APPLICATION OF MILLISECOND PULSAR TIMING TO THE LONG-TERM STABILITY OF CLOCK ENSEMBLES

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Abstract

We review the application of millisecond pulsars to define a precise long-term time standard and positional reference system in a nearly inertial reference frame. We quantify the current timing precision of the best millisecond pulsars and define the required precise time and time interval (PTTI) accuracy and stability to enable time transfer via pulsars. Pulsars may prove useful as independent standards to examine decade-long timing stability and provide an independent natural system within which to calibrate any new, perhaps vastly improved atomic time scale. Since pulsar stability appears to be related to the lifetime of the pulsar, the new millisecond pulsar J1713+0747 is projected to have a 100-day accuracy equivalent to a single HPS071 cesium standard. Over the last five years, dozens of new millisecond pulsars have been discovered. A few of the new millisecond pulsars may have even better timing properties.

INTRODUCTION

Regular timing measurements of millisecond pulsars provide a unique metrology data set produced by natural sources in a nearly inertial reference frame. Since the discovery of the first millisecond pulsar in 1982, the utility of these sources as precise time standards has been clearly recognized[1,2]. Decade-long observations of the first two millisecond pulsars now show irregularities in the timing residuals that are most likely due to intrinsic rotational instability in the star itself[3]. With terrestrial time standards generally improving an order of magnitude every seven years or so, questions have been raised about the utility of radio pulsars for timekeeping applications. This paper will attempt to address some of the fundamental limitations of pulsar data.

Despite a clear indication on the limit to the precision of millisecond pulsars, they may have great utility in anchoring an inertial space-time reference frame free from the gravitational effects of our solar system. Cesium time scales show irregularities due to physical limitations of the atomic clocks beyond about six months. Specifically, International Atomic Time (TAI) now
has a measured stability of about $2.5 \times 10^{-16}$ on a time scale of 1 month. The use of different clock ensemble time scales results in measurable changes in millisecond pulsar timing data (e.g. [4]). To date, only two millisecond pulsars have been timed reliably for over five years[3]. Table 1 gives the current best estimates of the timing precision and astrometric accuracy of the two longest timed millisecond pulsars (PSR B1855+09 & B1937+21) plus four additional millisecond pulsars that have been timed for shorter time spans.

<table>
<thead>
<tr>
<th>Pulsar Name</th>
<th>Observation Duration (years)</th>
<th>Timing Precision (us)</th>
<th>RA Precision (mas)</th>
<th>DEC Precision (mas)</th>
<th>Reference</th>
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<tr>
<td>B1937+21</td>
<td>8.2</td>
<td>0.2</td>
<td>0.03</td>
<td>0.06</td>
<td>[5]</td>
</tr>
<tr>
<td>B1855+09</td>
<td>6.9</td>
<td>1.0</td>
<td>0.07</td>
<td>0.12</td>
<td>[3]</td>
</tr>
<tr>
<td>J1713+0747</td>
<td>1.8</td>
<td>0.4</td>
<td>0.2</td>
<td>0.3</td>
<td>[6]</td>
</tr>
<tr>
<td>B1257+12</td>
<td>2.6</td>
<td>2.3</td>
<td>0.4</td>
<td>1.0</td>
<td>[6]</td>
</tr>
<tr>
<td>J2322+2057</td>
<td>2.3</td>
<td>2.9</td>
<td>1.0</td>
<td>2.0</td>
<td>[7]</td>
</tr>
<tr>
<td>J2019+2425</td>
<td>2.7</td>
<td>3.0</td>
<td>0.6</td>
<td>0.9</td>
<td>[7]</td>
</tr>
</tbody>
</table>

**TIME SCALE IMPROVEMENTS**

It is not practical to derive exact numerical values for the stability of pulsars because the pulsar timing data are actually residuals to a multi-parameter fit to pulsar parameters (such as the period, spin-down rate, position, and proper motion), whose covariance depends upon the length of the data set. In the case of the original millisecond pulsar B1937+21, we chose to estimate the instability by generating a series of random numbers characterized by white and random walk frequency noise, removing a solution to pulsar parameters, and observing the differences between the true and simulated residuals. A similar technique was applied to cesium and maser data, except that only an overall frequency offset and drift were removed; the terrestrial clock characteristics derived this way proved consistent with those of Breakiron[8], which extend only out to time scales of a few months. Using this comparison, the pulsar B1937+21 proved to be about 6 times noisier than an individual HP5071 cesium time standard. The comparison becomes even less exact for time scales longer than several months, as higher-order terms would be masked in the data sets.

Another consideration is that the available millisecond pulsar data for B1937+21 extend back to 1984. Through the parameter-fitting process, all the pulsar timing data are influenced by instabilities in TAI (or TT) since then (see [9] for an explanation of the current definition of various time scales). This has the effect of masking the recent dramatic improvements in TAI (see Figure 1). Future pulsar data will benefit from the improved TAI since about 1990. With long enough data sets it may be possible to take this into account, so as to improve our determination of apparent pulsar irregularities and increase the utility of a pulsar time scale.

**A NEW BEST PULSAR CLOCK**

An example of one of the new generation of millisecond pulsars found as part of the recent large scale sky surveys is PSR J1713+0747. The pulsar was discovered in a systematic survey
of the radio sky using the Arecibo radio telescope\cite{10,11}. After 22 months of timing, the binary millisecond pulsar J1713+0747 appears to be the most stable millisecond pulsar yet observed\cite{12}. Current timing precision from this pulsar now rivals the best results obtained from pulsars B1937+21 and B1855+09. Figure 2 shows the fractional frequency stability of PSR 1713+0747 compared to PSR B1855+09 and B1937+21. The stability parameter is defined as 
\[ \sigma_r(t) = \left( \frac{1}{T} \sum_{m} S_m \right)^{1/2} \]
where \( T \) is the data span, \( S_m \) is the spectral density, and \( m = 1, 2, 4, ... \) is a sub-interval\cite{12}. The timing solution for this pulsar includes the measurement of the pulsar’s annual parallax, proper motion, and its relativistic “Shapiro delay” due to the light travel delay through the gravitational potential of the companion star\cite{13}.

These observations are used to place limits on the mass of the neutron star and its companion. With a post-fit weighted root-mean-square timing residual of approximately 0.4 \( \mu \text{s} \), and a characteristic age of roughly 9 billion years, this pulsar may prove to be an important celestial clock in the construction of a pulsar timing array. The source has a large enough radio flux density that it can be used in the future as a target source for observations with the Very Long Baseline Array (VLBA).

The above-mentioned millisecond pulsar was found with the Arecibo radio telescope prior to the current major upgrade. The construction has resulted in a complete loss of observing potential above 430 MHz since late 1994 and at all frequencies since early 1995. The current construction schedule does not call for resumed operations until the end of 1996 or possibly early 1997. Irregular observations of PSR 1713+0747 and other Arecibo pulsars are going on at other major radio telescopes around the world, but their low sensitivity and sporadic schedules will greatly diminish the quality of pulsar data until the Arecibo upgrade is completed.

LIMITATIONS TO PULSAR CLOCK AND PULSAR TIME

The limitations of millisecond pulsars to establishing a stable spatial-temporal reference frame can come from the following sources: (1) instabilities in the pulsar itself, (2) uncertainties in the Earth-based atomic time scale, (3) source position errors due to planetary ephemeris errors or errors in the pulsar position, (4) additional noise sources including propagation effects from the interstellar medium and the gravitational wave background, (5) detector hardware noise sources, (6) errors in the time-transfer system, and (7) unmodeled binary orbit effects. Many of the effects are discussed in recent reviews of pulsar timing, including Backer\cite{13,14}, Taylor\cite{15}, and Backer and Hellings\cite{16}.

Primary interest in millisecond pulsar stability comes from limitations set by intrinsic rotational instabilities driven by physics in the neutron star interior. Recent stability analysis of a large population of pulsars, including millisecond pulsars by Arzoumanian et al. \cite{17}, indicate a strong correlation between pulsar timing noise and pulsar period derivative (this could equivalently be correlated with pulsar age assuming spin-down due to magnetic braking). A best linear fit through the timing stability-period derivative plane for 139 pulsars, including a number of millisecond pulsars, indicates a stability trend that can be parameterized as

\[ \Delta_8 = 6.6 + 0.6 \log P, \]

where \( \Delta_8 \) is a fixed time interval stability parameter over a reference time interval of \( 10^6 \) seconds. The stability equation is defined from:

449
\[ \Delta(t) = \log \left( \frac{1}{\nu} ||\dot{\nu}|| t^2 \right), \]  

where \( \nu \) is the rotational frequency, and \( \dot{\nu} \) is the second derivative of the rotational frequency of the Taylor expansion of the pulsar's rotational phase:

\[ \phi = \phi_0 + \nu t + \frac{1}{2} \dot{\nu} t^2 + \frac{1}{6} \ddot{\nu} t^3 + \ldots \]

Assuming that millisecond pulsars spin-down due to magnetic dipole braking, then the pulsar age correlates with the period derivative, implying that older pulsars with smaller period derivatives are more stable.

A GREAT SUCCESS

The successful discovery of numerous field millisecond pulsars by various all sky pulsar surveys (e.g. [18,11,18-27]). Since 1989 a number of major pulsar search efforts have increased the total number of known millisecond pulsars in the Galactic field to 29 (see Figures 3 and 4).

More than 75 percent of these new sources are in binary systems with white-dwarf companions. Deep optical observations with ground- (Keck 10-m telescope) and space-based (Hubble Space Telescope) instruments along with ground-based astrometric observations, will allow determination of the pulsar optical position to better than 30 milliarcseconds (mas). With the new Very Long Baseline Array (VLBA), several of these pulsars will have a large enough flux densities to establish radio positions with respect to the extragalactic reference frame. Using pulsar timing data, radio interferometric data and optical astrometry, we can use these new millisecond pulsars to tie together the radio, dynamical, and optical reference frames to better than 30 mas. Over the next five years, data from the dozens of new millisecond pulsars should provide a large ensemble of pulsars for use in establishing an inertial spatial-temporal reference frame.

THE FUTURE OF PULSAR TIMING

The initial goal of establishing a pulsar timing array to define an inertial space-time reference frame will require timing about 10 millisecond pulsars with 100 ns timing precision for at least 10 years. The PTTI requirements to reach this goal are an absolute timing accuracy of 10-20 ns, and a stability of 10 ns over 1 year, or a fractional frequency stability of \( 3 \times 10^{-16} \) (this will give the atomic clock time scale an order of magnitude improvement over the best pulsar clock). If these requirements can be met and maintained, then TT will not be the limiting source of precision in the long-term timing of millisecond pulsars. With a sufficient number of radio pulsars being timed, the use of UTC as an absolute reference time scale could be eliminated. UTC would only be needed as the carrier time to transfer pulsar arrival time measurements between successive observations. The pulsars themselves could be used as the ultimate reference time scale.

On an even longer-term basis, a pulsar ensemble could be used to time a hundred millisecond pulsars over a hundred years at better than 100 ns timing precision, given improvements in radio telescope receiver hardware, and the development of better pulsar timing systems. Millisecond pulsars will in the future have an important role to play in tying together three distinct reference frames: the radio, the dynamical, and the optical. These natural clocks will
also offer an independent means for verification of terrestrial time scales, testing for relativistic
effects in the solar system, and of models of planetary ephemerides. The proper maintenance
of pulsar timing data will be critical to enable pulsar time to be applied over a hundred-year
astronomical program by a future generation of astronomers. Because of the precision of
millisecond pulsars, we note that they could in fact be used to recover the absolute phase
of the atomic time scale, if it were ever lost due to some unforeseen global mishap. A pulsar space-time reference frame defined with significant precision could be used to establish
a space-time coordinate system useful for passive interplanetary spacecraft navigation, without
need for two-way communication.

The next generation of pulsar timing programs will have to wait until sometime in 1997 to begin
full scale operation. Two of the major northern hemisphere radio telescope facilities, the Arecibo
Observatory (AO) radio telescope and the Green Bank Telescope (GBT), are undergoing a
major upgrade or are under construction. Both instruments are currently scheduled to start
regular operations in 1997. Until that time, smaller telescopes like the NRAO 42-m, Jodrell
Bank, the Bonn 100-m, the Very Large Array, and the Nançay radio telescope will have to
carry on with limited pulsar observations.

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REFERENCES

369-380.


Fig. 1.— Frequency difference between free-running timescales of TAI, PTB, and USNO (A.1) in fs/s. The USNO and PTB timescales are independent, but TAI is gently steered towards the PTB, whereas the USNO clocks' contribution to TAI has increased to 40%. Nevertheless, improvement since 1990 is obvious. The lowest panel shows the difference between TT and TAI, where TT represents TAI recomputed using hindsight corrections to clock weights, offsets, and drifts. A constant frequency offset has been removed from all plots. Data were obtain from the BIPM by anonymous ftp to address 145.238.2.2. The time range shown is from January 1982 to December, 1993. The figure is from Matsakis and Foster (1995) [4]

Fig. 2.— Twenty-two months of pulsar timing data from PSR J1713+0747 show that it has better long-term timing accuracy than either PSR B1855+09 or PSR B1937+21. (This figure is from Foster, Camilo, & Wolszczan (1995) [12]
Fig. 3.— A plot in galactic coordinates of the known millisecond pulsars in 1989 (large hexagons). All the known non-millisecond pulsars in the galactic field are plotted as dots. Note the location of the millisecond pulsars in the region of the galactic plane visible using the Arecibo telescope from -1 to +38 degrees declination (solid lines).

Fig. 4.— The same plot as in Figure 3 expect now the 29 known millisecond pulsars (as of November 1995) in the galactic field are plotted. Note the wide distribution around the sky indicative of a large galactic scale height.
Questions and Answers

JUDAH LEVINE (NIST): You talked about stabilities of something in the $10^{-15}$ for a year; but most of the commercial time scales that you could get at are much worse than that, USNO or NIST, or something, are probably above a part in $10^{14}$ in a year. Could you talk about how you know where you are when you think you are?

ROGER FOSTER (NRL): Well, I think we don't. And I think that the way that we're going to perhaps improve this is that we're going to independently couple in some of the astrometry aspects of this — at least, on the time scales less than a decade where the astrometry issue becomes important, we can get independent astrometry measurements, independent reference frame times, that we'll be able to at least remove those variables from the equation and improve it. We also have the opportunity with multiple pulsars of timing the pulsars against themselves.

JUDAH LEVINE (NIST): I don't know the exact analysis that you made, but I thought it was necessary to remove the electron content along the path; that within an independent parameter, it's kind of an adjust for.

ROGER FOSTER (NRL): That's correct. The electron column is fitted out, and that does come into one of the issues of stability. One of the interesting aspects of the population of galactic field pulsars is that because they're high latitudes, they're basically outside of the galactic plane; and the electron columns tend to be rather low where these effects are minimal.

GERARD PETIT (BIPM): Could you comment on the number of pulsars that could be found and probably galactic?

ROGER FOSTER (NRL): I think the number over the next decade or so will probably approach several hundred of these millisecond pulsars. And that has to do with just extrapolation of the surface density in which they have been detected in general surveys. Roughly, about one per 200 or 300 square degrees; and there are about 42,000 square degrees.

DAVID ALLAN (ALLAN'S TIME): I'm excited about the possibilities that millisecond pulsar timing can bring from the physics point of view. I think we have to be careful how we utilize the data or how we think about the data. For example, we have a report from the LPTF on the cesium fountain of $3 \times 10^{-15}$ accuracy. Accuracy has intrinsically no tau value. And so, it can carry frequency in the absolute sense for any integration time you wish. We expect that these accuracies will improve in other standards. As we do the same extrapolation that you've done for pulsars for the terrestrial clock community, we expect within a couple of decades to be in parts in $10^{18}$. If you now look at the fundamental limits of pulsars, millisecond pulsar timing, due to interstellar medium, et cetera, this requires integration times from fundamental limits of about 200 years, just for the first tau value. In order to have a confidence on that, a millennium of data would be needed.

So I think we have to be careful as we make big plans, and this is a planning meeting. I think we can never expect millisecond pulsar timing to contribute in a significant way to an understanding of the uniform time scale and accuracy, in terms of stability, setting time scales (as you've alluded to), studying physics. Looking for gravitational waves, there is much excitement and a good reason to proceed with these studies. But I think we have to be a little careful in how we couch it.