Optimization Methodology for Unconventional Rocket Nozzle Design

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Abstract

Several current rocket engine concepts such as the bell-annular tripropellant engine, and the linear aerospike being proposed for the X-33, require unconventional three-dimensional rocket nozzles which must conform to rectangular or sector-shaped envelopes to meet integration constraints. These types of nozzles exist outside the current experience database, therefore, development of efficient design methods for these propulsion concepts is critical to the success of launch vehicle programs.

Several approaches for optimizing rocket nozzles, including streamline tracing techniques, and the coupling of CFD analysis to optimization algorithms are described. The relative strengths and weaknesses of four classes of optimization algorithms are discussed: Gradient based methods, genetic algorithms, simplex methods, and surface response methods. Additionally, a streamline tracing technique, which provides a very computationally efficient means of defining a three-dimensional contour, is discussed.

Gradient based schemes generally rely on a gradient evaluation at the current design point to determine the search direction required for objective function minimization with either constrained or unconstrained design variables. This type of technique is good at rapidly achieving an optimum if the objective function is well behaved. However, it can easily be trapped in local optima or by constraints in a region far from the optimal design.

Genetic algorithms are adaptive search procedures based on the biological concept of evolution. They start with an initial set, or population, of design points and use the genetic operators of selection, crossover and mutation to converge on an optimal design. Since this method searches from a set of designs rather than a single design, it can uncover different "families" of good designs.

Surface response methods utilize a limited number of runs to construct a model of the design space. This model is then used to determine an optimal design. Examples of such methods include Taguchi, neural networks, and regression models. If accurate models of the design space can be constructed, these methods become very attractive for problems where function evaluations are computationally expensive.

Streamline tracing provides a rapid means of determining a nozzle design which exhibits good performance but has an arbitrary exit shape. By using an axisymmetric optimum or ideal nozzle as a baseline, a shape can be inscribed onto the exit plane of the nozzle and numerous streamlines may be traced back to the plenum. These streamlines may then be used to define a nozzle contour, which will exhibit the same inviscid flow characteristics as the nozzle from which it was traced.

The performance of the various optimization methods on thrust optimization problems for tripropellant and aerospike concepts is assessed and recommendations are made for future development efforts.
OPTIMIZATION METHODOLOGY FOR UNCONVENTIONAL ROCKET NOZZLE DESIGN

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OPTIMIZATION METHODOLOGY FOR UNCONVENTIONAL ROCKET NOZZLE DESIGN

• WHY OPTIMIZATION?

• EFFICIENT AERO DESIGN TOOLS CURRENTLY EXIST FOR 2-D AND AXISYMMETRIC NOZZLES

• CURRENT ROCKET ENGINE CONCEPTS CAN BENEFIT FROM 3-D NOZZLE DESIGNS

• A POWERFUL DESIGN TOOL IS BEING CREATED BY COUPLING CFD TO OPTIMIZATION METHODS

• CHARACTERISTICS OF THRUST OPTIMIZATION PROBLEM:

• RELATIVELY LITTLE ROOM FOR IMPROVEMENT (<3%) - NEED ACCURATE FLOWSOLVER

• MULTIPLE OPTIMA

• ROUGHNESS
OBJECTIVE

- EVALUATE EFFECTIVENESS OF VARIOUS OPTIMIZATION ALGORITHMS WHEN APPLIED TO THRUST PROBLEM
  - GRADIENT BASED METHODS
  - GENETIC ALGORITHMS
  - SIMPLEX METHOD
  - SURFACE RESPONSE METHODS
  - STREAMLINE TRACING
OPTIMIZATION METHODOLOGY DESCRIPTION

- **GRADIENT BASED METHODS**
  - PERTURB EACH DESIGN VARIABLE SLIGHTLY TO GENERATE FINITE DIFFERENCE GRADIENT
  - PROS - RAPID SOLUTION FOR SMOOTH FUNCTIONS
  - CONS - MAY GET TRAPPED IN LOCAL OPTIMA
    - DOES NOT HANDLE ROUGHNESS WELL

- **GENETIC ALGORITHMS**
  - UTILIZE GENETIC OPERATORS OF MUTATION, CROSSOVER, & SELECTION TO EVOLVE TOWARDS OPTIMUM DESIGN
  - PROS - ROBUST METHOD GOOD AT FINDING GLOBAL OPTIMUM AMIDST LOCAL OPTIMA
    - HANDLES ROUGHNESS WELL
    - CAN FIND "FAMILIES" OF GOOD DESIGNS
  - CONS - COMPUTATIONALLY EXPENSIVE
OPTIMIZATION METHODOLOGY DESCRIPTION

- **SIMPLEX METHOD**
  - CONSTRUCT SIMPLEX FROM N+1 POINTS
  - EVALUATE ALL POINTS - MOVE WORST ONE TO OPPOSITE SIDE
  - IF NEW POINT IS STILL WORST, SHRINK SIMPLEX

- PROS - CAN HANDLE ROUGHNESS

- CONS - MAY GET TRAPPED IN LOCAL OPTIMA
  - SLOW ON SMOOTH FUNCTIONS
  - NOT NATURALLY PARALLEL

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STEP 3: SHRINK

VARIABLE 2

VARIABLE 1

Rockwell Aerospace
Rocketdyne

CFD Technology Center
OPTIMIZATION METHODOLOGY DESCRIPTION

- SURFACE RESPONSE METHODS
  - TAGUCHI METHODS
    - ORTHOGONAL ARRAYS ASSUME GLOBAL LINEARITY
      - EFFECT OF A + EFFECT OF B = EFFECT OF A+B
  - NEURAL NETS
    - GOOD FOR MODELING NONLINEAR RESPONSES
  - REGRESSION MODELS - FIRST ORDER, SECOND ORDER

- PROS - SIMULATES DESIGN SPACE WITH SMALL NUMBER OF CFD EVALUATIONS
  - CAN FILTER OUT ROUGHNESS

- CONS - MODEL MAY NOT ACCURATELY SIMULATE DESIGN SPACE
COUPLING CFD TO OPTIMIZATION ALGORITHMS IN THE REAL WORLD

- CFD AUTOMATION
  - GRID GENERATION
  - CONVERGENCE CHECKING
  - FLEXIBLE SOLUTION STRATEGY

- PARALLEL PROCESSING
  - MOST OPTIMIZATION SCHEMES ARE NATURALLY PARALLEL
  - DISTRIBUTED HETEROGENEOUS ENVIRONMENT
  - FAULT TOLERANCE

- COPING WITH CODE CRASHES
  - ASSIGNING PERFORMANCE LEVEL
  - SWITCH OPTIMIZATION ALGORITHMS
## COMPARISON OF OPTIMIZATION METHODS FOR NOZZLE DESIGN APPLICATIONS

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<thead>
<tr>
<th></th>
<th>GRADIENT BASED</th>
<th>GENETIC ALGORITHM</th>
<th>SIMPLEX</th>
<th>TAGUCHI</th>
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- NO ALGORITHM IS PERFECT, ALL HAVE STRENGTHS AND WEAKNESSES
- AN INTELLIGENT HYBRID COULD TAKE ADVANTAGE OF THE BEST ASPECTS OF EACH TECHNIQUE
3-D THRUST CELL OPTIMIZATION

- OBJECTIVE
  - MAXIMIZE: THRUST(THRUSTER ONLY) / SYSTEM WEIGHT
  - MINIMIZE PEAK HEAT LOAD

- OPTIMIZATION METHODS
  - TAGUCHI L32 & L64 MATRICES
  - GENETIC ALGORITHM
  - 15-21 DESIGN VARIABLES

- 3-D MOC EVALUATIONS
  - 460 FOR TAGUCHI
  - 1000 FOR GENETIC

- IMPROVEMENT OVER BASELINE
  - 4.6% IN THRUST / WEIGHT

Thrust Cell Taguchi Optimum
Mach Number Contours

Mach Number Minimum = 1.0
Mach Number Maximum = 4.0

Mach Number Minimum = 3.0
Mach Number Maximum = 4.0

Min Mach Number Max

TAGUCHI

CONVERGENCE HISTORIES

GENETIC

CFD Technology Center

Rockwell Aerospace
Rocketdyne
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CONCLUSIONS

- THRUST OPTIMIZATION IS DIFFICULT DUE TO MULTIPLE LOCAL OPTIMA AND ROUGHNESS OF THE OBJECTIVE FUNCTION
  - GRADIENT BASED METHODS HAVE PROVEN INEFFECTIVE
  - SIMPLEX METHODS ARE MEDIocre
  - GENETIC ALGORITHMS ARE ROBUST, BUT COSTLY
  - SURFACE RESPONSE METHODS CAN DRASTICALLY REDUCE NUMBER OF CFD RUNS REQUIRED, BUT ONLY IF MODEL IS ACCURATE

- NO SINGLE SCHEME IS A "MAGIC BULLET"
  - CURRENTLY INVESTIGATING FILTERING METHODS FOR GRADIENT BASED METHODS, INTELLIGENT HYBRID SCHEMES

- MULTI-DISCIPLINARY OPTIMIZATION IS ESSENTIAL