

*Handwritten signature or initials*

**MODAL CORRECTION METHOD FOR  
DYNAMICALLY INDUCED ERRORS  
IN WIND-TUNNEL MODEL ATTITUDE  
MEASUREMENTS**

R. D. Buehrle  
NASA Langley Research Center  
Hampton, VA

C. P. Young, Jr.  
North Carolina State University  
Raleigh, NC

**13th International Modal Analysis Conference**  
February 13-16, 1995 / Nashville, TN

Sponsored by:  
**Society for Experimental Mechanics, INC.**  
Bethel, CT 06801  
and  
**Union College**  
Schenectady, NY 12308



# Modal Correction Method for Dynamically Induced Errors in Wind Tunnel Model Attitude Measurements

Ralph D. Buehrle  
Aerospace Engineer  
NASA Langley Research Center  
Hampton, Virginia 23681

Clarence P. Young Jr.  
Visiting Associate Professor  
North Carolina State University  
Raleigh, North Carolina

## ABSTRACT

This paper describes a method for correcting the dynamically induced bias errors in wind tunnel model attitude measurements using measured modal properties of the model system. At NASA Langley Research Center, the predominant instrumentation used to measure model attitude is a servo-accelerometer device that senses the model attitude with respect to the local vertical. Under smooth wind tunnel operating conditions, this inertial device can measure the model attitude with an accuracy of 0.01 degree. During wind tunnel tests when the model is responding at high dynamic amplitudes, the inertial device also senses the centrifugal acceleration associated with model vibration. This centrifugal acceleration results in a bias error in the model attitude measurement. A study of the response of a cantilevered model system to a simulated dynamic environment shows significant bias error in the model attitude measurement can occur and is vibration mode and amplitude dependent. For each vibration mode contributing to the bias error, the error is estimated from the measured modal properties and tangential accelerations at the model attitude device. Linear superposition is used to combine the bias estimates for individual modes to determine the overall bias error as a function of time. The modal correction model predicts the bias error to a high degree of accuracy for the vibration modes characterized in the simulated dynamic environment.

## NOMENCLATURE

AOA	angle of attack
$A_{fil}$	filtered AOA signal
$A_r$	peak acceleration for $r^{th}$ vibration mode
$a_n(t)$	time dependent normal acceleration
$a_t(t)$	time dependent tangential acceleration
$a_x(t)$	time dependent longitudinal acceleration

$A_{unf}(t)$	time dependent unfiltered AOA signal
$g$	gravitational constant
Hz	Hertz
$\rho_r$	effective radius of $r^{th}$ vibration mode
$m$	number of included modes
$r$	current mode number
$t$	time in seconds
$v_r(t)$	time dependent velocity for $r^{th}$ mode
$V_r$	peak velocity for $r^{th}$ mode
$\omega_r$	circular frequency of $r^{th}$ mode
$y_r(t)$	time dependent displacement for $r^{th}$ mode
$Y_r$	peak displacement for $r^{th}$ mode
$\frac{d}{dt}$	derivative with respect to time
$\alpha$	angle of attack

## 1. INTRODUCTION

The predominant instrumentation used to measure model attitude or angle of attack (AOA) in wind tunnel testing at NASA Langley Research Center is described in reference 1. Figure 1 shows a schematic diagram of a typical model configuration with the AOA package installed in the nose of the model. The AOA package uses a servo-accelerometer with its sensitive axis parallel with the longitudinal axis of the model to determine the change in model attitude relative to the local vertical. For quasi-static conditions, this inertial sensor provides a model attitude measurement with respect to the local gravity field to an accuracy of 0.01° over a range of ±20°. Most wind tunnels use a cantilever arm (sting) to support the model. The model mounted at the end of the sting experiences dynamic oscillations due to unsteady flows that induce centrifugal accelerations on the inertial

AOA package and result in angle of attack measurement errors. The problem of the inertial device sensitivity to model vibrations is briefly discussed in reference 1.

The National Transonic Facility (NTF) [2] is a cryogenic transonic wind tunnel located at NASA Langley Research Center and has the capability for testing models at Reynolds number up to 140 million at Mach 1 and dynamic pressure up to 3.3 atmospheres. Severe model vibrations have been encountered on a number of models since the tunnel began operation. References 3 through 6 document studies of model and model support vibrations in the facility. Reference 7 documents results of an experimental study conducted in 1993 at the NTF to study the inertial angle of attack (AOA) sensor response to a simulated dynamic environment. The experimental study [7] clearly established that AOA bias error is due to centrifugal forces associated with model vibration. During wind-off dynamic tests, bias errors over an order of magnitude greater than the desired device accuracy of 0.01 degree were measured. The bias error was found to be dependent on the vibration mode and amplitude. The study revealed the complexity of the problem when multiple vibration modes were present involving both pitch and yaw motions. A first-order correction model was devised from the problem physics which gave good estimates of bias error when compared with test results.

Although the reference 7 study was conducted at the NTF, the AOA measurement error due to dynamics is not unique to this facility or to cryogenic facilities. The problem exists anytime model attitude is being measured by an inertial device in the presence of significant model system vibrations. The amount of error in the inertial model attitude measurement is dependent on the model system dynamics (i.e. will vary for each model system) and is very difficult to quantify during actual wind tunnel tests.

This report describes a time domain method for correcting the dynamically induced bias errors in wind tunnel model attitude measurements using measured modal properties and tangential accelerations at the model attitude device. The physics of the problem are detailed along with the correction methodology. The experimental setup and results for wind-off ground vibration tests on a model system are presented to validate the technique.

## 2.0 Physics of Problem

The physics of the problem are studied by considering the response of a single yaw mode as simple harmonic motion as depicted in figure 2. For the system shown with natural frequency of oscillation,  $\omega_r$ , the AOA package displacement,  $y_r(t)$ , and velocity,  $v_r(t)$ , can be written:

$$y_r(t) = Y_r \sin(\omega_r t) \quad (1)$$

$$v_r(t) = \frac{dy_r}{dt} = V_r \cos(\omega_r t); \text{ where } V_r = Y_r \omega_r \quad (2)$$

The corresponding tangential and normal acceleration components,  $a_t$  and  $a_n$ , are:

$$a_t(t) = \frac{dv_r}{dt} = A_r \sin(\omega_r t); \text{ where } A_r = -Y_r \omega_r^2 \quad (3)$$

$$a_n(t) = \frac{v_r^2(t)}{\rho_r} = \frac{V_r^2}{2\rho_r} (1 + \cos(2\omega_r t)) \quad (4)$$

Rewriting equation (4) in terms of peak acceleration gives,

$$a_n(t) = \frac{A_r^2}{2\omega_r^2 \rho_r} (1 + \cos(2\omega_r t)) \quad (5)$$

The vibration induced normal acceleration results in the AOA package sensing a centrifugal acceleration coincident with its sensitive axis (i.e. the longitudinal axis of the model). The AOA package output prior to filtering becomes:

$$A_{unf}(t) = g \sin \alpha - \frac{V_r^2}{2\rho_r} (1 + \cos(2\omega_r t)) + a_x(t) \quad (6)$$

The first term on the right hand side of the equation is the gravitational acceleration due to the model attitude,  $\alpha$ , relative to the local vertical. The second term is the centrifugal acceleration (from equation (4)) caused by the model yaw motion. Accelerations resulting from flow induced longitudinal model vibrations (typically high frequency) are represented by the third term. To obtain the mean angle, the AOA signal is low pass filtered.

$$A_{fil} \approx g \sin \alpha - \frac{V_r^2}{2\rho_r} \quad (7)$$

It is important to note that the model vibration causes a bias error in the model attitude measurement that cannot be removed by filtering or data averaging. Pitch motion results in a corresponding bias error. If the vibration response is composed of multiple yaw and pitch modes, the total bias error will be a linear summation of the error contributions for the  $m$  modes.

$$A_{fil} \approx g \sin \alpha - \sum_{r=1}^m \frac{V_r^2}{2\rho_r} \quad (8)$$

Or, in terms of the peak acceleration, from equation (5),

$$A_{fil} \approx g \sin \alpha - \sum_{r=1}^m \frac{A_r^2}{2\omega_r^2 \rho_r} \quad (9)$$

The above discussion is based on the case of continuous sinusoidal model motion. In the wind tunnel, the data is non-stationary and random in nature. This results in a time varying bias error that is dependent on the number of modes participating and the amplitudes of motion for those modes (i.e.  $V_r(t)$ ,  $A_r(t)$ ,  $A_{fil}(t)$ ).

### 3.0 Bias Error Correction

Several methods have been proposed to correct for this bias error using measured tangential accelerations at the AOA sensor location due to model yaw and pitch motion. One method is to measure the natural frequencies from the frequency spectrum of the tangential accelerations and then determine the bias magnitudes by measuring the magnitude of the second harmonic components from the frequency spectrum of the unfiltered AOA signal (see equation (6)). This technique may be difficult to implement due to the participation of multiple modes and the required data accuracy to measure the small magnitudes at the second harmonic frequency. Young et. al. [7] determine the natural frequencies,  $\omega_r$ , and corresponding peak acceleration magnitudes,  $A_r$ , from the frequency spectrums of the yaw and pitch acceleration measurements. The bias error is then calculated from the second term of equation (9). The vibration mode effective radius,  $\rho_r$ , was estimated from sinusoidal input-output data taken during wind-off ground vibration tests and later confirmed with measured mode shapes. Both techniques use frequency domain signal processing which is suitable for the stationary data observed in the wind-off tests but is questionable when evaluating the non-stationary data observed during wind tunnel testing.

A third bias correction technique was developed and used at the National Aerospace Laboratory (NLR) in the Netherlands. This technique was developed for one vibration mode in each the yaw and pitch plane. Two additional accelerometers are used to measure the tangential accelerations due to the yaw and pitch motion of the model. The tangential accelerations are integrated to obtain velocity, squared, and divided by a scale factor to compensate for the effective radius of the vibration mode. This signal is then added to the unfiltered AOA output to cancel the second term of equation (6). The corrected signal is then low pass filtered to obtain the corrected mean AOA measurement. The mode radius in the yaw and pitch plane is determined by tuning a potentiometer while manually exciting the model in the yaw and pitch plane, respectively. This technique does not address the case where multiple yaw and pitch vibration modes are present.

During dynamic testing of a model system [7], it became evident that multiple modes may be contributing to the vibration induced bias error. The "modal correction method" is being developed to extend the time domain technique used at NLR to compensate for multiple yaw and pitch vibration modes. This technique estimates the mode effective radii using measured modal properties of the model system. Accelerations measured tangent to the AOA package sensitive axis are band-pass filtered to isolate individual modes. The filtered signal is then integrated, squared, and divided by the corresponding mode effective radii to determine the bias error for a particular mode

(second term of equation (6)). To compensate for multiple modes, a linear superposition of the individual mode effects is used to estimate the total bias error as a function of time. The estimated total bias error is then added to the AOA output prior to filtering. This corrected signal is low pass filtered to obtain the corrected mean AOA measurement.

The mode radii are estimated by assuming the fuselage moves as a rigid body and using a least square fit of the fuselage mode shape coefficients to determine an effective point of rotation for the mode. The vibration mode effective radii are then taken to be the distance from the point of rotation to the inertial AOA sensor location in the model fuselage. This may be more easily understood by examining the measured mode shapes that are shown in figures 3 and 4. Figure 3 shows the 9.0 Hz model/sting bending mode in the yaw plane with a projected point of rotation aft of the model fuselage. Figure 4 shows the 29.8 Hz model yaw mode with a projected point of rotation forward of the AOA package. For the 29.8 Hz mode, the effective radii is negative. Previously, it was assumed that the centrifugal accelerations for all modes would act in the same direction, i.e. the point of rotation was always aft of the AOA package. However, this data indicated that the dynamically induced errors could be positive or negative dependent on the mode shape (mode effective radii).

The rigid body assumption used in the mode radii estimation appears to be satisfactory for the low frequency (<50 Hz) modes that are being evaluated. A second assumption is that the mode shapes do not change significantly under the wind tunnel test conditions. This enables wind-off estimates of the mode effective radii to be used for on-line correction of the model attitude measurement during wind tunnel testing. Further work is required to validate this assumption.

## 4.0 EXPERIMENTAL VALIDATION

A model system that experienced high levels of model vibration [6] during previous wind tunnel tests was selected to investigate the effects of dynamics on the inertial AOA package. Ground vibration tests were conducted under wind-off conditions to better understand the dynamic effects. The test setup, procedures and results for the ground vibration test are described in the following sections.

### 4.1 TEST SETUP

The model system was installed in a model assembly bay at the NTF. The mounting consisted of a "rigidly" supported cantilever (sting), that is positioned by a pitch-roll-translation mechanism. The model is attached to the sting through a six component strain gage balance (see figure 1).

The model was instrumented with an inertial AOA package [1] maintained at a constant temperature of 165°F. The signal conditioner for the AOA package provides both an

unfiltered (0-300 Hz bandwidth) and filtered (0-0.25 Hz bandwidth) signal. Two miniature accelerometers were installed on the face of the AOA package to measure yaw and pitch motions. In addition, four accelerometers were used to measure the model fuselage yaw and pitch motion.

An electrodynamic shaker was used to excite the model/sting through a single point force linkage attached to the model approximately 36 inches forward of the balance moment center. Sine, modulated sine and band limited random shaker input were used. A dynamic signal analyzer was used to provide the shaker stimulus and record the shaker force input, model force balance outputs, AOA filtered (static) and unfiltered (dynamic) outputs, and model accelerations. This system was used to monitor the model yaw and pitch moments which established the dynamic test conditions for acquiring model attitude measurements.

## 4.2 TEST PROCEDURE

The model was set at a prescribed angle of attack under static conditions. The model system natural frequencies were identified using sine sweep excitation of the model in the pitch and yaw planes. For each natural frequency of interest, a sinusoidal forced response test was conducted by controlling the shaker input amplitude to provide a defined peak to peak pitch or yaw moment on the model force balance. The control test variables were pitch moment for modes that had predominantly pitch motion, and yaw moment for modes that had predominantly yaw motion. The model attitude was measured at a series of moment amplitude levels for sinusoidal excitation at a prescribed natural frequency of the model system. Sinusoidal forced response tests were conducted with the model set at angles of 0°, 4.3° and 6°.

In addition to the sinusoidal forced response tests, the model attitude was measured for modulated sine and random excitation tests. The modulated sine and random excitations/responses are more representative of the model dynamics observed in actual wind tunnel tests. The majority of the modulated sine tests were conducted with a 0.25 Hz modulation of the first natural frequency in the pitch and yaw planes. The random excitation was only evaluated in the pitch plane for the model at a nominal angle of 0°. In each case, the inertial AOA package was used to measure the model attitude for a series of moment amplitude levels.

## 4.3 DISCUSSION OF TEST RESULTS

An attempt was made to excite three modes in each of the yaw and pitch planes of the model system. The modes of interest were determined from a modal survey of the model system and are listed in Table 1. Measurements taken during previous wind tunnel tests indicated that the primary modes being excited were at approximately 8-10 Hz and 28-30 Hz [6]. The radii for the vibration modes were

estimated using a least square fit of the modal deformations as described in section 3.0. These predictions indicated that the radii may be positive or negative dependent on the vibration mode shape. For the case where the radius is positive, the point of rotation for the vibration mode is ahead of the AOA package. The significance of the sign of the radii is that the bias error effect may be positive or negative dependent upon the vibration mode being excited. This is demonstrated by the response for the yaw plane modes shown in figures 5 and 6. For the 9.0 Hz yaw mode, the indicated model angle change is negative when the model is being driven with sinusoidal excitation at the mode natural frequency and then returns to its nominal angle when the shaker system is shutoff. The 29.8 Hz yaw mode (which has a negative radius value) shows an indicated positive angle change when the model is being driven with sinusoidal excitation at the mode natural frequency and then returns to its nominal angle when the shaker system is shutoff. The excitation system was adequate to show the above trends, however, the higher frequency modes (>10 Hz) were difficult to drive and only the first mode in each the yaw and pitch planes were excited to levels that showed significant shifts in the indicated model attitude from the onboard inertial AOA package. Difficulty in driving the higher frequency modes is attributed to the rigid backstop support in the model assembly bay. During previous wind-tunnel tests[6], the model coupled with the model support structure resulting in high dynamic yaw moments with energy in the 28-30 Hz band. This points out the need to do dynamic testing with the model installed in the tunnel.

The results of sinusoidal excitation tests (model at nominal angle of 0°) for the first mode in each the yaw and pitch plane is shown in figures 7 and 8. For a set excitation level, time domain data were acquired and stored using the dynamic signal analyzer. These data were transferred to a personal computer where a software routine, written as an M-file in the MATLAB [8] language, was used to estimate the bias error in the inertial device. The yaw and pitch plane accelerations were bandpass filtered, integrated to obtain velocity, squared, divided by the mode effective radii, and scaled to obtain the equivalent angle change in degrees. The estimated bias error is in good agreement with the indicated mean angle change measured with the onboard inertial AOA sensor. If the estimated bias error is subtracted from the indicated model angle change the error would be reduced from a maximum of -0.146° to -0.009° for the first mode in the yaw plane (y) and from -0.175° to -0.006° in the pitch plane(z). These corrections are within the AOA accuracy requirement of 0.01°. Similar results were obtained for the sinusoidal input tests with the model at nominal angles of 4.3° and 6°.

In addition to the sinusoidal tests, the bias error was examined for modulated sine and random inputs. Figure 9 shows the estimated and measured bias error as a function of time for a 9.2 Hz pitch excitation with a 0.25 Hz

modulation. Excellent agreement is obtained with the difference between the estimated and measured bias being less than 0.005°. Modulated sine tests were conducted for the first mode in each the y and z axes at several excitation amplitude levels and consistent results were obtained between the measured and predicted bias errors for all cases. In the pitch plane, two levels of random excitation were input to the model. An eight second record of the inertial AOA sensor response for the highest level random excitation is shown in figure 10. The random response measured by the pitch accelerometer on the face of the AOA package was composed of primarily 9.2 Hz response. The bias estimate based on only the 9.2 Hz mode contribution is also shown in figure 10. Again, the estimated and measured bias are in very good agreement.

### 5.0 CONCLUDING REMARKS

A method for correcting the dynamically induced bias errors in inertial wind-tunnel model attitude measurements is presented. This modal correction method uses the measured modal properties of the model system and accelerations tangent to the AOA sensor to estimate the bias errors. Multiple modes are addressed by separating the mode effects using band-pass filtering and then using a linear superposition of the estimated bias errors for the individual modes. Very good agreement between the estimated and measured bias errors was found for the sinusoidal, modulated sinusoidal, and random input wind-off vibration tests. For these tests, the modal correction method was used to estimate the bias errors to an accuracy within the AOA device accuracy of 0.01°.

Vibration mode effective radii are estimated using measured modal properties. It was found that the mode effective radii can be positive or negative dependent on the mode shape. Previously it was assumed that the bias errors for all modes would act in the same direction (i.e. positive radii; point of rotation always aft of the AOA package). However, data are presented that shows the dynamically induced errors may be positive or negative depending on the mode shape.

It was found that some of the model system vibration modes excited to high levels during wind tunnel tests could not be excited very well in the model assembly bay. This is attributed to boundary conditions at the assembly bay model support which are quite different from the wind-tunnel model support system. This points out the need for doing dynamic testing on the model system after installation in the tunnel to assure the most accurate measurement of the vibration mode radii and for dynamic calibration of AOA response characteristics. Further work is required to test the modal bias correction method for multiple mode excitation/response and validate the method during actual wind-tunnel tests. The assumption that the mode shapes do not change significantly under wind tunnel flow conditions also needs to be validated.

### REFERENCES

1. Finley, T., and Tchong, P.: Model Attitude Measurements at NASA Langley Research Center. AIAA-92-0763, 1992.
2. Fuller, D. E.: Guide to Users of the National Transonic Facility. NASA TM-83124, July 1981.
3. Strganac, T. W.: A Study of the Aeroelastic Stability for the Model Support System of the National Transonic Facility. AIAA-88-2033, 1988.
4. Whitlow, W., Jr.; Bennet, R. M.; and Strganac, T. W.: Analysis of Vibrations of the National Transonic Facility Model Support System Using a 3-D Aeroelastic Code. AIAA-89-2207, 1989.
5. Young, C. P., Jr.; Popernack, T. G.; Gloss, B. B.: National Transonic Facility Model and Model Support Vibration Problems. AIAA-90-1416, 1990.
6. Buehrle, R. D.; Young, C. P., Jr.; Balakrishna, S.; and Kilgore, W. A.: Experimental Study of Dynamic Interaction Between Model Support Structure and a High Speed Research Model in the National Transonic Facility. AIAA-94-1623, 1994.
7. Young, C. P., Jr.; Buehrle, R. D.; Balakrishna, S.; and Kilgore, W. A.: Effects of Vibration on Inertial Wind-Tunnel Model Attitude Measurement Devices. NASA TM 109083, July 1994.
8. MATLAB Reference Guide, The Math Works Inc., August, 1992

Table 1  
Model System Vibration Mode Characteristics

Mode No.	Natural Frequency (Hz)	Effective Radius (centimeter)	Mode Description
1	9.0	78.7	Sting Bending in Yaw
2	9.2	76.7	Sting Bending in Pitch
3	19.5	0.46	Model Yaw & Sting 1st Bending
4	20.3	-2.74	Model Pitch & Sting 1st Bending
5	29.8	-18.1	Model Yaw & Sting 2nd Bending
6	34.2	-19.4	Model Pitch & Sting 2nd Bending

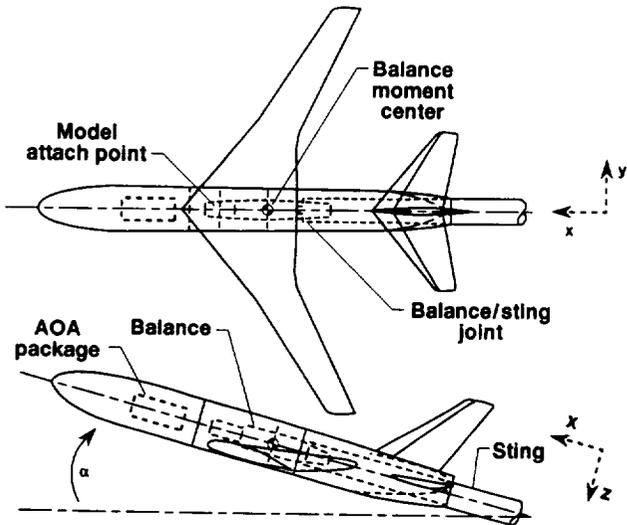


Figure 1.- Schematic of wind tunnel model system.

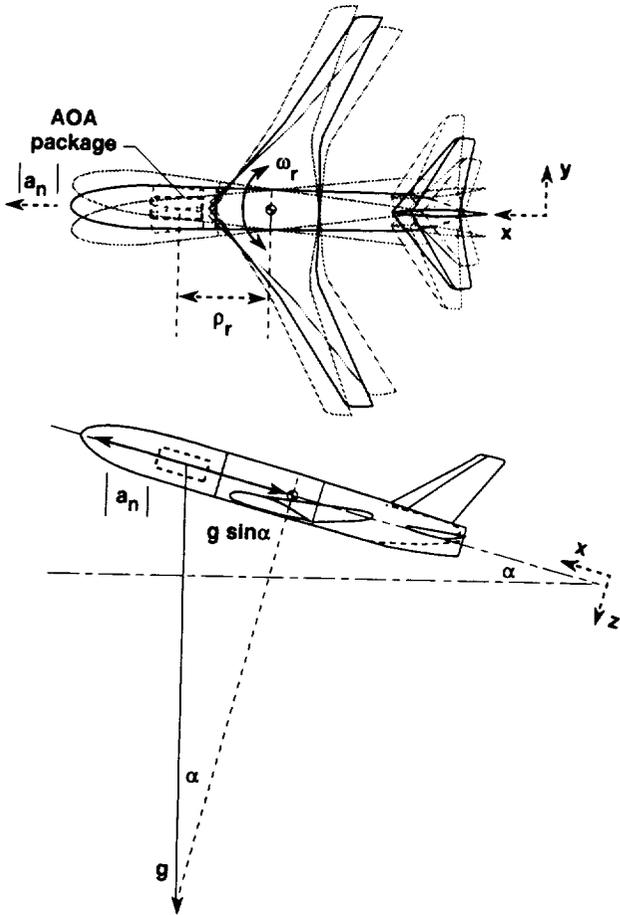


Figure 2.- Effect of vibration on inertial model attitude measurement.

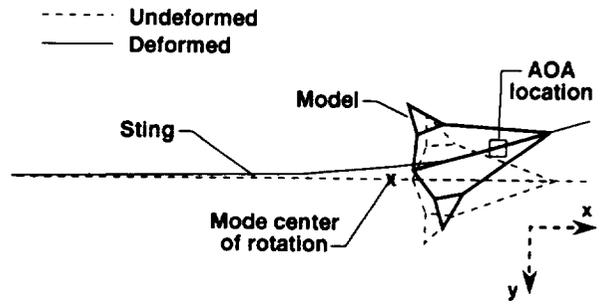


Figure 3.- Sting bending in yaw plane, 9.0 Hz vibration mode.

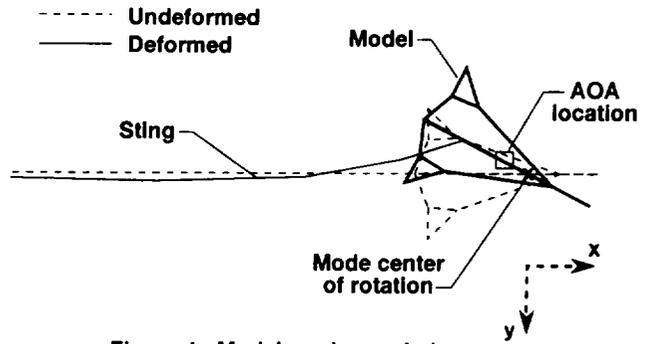


Figure 4.- Model yawing on balance, 29.8 Hz vibration mode.

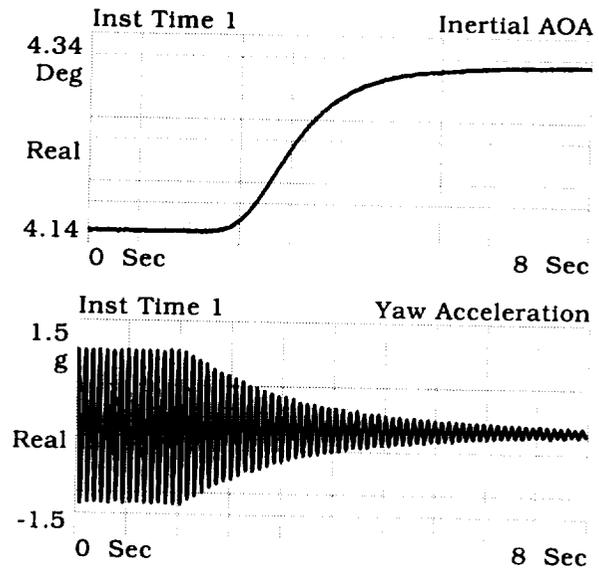


Figure 5. Inertial AOA measurement and yaw acceleration versus time for 9.0 Hz sinusoidal input in yaw plane.

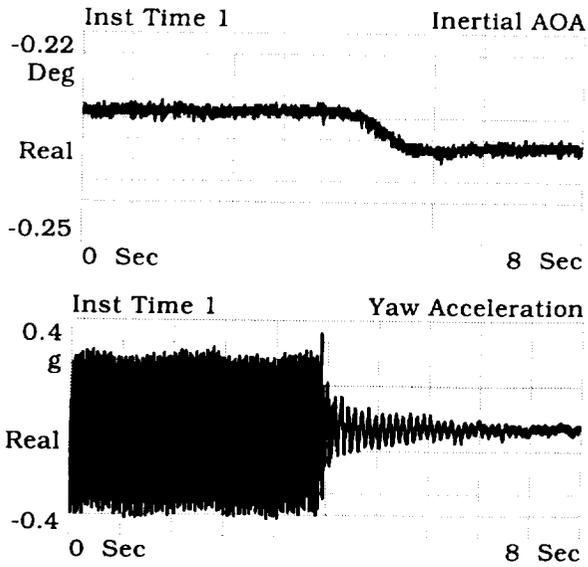


Figure 6. Inertial AOA measurement and yaw acceleration versus time for 29.8 Hz sinusoidal input in yaw plane.

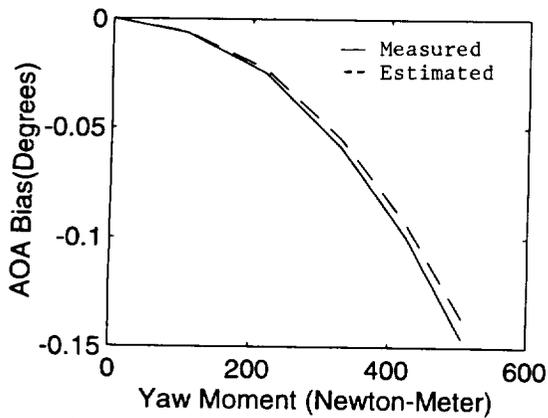


Figure 7. Measured and estimated AOA bias error versus yaw moment for 9.0 Hz yaw mode.

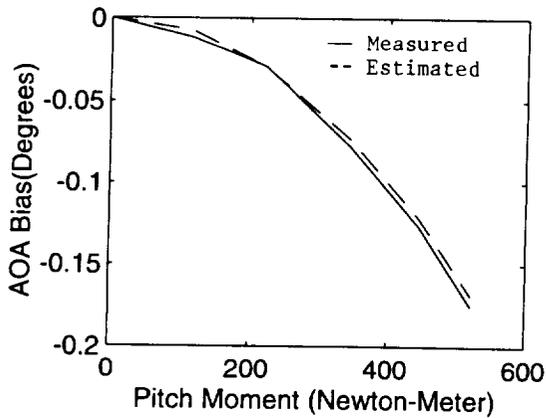


Figure 8. Measured and estimated AOA bias error versus pitch moment for 9.2 Hz pitch mode.

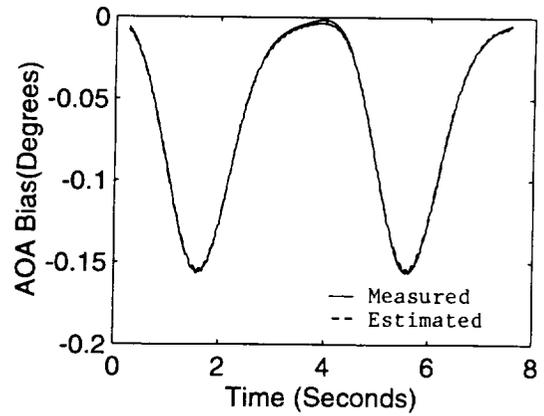


Figure 9. Measured and estimated AOA bias error versus time for 9.2 Hz sinusoidal excitation in pitch with 0.25 Hz modulation, maximum pitch moment of 509 Newton-meters.

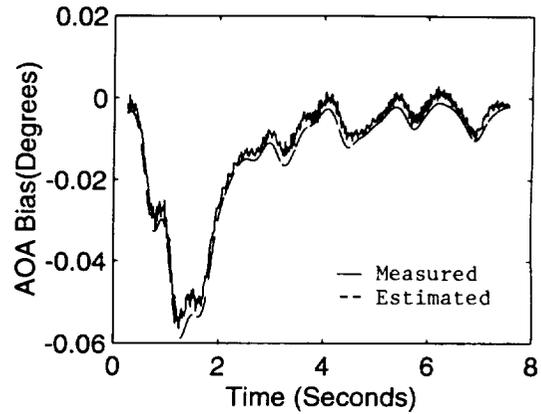


Figure 10. Measured and estimated AOA bias error versus time for random excitation in pitch.





