Effects of the Stark shift on the evolution of the field entropy and entanglement in the two-photon Jaynes-Cummings model

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Abstract

I have investigated the evolution of the field entropy in the two-photon JCM in the presence of the Stark shift and examined the effects of the dynamic Stark shift on the evolution of the field entropy and entanglement between the atom and field. My results have shown that the dynamic Stark shift plays an important role in the evolution of the field entropy in two-photon processes.

1 Introduction

The two-photon Jaynes-Cummings model [1] describing the interaction of a single-mode quantized field with a two-level atom through intermediate state involving the emission or absorption is one of the most intensively studied models in quantum optics. In this model, when the two atomic levels are coupled with comparable strength to the intermediate relay level, the Stark shift becomes significant and cannot be ignored [2-5]. Puri and Bullough [3], A.Josh [4], Tahira Nasreen and Razmi [5] studied the influences of the Stark shift terms on the atomic inversion and dipole squeezing. Tahira Nasreen and Razmi [5,8] discussed the effect of the Stark shift on the Atomic emission and cavity field spectra in the two-photon JCM. These works have shown that the dynamic Stark shift plays an important role for the properties of dynamics in two-photon JCM. On the other hand, recently much attention has been focused on the properties of the entanglement between the field and atom in the Jaynes-Cummings model (JCM) [9-15]. Phoenix and Knight [9] have shown that the partial entropy is a convenient and sensitive measure of entanglement between the atom and field. The time behavior of the field (atomic) entropy reflects time behavior of the degree of entanglement between the field and atom in JCM. The higher the entropy, the greater the entanglement, the information concerning the field is obtained by measurement performed on atoms. For the two-photon JCM, Phoenix and Knight [11], and Buzek [10] studied the evolution of the field entropy and the entanglement between the field and atom. The author [13] also examined the influence of atomic coherence on the evolution of field entropy in two-photon processes. However, these results are obtained in the case the Stark shift is ignored. In this paper, to make the two-photon JCM closer to the experimental realization, I include the effect of the dynamic Stark shift in studying the evolution of field entropy and entanglement. The results for the entropy evolution and entanglement incorporating the Stark shift are radically different from the results obtained in the absence of the Stark shift.
2 The reduced density operator and the field entropy calculation formalism for two-photon JCM in the presence of the Stark shift

In this paper, the model considered consists of a single-mode cavity field of frequency $\omega$ with an effective two-level atom of transition frequency $\omega_0$ through two-photon transitions in a lossless cavity. The excited and ground states of the atom will be designated by $| + \rangle$ and $| - \rangle$, respectively. I assume these states to have identical parity, whereas the intermediate states, labeled $| j \rangle$ ($j = 3, 4, \ldots$), are coupled to $| + \rangle$ and $| - \rangle$ by a direct dipole transition and so located as to give rise to a significant Stark shift. The effective Hamiltonian describing such a model has form [3]

$$\hat{H}_{\text{eff}} = \omega \hat{a}^\dagger \hat{a} + \omega_0 \hat{S}_z + \hat{a}^\dagger \hat{a}(\beta_1 | + \rangle\langle + | + \beta_1 | - \rangle\langle - |) + g(\hat{a}^\dagger \hat{S}_- + \hat{a} \hat{S}_+),$$  

(1)

where I have chosen units such that $\hbar = 1$. $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operator of the cavity field; $\hat{S}_z = | + \rangle\langle + | - | - \rangle\langle - |$, $\hat{S}_+ = | + \rangle\langle - |$, and $\hat{S}_- = | - \rangle\langle + |$ are the atomic flopping operators. $\beta_1$ and $\beta_2$ are the parameters describing the dynamic Stark shift of the two levels due to the virtual transitions to the intermediate relay level, and $g$ is the atom-field coupling constant. For simplicity, I consider on-resonance interaction, so that $\omega_0 = 2\omega$. By diagonalizing $\hat{H}_{\text{eff}}$ in the manifold of states $| +, n \rangle$ and $| -, n+2 \rangle$, the time-evolution operator in the interaction picture can be obtained [5]

$$\hat{U}_f(t) = \left( \begin{array}{cc} U_{11}(n) & U_{12}(n) \\ U_{21}(n) & U_{22}(n) \end{array} \right),$$  

(2)

where I have written

$$U_{11}(n) = \sin^2(\theta_n) \exp(-i\lambda_+^n t) + \cos^2(\theta_n) \exp(-i\lambda_-^n t)$$

$$U_{12}(n) = \frac{1}{2} \sin(2\theta_n)[\exp(-i\lambda_+^n t) - \exp(-i\lambda_-^n t)] = U_{21}(n)$$

$$U_{22}(n) = \sin^2(\theta_n) \exp(-i\lambda_-^n t) + \cos^2(\theta_n) \exp(-i\lambda_+^n t),$$  

(3)

With

$$\sin(\theta_n) = \frac{1}{\sqrt{2}}(1 + \frac{\xi_n}{\Omega_n})^{1/2}$$

$$\lambda_+^n = g \frac{n(1+r^2) + 2r^2}{2r} \pm \Omega_n$$

$$\Omega_n = [g^2(n+1)(n+2) + \xi_n^2]^{1/2}$$

$$\xi_n = \frac{g}{2r}[n(1-r^2) - 2r^2]$$

$$r = (\beta_1/\beta_2)^{1/2},$$  

(4)

I consider the at time $t=0$ the atom is in a coherent superposition of the excited and ground states

$$| \theta, \varphi \rangle = \cos(\frac{\theta}{2}) | + \rangle + \exp(-i\varphi) \sin(\frac{\theta}{2}) | - \rangle,$$  

(5)
and the field is an arbitrary superposition of Fock states, so that at time $t=0$ the density operator for system

$$\rho(0) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{n,m} \{ \cos(\frac{\theta}{2}) | +, n \rangle + \exp(-i\varphi) \sin(\frac{\theta}{2}) | -, n \rangle \} \times \{ \cos(\frac{\theta}{2}) | m, + \rangle + \exp(i\varphi) \sin(\frac{\theta}{2}) | m, - \rangle \}.$$  

(6)

where $\theta$ is the degree of excitation, $\varphi$ is the relative phase of the two atomic levels, $F_{n,m} = F_n F_m^*$ and $F_n$ are coefficients in the Fock-state. At any time $t > 0$ the reduced field density operator for the system is given by

$$\rho_f(t) = Tr_{atom}\{ \hat{U}_f(t) \rho(0) \hat{U}_f^+(t) \}$$

$$= \cos^2(\frac{1}{2} \theta) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{n,m} \{ U_{11}(n)U_{11}^*(m) \langle n | m \rangle + U_{21}(n)U_{21}^*(m) \langle n+2 | m+2 \rangle \}$$

$$+ \frac{1}{2} \sin(\theta) \exp(i\varphi) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{n,m} \{ U_{12}(n)U_{12}^*(m-2) \langle n | m-2 \rangle + U_{22}(n-2)U_{22}^*(m) \langle n-2 | m \rangle \}$$

$$+ \frac{1}{2} \sin(\theta) \exp(-i\varphi) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{n,m} \{ U_{11}(n-2)U_{11}^*(m) \langle n-2 | m \rangle + U_{21}(n-2)U_{21}^*(m-2) \langle n-2 | m-2 \rangle \}$$

$$+ \sin^2(\frac{1}{2} \theta) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{n,m} \{ U_{22}(n-2)U_{22}^*(m-2) \langle n-2 | m-2 \rangle + U_{21}(n-2)U_{21}^*(m) \langle n-2 | m \rangle \}. \quad (7)$$

The reduced density matrix Equation (7) has included the influences of the Stark shift and atomic coherence. Following the work of Phoenix and Knight [9], the eigenvalues and eigenstates of the reduced field density operator Equation (7) may be obtained

$$\pi_f^\pm(t) = \langle U_1^{*1} U_1' \rangle \pm \exp(\mp \delta) \langle U_1^{*1} U_2' \rangle$$

$$= \langle U_2^{*1} U_2' \rangle \pm \exp(\pm \delta) \langle U_1^{*1} U_2' \rangle, \quad (8)$$

$$| \psi_f^\pm(t) \rangle = \frac{1}{\sqrt{2\pi_f^\pm(t) \cosh(\delta)}} \{ \exp[\frac{1}{2}(i\xi \pm \delta)] | U_1' \rangle \pm \exp[-\frac{1}{2}(i\xi \pm \delta)] | U_2' \rangle \}.$$  

(9)

where

$$| U_1' \rangle = \cos(\frac{1}{2} \theta) \sum_{n=0}^{\infty} F_n U_{11}(n) | n \rangle + \sin(\frac{1}{2} \theta) \exp(-i\varphi) \sum_{n=0}^{\infty} F_n U_{12}(n-2) | n-2 \rangle,$$
\[ | U_{21}^\prime \rangle = \cos(\frac{1}{2} \theta) \sum_{n=0}^{\infty} F_n U_{21}(n + 2) + \sin(\frac{1}{2} \theta) \exp(-i\varphi) \sum_{n=0}^{\infty} F_n U_{22}(n - 2) | n \rangle. \]

\[ \chi = \frac{1}{2} | \langle U_{11}^\dagger U_{21} \rangle - \langle U_{21}^\dagger U_{21}^\prime \rangle |, \]

\[ \delta = \sin h^{-1}(\chi). \]  

(10)  

(11)

I can obtained calculation formalism of the field entropy \( S_f(t) \) in terms of the eigenvalue \( \pi_f \) of the reduced field density operator

\[ S_f(t) = -[\pi_f^+(t) \ln \pi_f^+(t) + \pi_f^-(t) \ln \pi_f^-(t)]. \]  

(12)

The field entropy \( S_f(t) \) given by Equation (12) shows that the field entropy depends not only on the field statistics parameters \( F_{n,m} \), but also on the Stark shift parameter \( r = (\beta_1/\beta_2)^{1/2} \) and the initial state of atom. Especially when \( \beta_1 = \beta_2 = 0 \), Eq(12) give the results of Refs[11-13] for \( K=2 \).

3 Numerical results

I now discuss the numerical results for the field entropy \( S_f(t) \) is given by Equation (17) when the initial field state is in the coherent state \( | \alpha \rangle \)

\[ F_{n,m} = \exp(- | \alpha |^2) \alpha^n \alpha^m / (n!m!)^{1/2}. \]  

(13)

with \( \alpha = | \alpha | \exp(i\beta) \) and \( | \alpha |^2 = \bar{n} \), where \( \beta \) is the initial phase of the field and \( \bar{n} \) is the average number of photons in the coherent state. Here I hope to learn about the roles played by the Stark shift. The numerical results of equation (12) are show in Figs.1-3 for different values of the Stark shift parameter \( r \) and different the initial states of the atom with \( \bar{n} = 20 \).

**Figure1** Figs.1 Effects of the Stark shift on the evolution of the field entropy. \( \theta = 0 \), atom initially in excite state, field in the coherent state with mean photon numbers \( \bar{n} = 20 \). (a), no Stark shift \( (\beta_1 = \beta_2 = 0 \); (b), \( r = 1(\beta_1 = \beta_2) \); (c), \( r = 0.5 \); (d), \( r = 0.3 \).

**Figure2** Figs.2 Effects of the Stark shift on the evolution of the field entropy. \( \theta = \pi/2 \), \( \phi - 2\beta = 0 \), atom initially in trapping state, the rest parameters as Figs.1. (a), no Stark shift \( (\beta_1 = \beta_2 = 0 \); (b), \( r = 1(\beta_1 = \beta_2) \); (c), \( r = 0.5 \); (d), \( r = 0.3 \).

**Figure3** Figs.3 Effects of the Stark shift on the evolution of the field entropy. The same as Figs.2 but \( \phi - 2\beta = \pi/2 \). (a), no Stark shift \( (\beta_1 = \beta_2 = 0 \); (b), \( r = 1(\beta_1 = \beta_2) \); (c), \( r = 0.5 \); (d), \( r = 0.3 \).

3.1 Atom Initially in the excited state In Figs.1(a), I have the case \( \theta = 0 \) (i.e. the atom is in the excited state) and \( \beta_1 = \beta_2 = 0 \) (i.e. in the absence of the Stark shift), corresponding to the evolution of the field entropy in the standard two-photon JCM obtained by Ref[11-13]. I note that the field entropy evolves at periods \( \pi/g \), when \( t = n\pi/g \) (\( n = 0, 1, 2, 3, \ldots \)), \( S_f(t) \) evolves to the zero values and the field is completely disentangled with the atom, while when \( t = (n + 1/2)\pi/g \), \( S_f(t) \) evolves to the maximum value, and the field is strongly entangled with the atom. The results for the evolution of the field entropy \( S_f(t) \) in the presence of the Stark shift are plotted in Figure 1(b)-(d). In Figs.1(b), the Stark shift parameter \( r \) is given as 1 (namely \( \beta_1 = \beta_2 \)), this correspond to the case the two levels of the atom are equally strongly couple with the intermediate relay level. By making a comparison between Figs.1(a)
and Figs.1(b), I find that the evolution of the entropy is almost similar for both cases. This result corresponds with the fact that in two-photon processes, the Stark shift creates an effective intensity dependent detuning $\Delta_N = \beta_2 - \beta_1$[16]. When $r = 1.0$, (namely $\beta_1 = \beta_2$) thus $\Delta_N = 0.0$, the Stark shift does not affect the time evolution of the field entropy. In Figs.1(c), I show the case $r=0.5$, in which the two levels have the unequal Stark shifts ($\beta_1 < \beta_2$). I note that the Stark shift leads to decreasing the values of maximum field entropy and increasing the values of field minimum entropy. It also results in increasing frequency of the field entropy vibration. As the parameter $r$ further decreased (e.g., $r=0.3$, see Figs.1(d)), the values of the maximum field entropy and the degree of entanglement of the field-atom further reduced.

3.2 Atom initially in the superposition states When $\theta = \pi/2$ and $\varphi = -2\beta = 0$, the atom is initially in trapping state, the evolution of the field entropy are plotted in Figs.2. Figs.2(a) show the evolution of the field entropy in the absence of the Stark shift while Figs.2(b) show the case that $r=1.0$. I can see that under the condition the atom is initially in trapping state, the field entropy obviously reduced with comparison Figs.1 (a), (b) in where the atom is initially in excited state, and the evolution of field entropy in the case $r=1.0$ is almost same as that in the absence of the Stark shift. Figs.2(c) and Figs.2(d) show the evolution of $S_f(t)$ in two cases that $r=0.5$ and $r=0.3$, respectively. As is visible from the figures, the effects of the dynamic Stark shift are more pronounced when $r$ deviates from unity. On the other hand, when atom in initially trapping state, as $r$ decreased, the values of maximum field entropy increased, indicating that the Stark shift leads to increasing the degree of entanglement between the field and atom, which is contrary to the case the atom is initially excited state. The results for the evolution of the field entropy $S_f(t)$ as $\theta = \pi/2$, $\varphi = -2\beta = \pi/2$ and various values of Stark shift parameter $r$ are presented in Figure 3. In these cases, I note that the maximum field entropy always remains at its maximum values, regardless of the chosen value of $r$. A possible explanation for above behavior of field entropy evolution can be performed in terms of the Bloch vector in semiclassical theory[17] in the next section.

4 Semiclassical interpretation of the evolution behavior of the field entropy

I can find that the larger the extent of the Bloch vector's motion, the greater the values of the maximum field entropy by examining the evolution of field entropy in semiclassical version. By replacing the field annihilation operator $a$ by the $c$ number $\nu = \sqrt{n} \exp[i(\omega t - \beta)]$ I get semiclassical version of eq.(1)

$$
H_{\text{eff}} = \tilde{n} \omega + \omega_0 S_z + \tilde{n}(\beta_2 | + \rangle \langle + | + \beta_1 | - \rangle \langle - |)
+ \tilde{n}g(S_+ \exp[-i2(\omega t - \beta)] + S_- \exp[i2(\omega t - \beta)]).
$$

(14)

The motion equations of operators $S_\xi (\xi = +, -, z)$ can be written

$$
\frac{dS_+}{dt} = i\omega_0 S_+ - i2\tilde{n}g \exp[i2(\omega t - \beta)]S_z + i\tilde{n}\Delta_N S_+ ,
$$

(15)

$$
\frac{dS_-}{dt} = -i\omega_0 S_- + i2\tilde{n}g \exp[-i2(\omega t - \beta)]S_z - i\tilde{n}\Delta_N S_- ,
$$

(16)
\[
\frac{dS_z}{dt} = ig\tilde{n}(\exp[i2(\omega t - \beta)]S_- - \exp[-i2(\omega t - \beta)]S_+) ,
\]

where

\[
\Delta_N = \beta_2 - \beta_1 = \frac{g(1 - r^2)}{r}.
\]

is an effective detuning created by the Stark shift. Define the two slowly varying operators which involve the coherence between the two atomic states

\[ S_x = \frac{1}{2}\{\exp[-i(\omega_0 t - \Phi)]S_+ + \exp[i(\omega_0 t - \Phi)]S_-\} , \]

\[ S_y = \frac{1}{2i}\{\exp[-i(\omega_0 t - \Phi)]S_+ - \exp[i(\omega_0 t - \Phi)]S_-\} . \]

where \( \Phi \) is a phase angle that may be chosen at will. The atom interacting with the field obeys the optical Bloch equation

\[
\frac{dS(t)}{dt} = \Omega_s(t) \times S(t).
\]

Where \( S(t) \) is the Bloch vector for the atom

\[ S(t) = \{S_x(t), S_y(t), S_z(t)\} . \]

and \( \Omega_s(t) \) is the driving field vector, which can be written by using eqs. (14)-(22)

\[
\Omega_s(t) = \{2g\tilde{n}\cos[(2\omega - \omega_0)t - (2\beta - \Phi)], 2g\tilde{n}\sin[(2\omega - \omega_0)t - (2\beta - \Phi)], \tilde{n}\Delta_N\} .
\]

Generally \( S(t) \) precesses in a cone about \( \Omega_s(t) \). The extent of Bloch vector's motion is largest when \( S(t) \) and \( \Omega_s(t) \) are orthogonal and minimum when \( S(t) \) and \( \Omega_s(t) \) are parallel or antiparallel. Thus the time evolution of the Bloch vector is quite different for different initial preparations. If the atom is initially coherent superposition state given by eq (5), the Bloch vector at time \( t=0 \) can be expressed [17]

\[ S(0) = \{\frac{1}{2}\sin(\theta)\cos(\Phi - \varphi), \frac{1}{2}\sin(\theta)\sin(\Phi - \varphi), \frac{1}{2}\cos(\theta)\} , \]

\[ \Omega_s(0) = \{2g\tilde{n}\cos(\Phi - 2\beta), 2g\tilde{n}\sin(\Phi - 2\beta), \tilde{n}\Delta_N\} . \]

For simplicity, I let \( \Phi = \varphi \), the initial Bloch vector in this case is on the x-z plane. When the atom is initially in excite state (\( \theta = 0 \)), the initial Bloch vector \( S(0) = \{0, 0, \frac{1}{2}\} \) and the vector \( \Omega_s(0) = \{2g\tilde{n}, 0, \tilde{n}\Delta_N\} \). Under this condition, the extent of Bloch vector's motion is dependent of the effective detuning \( \Delta_N \) created by the Stark shift. When \( \beta_1 = \beta_2 = 0 \) (in the absence of the Stark shift) or \( r = 1.0(\beta_1 = \beta_2) \), \( \Delta_N = 0 \), the vector \( S(0) \) and \( \Omega_s(0) \) are orthogonal and the extent of Bloch vector is the maximum for these two cases so that the maximum field entropy always remains at its maximum value and the degree of the entanglement of atom-field is the largest (see Figs.1(a) and (b)). With \( r \) reduced, \( \Delta_N \) increased, \( S(0) \) and \( \Omega_s(0) \) are no longer orthogonal and the extent of Bloch vector's motion is reduced. This leads to the values of the maximum field entropy and the degree of entanglement of atom-field decreased as the Stark shift parameter \( r \) reduced under the condition the atom is initially in excite state. When the atom is initially trapping state (namely \( \theta = \frac{\pi}{3}, \varphi = 2\beta = 0 \), Bloch vector \( S(0) = \{\frac{1}{2}, 0, 0\}, \Omega_s(0) = \{2g\tilde{n}, 0, \tilde{n}\Delta_N\} \). The motion extent of \( S(t) \) is also dependent of the effective detuning \( \Delta_N \). When
\( \beta_1 = \beta_2 = 0 \) (in the absence of the Stark shift) or \( r = 1.0(\beta_1 = \beta_2), \Delta_N = 0.0 \) and the vector \( S(0) \) is parallel to \( \Omega_s(0) \), the motion extent of the \( S(t) \) tends to zero, and the values of the maximum field entropy obviously reduced (see Figs. 2(a), (b)). With parameter \( r \) decreased, \( \Delta_N \) increased and the vector \( S(0) \) and \( \Omega_s(0) \) are no longer parallel. The greater the effective detuning \( \Delta_N \) (the smaller the parameter \( r \)), the larger the extent of the \( S(t) \), this leads to the values of maximum field entropy and the degree of entanglement of the atom-field increased as parameter \( r \) decreased (see Figs. 2(c), (d)) under the atom is initially in trapping state. Furthermore, when \( \theta = \pi/2 \), relative phase \( \varphi - 2\beta = \pi/2 \) is chosen, \( S(0) = \{ \frac{1}{2}, 0, 0 \}, \Omega_s(0) = \{ 0, 2g\bar{n}, \bar{n}\Delta_N \} \) are completely orthogonal irrespective of values of \( \Delta_N \). Therefore, at this case, the extent of \( S \)'s motion and the values of maximum field entropy are independent of the influences of the Stark shift, and are maximum at all (see Figs. 3(a)-(d)).

Acknowledgments

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References


The figure captions

Figs. 1 Effects of the Stark shift on the evolution of the field entropy. \( \theta = 0 \), atom initially in excited state, field in the coherent state with mean photon numbers \( \bar{n} = 20 \), (a), no Stark shift \((\beta_1 = \beta_2 = 0)\); (b), \( r = 1(\beta_1 = \beta_2) \); (c), \( r = 0.5 \); (d), \( r = 0.3 \).

Figs. 2 Effects of the Stark shift on the evolution of the field entropy. \( \theta = \pi/2 \), \( \varphi - 2\beta = 0 \), atom initially in trapping state, the rest parameters as Figs. 1. (a), no Stark shift \((\beta_1 = \beta_2 = 0)\); (b), \( r = 1(\beta_1 = \beta_2) \); (c), \( r = 0.5 \); (d), \( r = 0.3 \).

Figs. 3 Effects of the Stark shift on the evolution of the field entropy. The same as Figs. 2 but \( \varphi - 2\beta = \pi/2 \). (a), no Stark shift \((\beta_1 = \beta_2 = 0)\); (b), \( r = 1(\beta_1 = \beta_2) \); (c), \( r = 0.5 \); (d), \( r = 0.3 \).
Figs. 3