frequencies of γ₁ and γ₂. However, Δ(ω₁+ω₂)ΔT<1. As is well
known [1], in this case the situation in which both photons follow the long paths is
indistinguishable from the situation in which both photons follow the short paths. In
the present proposal a balanced Mach-Zehnder interferometer for photons γ₂, constituted
by H₃, H₂', M₂', and H₄ has been introduced.

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FIG. 1. Experiment proposed.

I will consider four different situations: (A) $H'_1$ and $H_4$ are removed; (B) $H'_1$ is in place and $H_4$ is removed; (C) $H'_1$ is removed and $H_4$ is in place; (D) $H'_1$ and $H_4$ are in place. The detections relevant to our discussion are only the coincident detections occurring at sites 1 and 2, 1 and 2', 1' and 2, and 1' and 2'. Naturally, the probability of coincident detections occurring at sites 1' and 2 in situation A, $P_\lambda'(1',2)=0$, since in situation A $\gamma_1(\gamma_2)$ has to follow the long(short) path to be detected at 1'(2).

The probability amplitude of coincident detections occurring at sites 1 and 2' in situation B is [6]

$$A_b^{(1,2')} = \sum_{\omega_1,\omega_2} c_{\omega_1,\omega_2} (\alpha_1\alpha_2 + \beta_1\beta_2), \quad (1)$$

where $c_{\omega_1,\omega_2}$ is the probability amplitude of having a photon $\gamma_1$ with a frequency $\omega_1$ and a photon $\gamma_2$ with frequency $\omega_2$, $\alpha_1 = 2^{-1/2} \exp(i\omega_1 t_s) T_1$ is the probability amplitude of having a photon $\gamma_1(\omega_1)$ following the short path, where $t_s$ is the time spent by light to follow the short path, $\alpha_2 = 2^{-1/2} T_2 i R_2 \exp[i\omega_2(t_s+t')]$ is the probability amplitude of having a photon $\gamma_2(\omega_2)$ following the short path, where $t'$ is the time spent by light from $H'_1$ to $H_4$, $\beta_1 = i2^{-1/2}\exp(i\phi_0)\exp(i\omega_1 t_L) i R_1$ is the probability amplitude of having a photon $\gamma_1(\omega_1)$ following the long path, where $t_L$ is the time spent by light to follow the long path, and $\beta_2 = i2^{-1/2}\exp(i\phi_2) T_2 \exp[i\omega_2(t_L+t')]$ is the probability amplitude of having a photon $\gamma_2(\omega_2)$ following the long path. Using (1) and the condition $\Delta(\omega_1 + \omega_2) \Delta T \ll 1$ we obtain

\[ A_{B}(1,2') = \frac{i}{2} \sum_{\omega_1, \omega_2} A_{\omega_1, \omega_2} (T_1 T_3 R_2 - BR_1 T_2) , \]  

where \( A_{\omega_1, \omega_2} = c_{\omega_1} c_{\omega_2} \exp \left[ i \omega_1 t_5 + i \omega_2 (t_5 + t') \right] \) and \( B = \exp \left[ i (\phi_1 + \phi_2) + i (\omega_{10} + \omega_{20}) \Delta T \right] \), where \( \omega_{10} \) and \( \omega_{20} \) are the central frequencies of \( \gamma_1 \) and \( \gamma_2 \). Choosing \( T_1 T_3 R_2 = R_1 T_2 \) and using the condition

\[ \sum_{\omega_1, \omega_2} |c_{\omega_1} c_{\omega_2}|^2 = 1 , \]  

we obtain

\[ P_{B}(1,2') = (1/2) (T_1 T_3 R_2)^2 (1 - \text{Re} B) . \]  

In an ideal situation we can have \( [P_{B}(1,2')]_{\text{min}} = 0 \) (\( \text{Re} B = 1 \)) and \( [P_{B}(1,2')]_{\text{max}} = (T_1 T_3 R_2)^2 \) (\( \text{Re} B = -1 \)). This follows from quantum mechanical nonlocality. But in a real situation this is not so. Let us then assume that \( \text{Re} B = 1 - \epsilon \) (\( \text{Re} B = -1 + \epsilon \)) in the minimum (maximum) case. Then we can introduce the visibility \( V_B \) given by

\[ V_B = \frac{[P_{B}(1,2')]_{\text{max}} - [P_{B}(1,2')]_{\text{min}}}{[P_{B}(1,2')]_{\text{max}} + [P_{B}(1,2')]_{\text{min}}} = 1 - \epsilon . \]  

Thus,

\[ [P_{B}(1,2')]_{\text{min}} = (1/2) (T_1 T_3 R_2)^2 (1 - V_B) . \]  

Using a similar reasoning, we obtain

\[ A_{C}(1,2') = \sum_{\omega_1, \omega_2} c_{\omega_1} c_{\omega_2} \delta (\rho_1 + \rho_2) . \]  

where \( \delta = 2^{-1/2} \exp(i \omega_1 t_5) \), \( \rho_1 = 2^{-1/2} T_3 i R_2 \exp [i \omega_2 (t_5 + t')] T_4 \), and \( \rho_2 = 2^{-1/2} i R_3 \exp(i \phi_3) \exp[i \omega_2 (t_5 + t')] i R_4 \), which leads to

\[ A_{C} = (1/2) \sum_{\omega_1, \omega_2} A_{\omega_1, \omega_2} (i T_3 R_2 T_4 - CR_3 R_4) , \]  

where \( C = \exp(i \phi_3) \). Thus, choosing \( T_3 R_2 T_4 = R_3 R_4 \) and using (3) we obtain

\[ P_{C}(1,2') = (1/2) (T_3 R_2 T_4)^2 (1 - \text{Im} C) . \]  

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As in (5) and (6), we can introduce the visibility $V_c$ to obtain

$$[P_c^\epsilon(1,2')]_{\text{min}} = (1/2)(T_3R_2T_4)(1-V_c) \ .$$

In an ideal situation we can have $V_c=1$ and $[P_c^\epsilon(1,2')]_{\text{min}}=0$. This follows from the wave like properties of light.

It is also easy to see that

$$A_1^\epsilon(1,2') = \sum_{\begin{subarray}{c} \omega_1 \omega_2 \\ \sigma_1 \sigma_2 \end{subarray}} c_{\omega_1 \omega_2} [\lambda_1(\sigma_1 + \sigma_2) + \lambda_2 \sigma_3] \ ,$$

where

$$\lambda_1 = 2^{-1/2} \exp(i\omega_1 t_3)T_1, \quad \sigma_1 = 2^{-1/2} T_3 i R_2 \exp[i\omega_2(t_3 + t')]T_4, \quad \sigma_2 = 2^{-1/2} i R_3 \exp(i\phi_3)$$

and

$$\lambda_2 = i 2^{-1/2} \exp(i\phi_1) \exp(i\omega_1 t_3) R_4, \quad \lambda_3 = i 2^{-1/2} \exp(i\phi_2) T_2 \\exp[i\omega_2(t_4 + t')]T_4, \quad \text{which leads to}$$

$$A_D^\epsilon = (i/2) T_1 T_3 R_2 T_4 \sum_{\begin{subarray}{c} \omega_1 \omega_2 \\ \omega_1 \omega_2 \end{subarray}} c_{\omega_1 \omega_2} (1-B+iC) \ .$$

Then, choosing $\phi_1$ and $\phi_2$ such that $P_B^\epsilon(1,2') = [P_B^\epsilon(1,2')]_{\text{min}}$, and $\phi_3$ such that $P_C^\epsilon(1,2') = [P_C^\epsilon(1,2')]_{\text{min}}$, we obtain

$$P_B^\epsilon(1,2') = (1/2)(T_3 R_2 T_4)^2 [(3/2) - V_B V_C - V_B V_C (1-V_B)^{1/2} (1-V_C)^{1/2}] \ .$$

To prove a Bell's theorem for two particles without using an inequality we can consider the ideal situation: $V_B = V_C = 1$. I will assign the value $i(1)$ for detections that occur at site $1$ and $2'$ (1' and 2). Thus, assuming there can be hidden variables states (HVS) of the photon pair which mimic quantum mechanics, we can only have: (A) $a_R^\epsilon(\lambda) b_R^\epsilon(\lambda) = i,-1$; (B) $a_R^\epsilon(\lambda) b_R^\epsilon(\lambda) = i,1$, from (6); and (C) $a_R^\epsilon(\lambda) b_R^\epsilon(\lambda) = i,1$, from (10). $a_R^\epsilon(\lambda)$ ($b_R^\epsilon(\lambda)$) represents the result of a measurement performed at 1,1' (2,2') when $H_1$ ($H_4$) is in place (removed), and so on, the superscript c refers to coincident detections, and $\lambda$ represents the HVS of the photon pair [7]. Assuming locality, that is, that $a_R^\epsilon(\lambda)$ is the same in A and C, for example, we see that $a_R^\epsilon(\lambda) b_R^\epsilon(\lambda) = 1$, in disagreement with $P_B^\epsilon(1,2') = (1/4)(T_3 R_2 T_4)^2$ (quantum mechanics), from (13).

Introducing some assumptions which are based on direct experimental observation the above argument can be extended to the case of a real (i.e., non-ideal) experiment. Let us initially consider situation C and select only those events in which detection at 2'
occurs. In this case, whenever a coincident detection at 1 occurs we know that \( \gamma_1 \) and \( \gamma_2 \) have followed the short paths. I will assume that: (A1) if \( H'_1 \) had been in place (sit.C \( \rightarrow \) sit.D) the number of photons following the short path that would be coincidently detected at 1 could not be greater than the number of photons coincidently detected at 1 when \( H'_1 \) is removed (I will return to this point). Therefore, the number of coincident detections at 1 and 2' in sit.D which correspond to the possibility in which \( \gamma_1 \) and \( \gamma_2 \) follow the short paths cannot be greater than \( N^c_1(1,2') \), the number of coincident detections at 1 and 2' in sit.C.

Let us now consider situation B and select only those events in which detection at 1 occurs. In this case, only the coincident detections at 1 and 2' can correspond to the possibility in which \( \gamma_1 \) and \( \gamma_2 \) follow the long paths. According to (A1), if \( H_4 \) had been in place (sit.B \( \rightarrow \) sit.D) the number of photons following path \( n \) that would be coincidently detected at 2' could not be greater than the number of photons coincidently detected at 2' when \( H_4 \) is removed. Therefore, the number of coincident detections at 1 and 2' in sit.D which correspond to the possibility in which \( \gamma_1 \) and \( \gamma_2 \) follow the long paths cannot be greater than \( N^c_1(1,2') \), the number of coincident detections at 1 and 2' in sit.B. Hence, \( N^c_1(1,2') \leq N^c_1(1,2') + N^c_2(1,2') \), or, in terms of probabilities,

\[
P^c_1(1,2') \leq P^c_1(1,2') + P^c_2(1,2'),
\]

since: (A2) coincident detections can only occur when photons of the emitted pair either (a) both follow the long paths, or (b) both follow the short paths.

Let us examine (A1) closer. It was assumed, when changing from situation C(B) to situation D, that the number of detections generated by photons \( \gamma_1(\gamma_2) \) following path \( S_1(n) \) could not be increased by placing a beam splitter \( H'_1(H_4) \) in front of the detectors. Although this may appear to be a nonenhancement assumption [8], this can be directly verified. For example, by blocking path \( L_1(q) \) in situation D. Now we are not assuming that for every HVS of a photon the probability of it being detected cannot be enhanced by placing a beam splitter in front of the detector. However, it might still be argued that when \( H'_1(H_4) \) is in the position represented in Fig.1, in which case photons from two different directions can impinge on it, its properties are modified, in such a way that photons coming via path \( S_1(n) \) become more "detectable" after impinging on \( H'_1(H_4) \) and being transmitted, whilst photons coming via path \( L_1(q) \) become less
"detectable" after impinging on $H_1^i(H_4)$ and being reflected [9]. However, this sounds as a much too contrived supposition.

To have a rough estimation of the expected disagreement between the local realistic and the quantum mechanical predictions in a real experiment, we can make $V_b=V_c=V$. Hence, using (6), (10), and (13), we see that in order to have a violation of (14) we must have

$$\frac{(T_1 T_4)^2[(1/2)-2V+2V^2]/[(T_4^2+T_1^2)(1-V)]}{1}. \quad (15)$$

Then, making $T_1=T_4=T, R_1=R_4=R$, which leads to $T_3=T_2=R_2=R_3=[-R+(1+3T^3)^{1/2}]/2T$, we obtain

$$\frac{(T^2/4)(1-4V+4V^2)/(1-V) > 1}{.} \quad (16)$$

We see that the minimum visibility we must have in order to violate (16) is given by $V > 0.87 \ (T=1)$. Apparently our best choice would be $T=1$. However, this corresponds to the situation in which $H_1^i$ and $H_4$ have been removed. In this case the probabilities drop to zero, and we would have to wait an infinite time to get any result. $V=0.90, T=1/(1.2)^{1/2} \rightarrow \text{l.h.s.}\ (16) > 1.3$. To have an idea of the time necessary to perform an experiment using these data we can calculate the ratio between the probability of having a coincident detection in a Franson’s experiment in the case of perfect correlations and the probability given by (13) in the ideal case ($V=1$). We easily see that we need about eleven times more time to have the same statistics as in a Franson’s experiment.

REFERENCES


