

## Spacecraft Expected Cost Analysis with k-out-of-n:G Subsystems

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### Nomenclature

$$\text{bin}(k-1;n,p) = \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}, \text{ the cumulative binomial function.}$$

C = the total of the cost of the subsystem itself plus the expected cost due to subsystem failure

$c_1$  = loss due to failure of the subsystem

$c_3$  = cost of a one module subsystem capable of full output

$c_4$  = cost of a module in a k-out-of-n:G (good) subsystem when k is fixed

g(k) = the function relating cost of the subsystem to the number of modules in subsystem

k = minimum number of good modules for the subsystem to be good

n = number of modules in the subsystem

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$p$  =probability that a subsystem module is good

$q$ =probability that a subsystem module fails or  $1-p$

$r$ =reliability of the whole system for other than failure of the subsystem

## INTRODUCTION

In designing a subsystem for a spacecraft, the design engineer is often faced with a number of options. These options can range from planning an inexpensive subsystem with low reliability to selecting a highly reliable system that would cost much more. How does a design engineer choose between competing subsystems? More particularly, what method can the engineer use to construct "models" that will take into consideration the various choices offered?

For example, in designing a power subsystem for a spacecraft, the engineer may choose between a power subsystem with .960 reliability and a more costly one with .995 reliability. When is the increased cost of a more reliable subsystem justified?

High reliability is not necessarily an end in itself but is desirable in order to reduce the statistically expected cost due to a subsystem failure. However, this may not be the wisest use of funds since this expected cost is not the only cost involved. The engineer should consider not only the cost of the subsystem but also assess the costs that would occur if the subsystem fails. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure, and choose the subsystem with the lowest total..

## K OUT-OF-N:G SUBSYSTEMS

We will now direct our attention to a specific type of subsystem, called a k-out-of-n:G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement for a spacecraft and may meet this requirement by having one large power module(k=1), two smaller modules(k=2), etc. If k = 4 then each module is 1/4 of the full required power. For example, an n = 6 and k = 4 subsystem would have 6 modules, each of 1/4 power and thus would have the output capability of 1.5 times the required power. The engineer chooses n and k. Selection of the different values of n and k results in different subsystems, each with different costs and reliabilities. Since each n and k yields different subsystems with different costs, we can choose the subsystem ( n and k ) which will minimize cost overall expected cost C.

The following two models illustrate the principles of the k-out-of-n:G subsystems designs. For Model 1, the following assumptions are necessary:

1. The probability of failure of any module in the system is not affected by the failure of any other module; i.e., the modules are independent.
2. Each of the modules has the same probability of success.

For Model 2 we have the assumptions noted, plus we are also free to choose k in our subsystem.

### MODEL 1

For model 1, we assume that k is fixed and that each module costs  $c_4$ . Here the engineer may choose only n. Now  $E\{\text{cost due to subsystem failure}\} = rc_1\Pr\{\text{subsystem}$

failure} =  $rc_1 \Pr\{X < k\}$ , where  $X$ , the number of good modules, has a binomial distribution with parameters  $n$  and  $p$ . Since  $C = \text{cost of subsystem} + E\{\text{cost due to subsystem failure}\}$ , then

$$C = nc_4 + rc_1 \text{bin}(k-1; p, n) \quad (1)$$

The authors have written a program CARRAC (Combined Analysis of Reliability, Redundancy and Cost- beta version available) which enables the engineer to select the best subsystems (i.e. ones with the lowest  $C$ 's) and graph  $C$  as a function of either  $p$  or  $c_1$ . Since these values are not often known precisely, this graph allows you to not only select the best subsystem for a particular value of  $p$  or  $c_1$  but also to view what happens to  $C$  for nearby values of  $p$  or  $c_1$ .

As an example, consider  $k = 1$ , that is only one module is required to be good for the subsystem to be good. The reliability of this single module is estimated to be .95 ( $p = .95$ ). Let the reliability of the system for other than failure of the subsystem be .9 ( $r = .9$ ). The cost of one module is 1 ( $c_4 = 1$ ) million dollars. The cost due to failure of this subsystem is 10 ( $c_1 = 10$ ) million dollars. Figure 1(from CARRAC) shows a plot of  $C$  for  $.79 < p < .99$  and  $n$ 's of 1 through 4. When the reliability of a single module is  $p = .95$ , then the  $n = 1$  subsystem has the lowest value of  $C$ . Therefore the best subsystem is the one with no spares.

insert figure 1

We can also see (fig. 1) that the  $n = 1$  subsystem has the lowest value of  $C$  for any  $p > .87$ . If  $p < .87$ , then  $n = 2$  (one spare) has the lowest value of  $C$ . Suppose instead that  $c_1$  (cost due to failure of the subsystem) is 50. Figure 2 shows the plot of  $C$  for  $c_1 = 50$ .

insert figure 2

We first note that if  $p = .95$ , then the  $n = 2$  subsystem (one spare) is the best. Comparing figs. 1 and 2 (at  $p = .95$ ) we see that the larger value of  $c_1$  (in figure 2) requires a larger value of  $n$ . In general, if the cost of subsystem failure increases, then more redundancy is required. If  $.84 < p < .98$ , fig. 2 shows that the  $n = 2$  subsystem is best. If  $p < .84$  then still more redundancy ( $n=3$ ) is required. If  $p > .98$ , then no redundancy ( $n=1$ ) is required.

## MODEL 2

Here the engineer is free to choose both  $n$  and  $k$ . As an example of model 2, suppose we are building a space electrical power subsystem. The engineer may build a one module subsystem ( $k=1$ ) capable of full power, a two module subsystem ( $k=2$ ) capable of full power, where each module is capable of  $1/2$  power, etc. Let  $g(k)$  represent the (generally) increased cost of building a subsystem consisting of  $k$  smaller modules rather than one large module. A rough rule of thumb says that the cost of smaller modules for a space electrical power subsystem is proportional to the electrical power raised to the .7, i.e.,

$$g(k) = k(1/k)^{.7} \quad (2)$$

(other cost functions,  $g(k)$ , are available in CARRAC). Suppose that the cost of building a single module capable of full power is 1 ( $c_3 = 1$ ). Then a subsystem consisting of a single module capable of full power would cost  $c_3g(1) = c_31(1/1)^{.7} = 1.0c_3$ , a subsystem consisting of 2 modules, each of  $1/2$  power, would cost  $c_3g(2) = c_32(1/2)^{.7} = 1.23c_3$  to build, etc. An  $n = 3$  and  $k = 2$  subsystem, i.e., one having 3 modules each of

1/2 power, would cost  $nc_3 g(k)/k = 3 \times 1.23c_3/2 = 1.85c_3$  to build.

Further suppose that the cost due to subsystem failure,  $c_1$ , is 240. Let the reliability of the system for other than failure of the subsystem be .9 ( $r = .9$ ). An estimate of  $p$ , the reliability of an individual module, is .96. For model 2,

$$C = nc_3g(k)/k + rc_1 \text{ bin}(k-1; p,n) \quad (3)$$

Figure 3 shows the best subsystems over  $p$  ranging from .89 to .99. From fig. 3, at  $p = .96$ , the  $n = 2, k = 1$  subsystem is best (lowest value of  $C$ ). If  $p < .95$ , the  $n = 4, k = 2$  subsystem is best. This flatter curve over the range of  $p$  indicates a low value for  $C$  over a wide range of  $p$ .

insert figure 3

## CONCLUSIONS

When a design engineer needs to choose among competing subsystems with differing costs and reliabilities, CARRAC serves as a useful tool for the engineer in selecting optimal  $k$ -out-of- $n$ :G subsystems. Graphs enable the engineer to explore competing near-optimal subsystems over a range of reliabilities and costs, since often these are not known precisely. CARRAC can be used to explore near optimal solutions for other cost models presented by Suich & Patterson<sup>1,2</sup>. These models are more complicated and cover time dependency, partial failures and situations with and without salvage value.

In selecting a subsystem, many factors (other than costs and reliabilities we have explored) enter into the final selection of a subsystem. However, the method of analysis we have presented is an important tool in making this final selection.

## REFERENCES

<sup>1</sup>Suich, R. and Patterson, R., "Balancing Reliability and Cost to Choose the Best Power Subsystem", NASA Technical Memorandum 104453, August, 1991.

<sup>2</sup>Suich, R. and Patterson, R., "K-out-of-n: G Systems: Some Cost Considerations", IEEE Transactions on Reliability, Vol. 40, No. 3, 1991, pp 259-264..

<sup>3</sup>Suich, R. and Patterson, R., "How Much Redundancy? - Some Cost Considerations, Including Examples for Spacecraft Systems", NASA Technical Memorandum 103197, August, 1990.

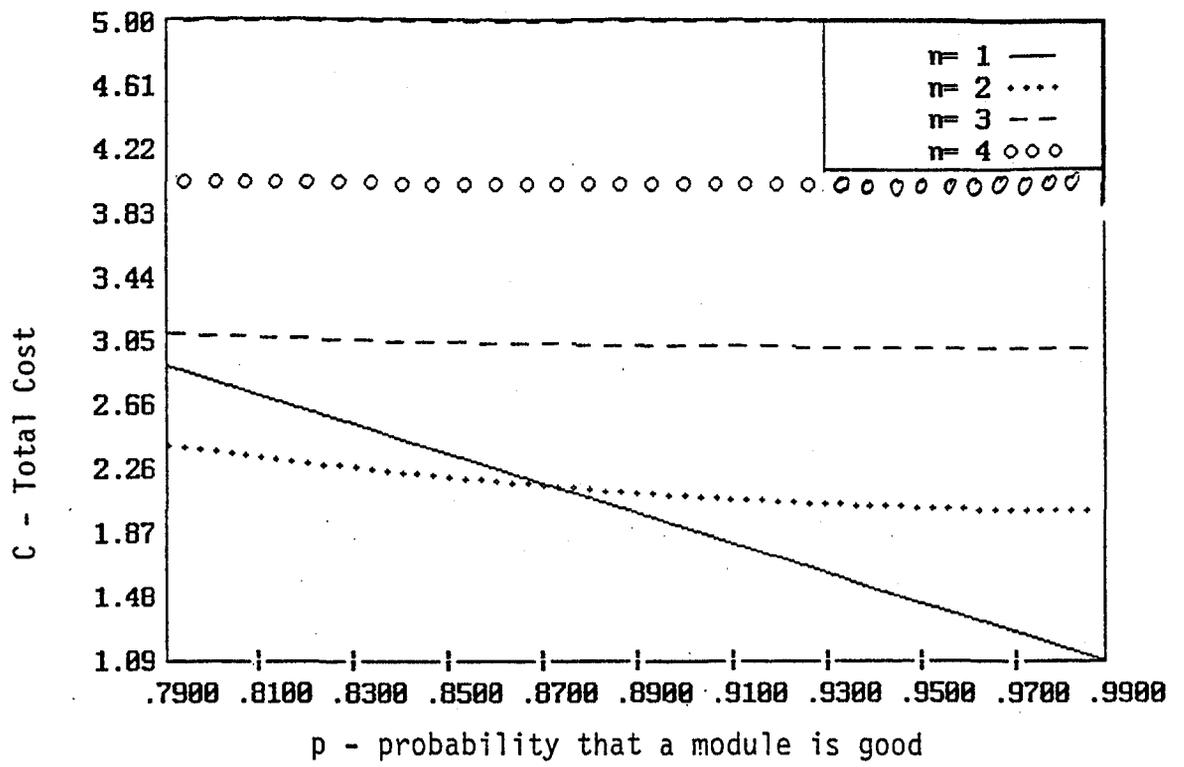


Fig. 1 - Model 1 with  $c_1 = 10$ ,  $c_4 = 1$ ,  $k = 1$ ,  $r = .9$ .

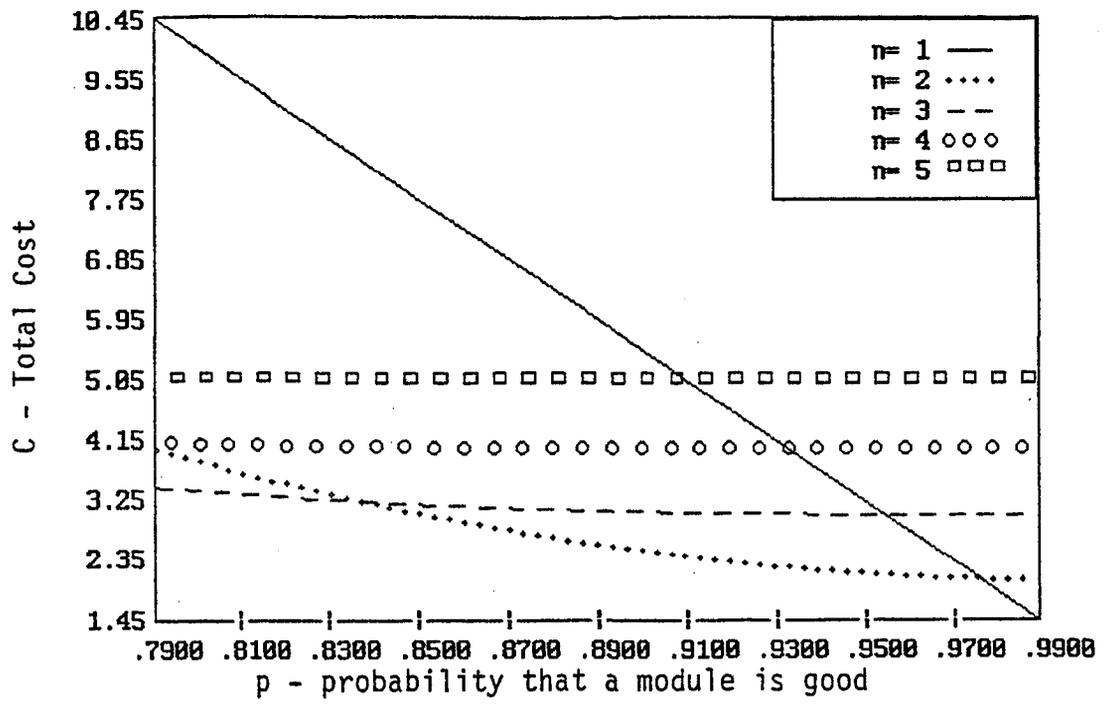


Fig. 2 - Model 1 with  $c_1 = 50$ ,  $c_4 = 1$ ,  $k = 1$ ,  $r = .9$ .

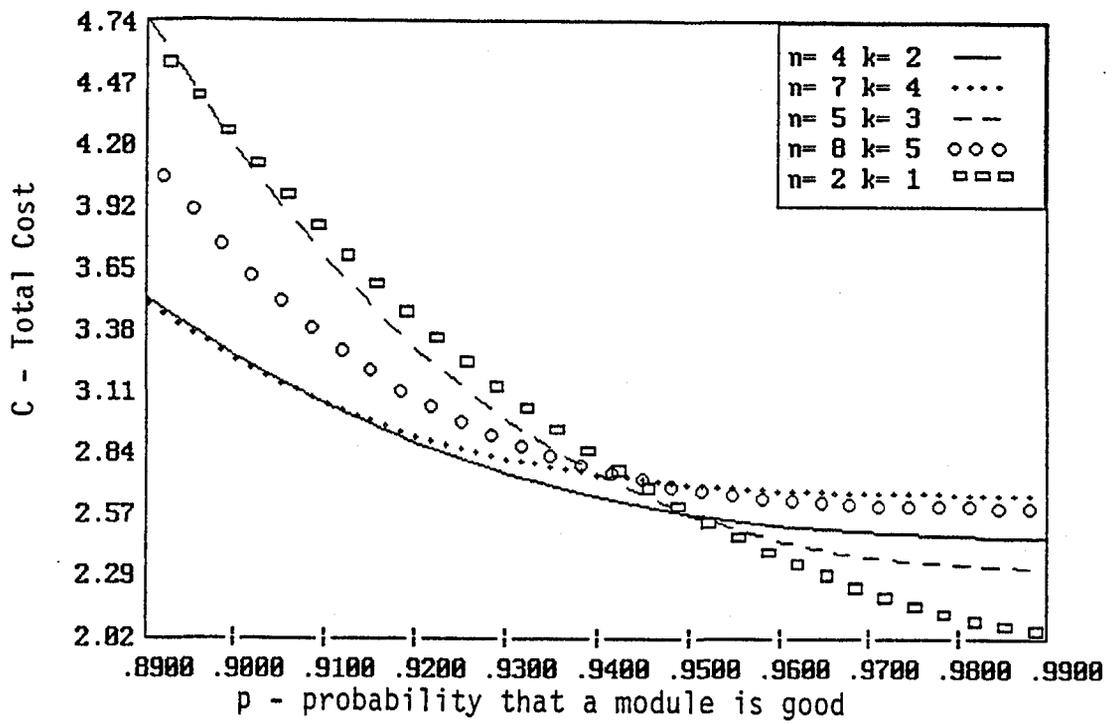


Fig. 3 - Model 2 with  $c_1=240$ ,  $c_3=1$ ,  $r = .9$ .