COMMENTS ON THE ROTATIONAL STATE AND NON-GRAVITATIONAL FORCES OF COMET 46/WIRTANEN

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Abstract. We apply our experience of modeling the rotational state and non-gravitational forces of comet 1P/Halley and other comets to comet 46P/Wirtanen. While the paucity of physical data on 46P/Wirtanen makes this process somewhat speculative, this comet's place as target for the important Rosetta mission gives significance to such a study. Our arguments are based on the summary of observational data provided by Jorda and Rickman (1995) and a comparative study of the behavior of other periodic comets. We find 46P/Wirtanen to have a level of surface activity relative to its mass that is dynamically more akin to that found in comet 1P/Halley than in a typical periodic comet. We show through an illustrative numerical example that this apparent fact should likely lead to an excited spin state for this comet and that significant changes in the spin period could occur in a single pass through perihelion. We argue that the available observations are not sufficient to substantiate the claim of Jorda and Rickman (1995) that the nucleus is undergoing retrograde rotation and it is possible that the rotation is either prograde as well as retrograde. The substantial requirements that must be placed on any future observing program necessary to determine the precise rotational state are outlined. We advocate an extended (~two month) southern hemisphere observing campaign to determine the nuclear rotational state in 1996 if possible before activity turns on.

Introduction
Given the paucity of physical data on comet 46P/Wirtanen (Jorda and Rickman, 1995) it is perhaps surprising that it was chosen by ESA as the target of the Rosetta mission to a cometary nucleus (ESA, 1994). Nevertheless, the decision is welcome and elevates comet 46P/Wirtanen to a level where even the uncertain results of a study of even the meager data set currently available may have important influences on aspects of mission design. It is in this spirit that we have undertaken a comparative study of the spin state of the comet. The results herein are based not only on the available facts (Jorda and Rickman, 1995), but on our experience in diagnosing and modeling the spin state of three comets: 1P/Halley, 29P/Schwassmann-Wachmann 1, and 10P/Tempel 2 (Samarasinha and Belton, 1995; Meech et al., 1993; and Mueller and Ferrin, 1995). While these comets are all very different from each other (and each of them, most probably, from 46P/Wirtanen), we are convinced that there is much to learn from a comparative study, and that the results may make future research on the spin state of 46P/Wirtanen more efficient and productive. Precise knowledge of the rotational state of a cometary nucleus is essential for the proper interpretation of any kind of remote sensing. The interpretation of some in situ measurements of cometary phenomena and, most certainly, the study of the evolution of cometary mantles (Belton, 1991; Samarasinha and Belton, 1995) depend upon it. Examples are: The untangling of the locations of sources of cometary activity on the nucleus from fan and jet structures as was successfully done for the first time with comet 1P/Halley (Belton et al., 1991); the interpretation of in situ observations of the coma as in the analysis of mass spectrometer measurements of particles flowing from the nucleus (e.g., compare the very different models of CO outflow in the coma of 1P/Halley in Eberhardt et
which ignores nuclear spin, and Samarasinha and Belton, 1994, which includes it); insights into the interior mass distribution of the nucleus through measurement of the ratios of the principal moments of inertia as was done for comet 1P/Halley (Belton et al., 1991) and, more recently, for the near-Earth asteroid Toutatis (Hudson and Ostro, 1995); finally, the interpretations of the non-gravitational forces that measurably modify the shape and orientation of the cometary orbit and which can, at least in principle, yield a crude estimate of the mass of the nucleus (Rickman, 1989; Samarasinha and Belton, 1995). In the case of 46P/Wirtanen, we shall show that the current knowledge is sufficient to support the expectation that the nucleus will likely be found in an excited spin state, i.e., the nucleus is undergoing complex rotational motion. In addition we find that this state may change on relatively short time scales and that a substantial and carefully thought out program of ground-based observations will be required to diagnose the periodicities that may ultimately be observed in its lightcurve.

The Size, Activity, and Shape of the Nucleus of 46P/Wirtanen

In their summary of what is known of 46P/Wirtanen and their interpretation of these facts, Jorda and Rickman (1995) find the following: (a) A perihelion production rate of $4 \times 10^{28}$ H$_2$O mol.sec$^{-1}$; (b) An upper limit for the nuclear radius of $1.8$ km assuming an albedo of 0.03; (c) Active areas could cover as much as 0.25 of the surface area; (d) Possible retrograde rotation of the nucleus. Also, in their compendium of the cometary activity Jorda and Rickman note at least one indication of a possible brightness flare (Morris, 1994) and the presence, on at least one occasion, of a fan-like tail (Mikuz, 1992). Generally the coma is diffuse sometimes with, and sometimes without, central condensation. We shall make use of these data in the discussion below.

Size and level of activity. At $1.8$ km, the effective radius of the comet is, by any measure, relatively small. Despite any observational selection effects, in Jewitt's (1991) review of what is known with reasonable observational certainty about the global physical properties of comet nuclei from cometary photometry, the mean effective radius of the comets listed is $5$ km. Other comets, more lately investigated, have even larger effective radii, e.g., 29P/Schwassmann-Wachmann 1 at $15$ km (Meech et al., 1993) and 95P/Chiron at $90$ km (Campins et al., 1994). Through inference from OH production rates Osip et al. (1992) have also listed radii estimates for 10 periodic comets - but these estimates depend in turn on the fractional coverage of the area which is, of course, itself an estimated quantity. For those comets on Osip et al.'s list for which an effective radius can be independently estimated (sometimes done by assuming a surface albedo near 0.03 or 0.04), 10P/Tempel 2 has only $2\%$ of its surface active, 2P/Encke has $1\%$ of its surface active - if the effective radius of $3.5$ km (Luu and Jewitt, 1990) is correct, and 1P/Halley has approximately $30\%$ of its surface active.

Jorda and Rickman (1995) give a water production rate for 46P/Wirtanen of $4 \times 10^{28}$ mol.sec$^{-1}$ at perihelion (1.08 AU). More recently, A'Hearn et al. (1995) obtained a water production rate of $10^{28}$ mol.sec$^{-1}$ based on an OH observation
of 46P/Wirtanen made few days after its 1991 perihelion passage. While, this latter estimate of the water production rate is more reliable than the estimate based on visual observations, unfortunately observational conditions did not allow A'Hearn et al. to make a series of OH observations thus preventing us from reaching any conclusion regarding the short term variability in the water production rate. Therefore, in this paper, we will assume the water production rate of Jorda and Rickman (1995) for calculations and subsequently we will discuss the implications for the water production rate of A'Hearn et al. The OH production rate is approximately 1/1.1 of the H2O production rate. For their comparative study, Osip et al. (1992) normalize the OH production rates to 1.5 AU using a r^3 (where r = heliocentric distance) dependency. This gives a production rate Q for 46P/Wirtanen of $1.35 \times 10^{28}$ mol.sec^{-1} compared to $1.6 \times 10^{29}$ mol.sec^{-1} for 1P/Halley. Since the surface area is proportional to the square of the radius, R,

$$s_w \approx \frac{Q_w}{Q_H} \left( \frac{R_H}{R_w} \right)^2$$

where the fractional active surface area is s and subscripts W and H stand for Wirtanen and Halley respectively. We have, therefore:

$$s_w \approx 0.8s_H$$

In contrast, 10P/Tempel 2 has roughly the same effective radius as 1P/Halley but only 2% of the surface are active according to Osip et al. (1992). Making the same comparison between 10P/Tempel 2 and 46P/Wirtanen we get:

$$s_w \approx 13s_{T2}$$

where subscript T2 stands for 10P/Tempel 2. Therefore, about 25% of the surface of 46P/Wirtanen is active. For comparison, the water production rate of A'Hearn et al. will yield a fractional active surface for Wirtanen smaller by a factor of 4.

Thus, as far as we can determine, 46P/Wirtanen appears to be physically smaller than the typical periodic comet, but has an active surface fraction which is more akin to that of 1P/Halley. Since the perihelion distance of 46P/Wirtanen was >1.6AU until its 1972 close encounter with Jupiter (however, we can not exclude the possibility that it was never closer to the Sun during its lifetime), it may have aged less than other short period comets and consequently has a comparatively larger active fraction.

1P/Halley is in a complex rotational state where as 10P/Tempel 2 appears to be fully relaxed in a simple flat spin. However, recent observations of 10P/Tempel 2 show that its rotational period has undergone a small, but significant, change between the 1988 and 1994 apparitions presumably due to torques caused by outgassing (Mueller and Ferrin, 1995). This indicates that not only can outgassing cause very active nuclei to be in excited rotational states, but even weakly active nuclei can show observable changes in their rotational states.

Nuclear Shape. With the exception of 95P/Chiron, the few cometary nuclei for which we have lightcurves have axial ratios close to 2:1 (Jewitt, 1991; Meech et al., 1993). The smaller lightcurve amplitude associated with the nucleus of 95P/Chiron (< 0.09 mag, Bus et al., 1989; Marcialis and Buratti, 1993) is
perhaps to be expected because of Chiron's size. A comparison with the shapes of asteroids and planetary satellites is instructive since it has been convincingly shown by Thomas (1989) that at a diameter near ~ 150 km, and for larger sizes, solar system objects become more spherical as self-gravity exerts its influence (however, examples for large asteroids with elongated shapes are not uncommon, e.g., Weidenschilling et al., 1979). Below this size, irregular shapes are the norm, although again exceptions, e.g., Deimos and Dactyl, occur (cf. Thomas et al., 1995). This effect is presumably what distinguishes the small amplitude of Chiron's lightcurve from the larger amplitudes found in smaller, more irregular comets. Because the nucleus of 46P/Wirtanen is very small, the weight of experience indicates that we should expect it to be irregular in shape. A further peculiarity about those cometary nuclei (and asteroids) that have been sufficiently studied by imaging and/or by analyzing well sampled lightcurves of the nuclei, is that their shorter axes (we visualize an ellipsoidal approximation to their shapes with semi-axes a,b,c) are usually found to be similar, i.e., a > b ~ c. The large amplitude variation (~2) of several well sampled nuclear lightcurves prompted A'Hearn (1988) and Jewitt (1991) to suggest that those cometary nuclei are close to a prolate shape - assuming a ground state rotational state around the short axis. In fact it has been suggested (Espinasse et al., 1993) that this shape is a natural result of mantle evolution in the case of cometary nuclei. In a counter example, Whipple and Sekanina (1979) have suggested an oblate nucleus for comet 2P/Encke, but this seems to be a spurious result coming from their particular model for how non-gravitational forces work. The results of Jewitt and Meech (1987) and Luu and Jewitt (1990) provide contrary photometric evidence to this model; and certain aspects of the model for non-gravitational forces that Whipple and Sekanina (and by Sekanina in subsequent papers) employed has been placed in doubt as it did not allow for changes in the principal axis rotation of the nucleus despite the accommodation of the "precession" of the spin axis. Furthermore, the non-uniqueness of the non-gravitational effects requires one to exercise caution when interpreting relevant observations (e.g., Yeomans and Chodas, 1989; Samarasinha and Belton, 1995).

The Nuclear Rotational State
The rotation state of a cometary nucleus shows itself in astronomical observations most clearly through periodicities in the lightcurve. The interpretation of these periodicities in terms of the rotation state has not proven to be easy, even with a multitude of data. This has been especially true in the case of a complex (i.e., excited) spin state - as in comet 1P/Halley (Belton, 1991). For example, in the case of 1P/Halley reliance on the wealth of ground and orbital photometric lightcurves of the coma alone would have yielded the incorrect result - we would almost certainly have been laboring under the impression that the spin was in the lowest energy state, i.e., principal axis rotation, with a period of ~ 7.4 days (Belton, 1990). It was only with extra information, such as the orientation of the nucleus during the spacecraft flybys and many observations of coma structures, that a resolution of the issues and
deduction of the true spin state was accomplished (Belton et al., 1991; Samarasinha and A'Hearn, 1991).

For 46P/Virtanen there is, at present, no internally consistent and precisely determined lightcurve suitable for extracting rotational periodicities. What is available (Kamel 1992; Morris 1994) are visual observations of apparent coma brightnesses from a multitude of observers. While these lightcurves tempt one to suggest that there are variations in the time scale of days (akin to what was observed for comet 1P/Halley during March/April of 1986), care in interpreting these variations is needed as the relatively coarse sampling of the lightcurve prohibits recognition of any variations at smaller time scales of the order of hours. On the other hand, the requirement for gravitational stability in the face of rapid rotation ensures that the minimum possible period without breaking up the nucleus (cf. Sekanina 1982) is of the order of few hours for this comet. Therefore, the observational programs designed to determine the rotational state should concentrate both on hourly time scales as well as on the daily time scales.

**Rotational Damping.** The damping time scale of a body undergoing complex rotation is inversely proportional to the square of the radius and the cube of the rotational angular velocity (Burns and Safronov, 1973; Harris 1994).

\[ \tau_{\text{damp}} \propto R^{-2} \omega^{-3} \]  

Peale and Lissauer (1989) estimate that for comet 1P/Halley, which has an effective radius three times as large as 46P/Virtanen (this early contribution used a 2.2 day spin period thought to be appropriate at the time), the damping time scale is of the order of $10^6$ to $10^8$ years. Therefore, for a similar spin period, the damping time scale for 46P/Virtanen would be one order of magnitude larger. It is therefore unlikely that the nucleus has retained a primordial complex rotational state based on this effect alone. However, since the damping time scale is large, the cumulative effects of the small sublimation caused torques (cf. Samarasinha and Belton 1995) may have allowed the nucleus to spin up into a complex rotational state.

**Rotational Excitation.** In what follows, we show by analytical means as well as from numerical simulations, that its level of activity coupled with its small size makes 46P/Virtanen a very likely candidate to exhibit rapid changes in its spin period as well and to be in a complex rotational state.

For simplicity, we assume that the outflow from the nucleus is in the form of directed outflow and not a symmetric hemispheric outflow. The observations of coma structure (Mikuz, 1992) and the flaring activity (Morris, 1994) gives some support to this position. Furthermore, we assume that initially the nucleus is in the principal axis rotation. Then the magnitude of the rate of change of angular velocity is given by:

\[ \dot{\omega} = \frac{N}{I} \]  

where \( N \) = torque, \( I \) = moment of inertia about the axis of rotation, therefore,

\[ \dot{\omega} \propto \frac{kRQ}{MR^2} \propto \frac{Q}{R^4} \]
where $kR =$ torque moment ($k$ is a fraction near 1), $Q =$ rate of outgassing, $M =$ mass of the nucleus, and $R =$ radius of the nucleus. Therefore, the timescale for the change of the rotational state of comet 46P/Wirtanen is given by:

$$
\tau_w \approx \frac{\omega_w}{\omega_H} = \left( \frac{\omega_H}{\omega_w} \right) \left( \frac{Q_H/Q_w}{(R_H/R_w)^4} \right)
$$

We have (Jewitt, 1991; Jorda and Rickman, 1995; Samarasinha and Belton, 1995):

$$
\frac{Q_{H,\text{peak}}}{Q_{W,\text{peak}}} \approx 30, \quad \frac{R_H}{R_w} \approx 3,
$$

and since (Samarasinha et al., 1986),

$$
\frac{\omega_H}{\omega_H} \approx 3 \text{ years},
$$

it follows that

$$
\tau_w \approx \frac{\omega_w}{\omega_H} \text{ years.}
$$

i.e., $\tau_w$ must be of the order of a few years. Therefore, a single apparition may be sufficient to make observable changes in the rotational state of 46P/Wirtanen. In addition, we have made numerical experiments that show, should the nucleus of 46P/Wirtanen be more prolate than oblate, that the presence of a dominant cometographic mid-latitude active area with a level of activity consistent with Jorda and Rickman calculations will rapidly excite the nucleus to a complex rotational state (Fig. 1). Even if the active areas on the surface are not suitably located to excite the nucleus to a state such as that shown in Fig. 1, states with small deviations from principal axis (PA) rotation (i.e., precession/nutation angles of the order of a degree) are distinctly possible.

Even such mildly excited rotational states may be important in the context of the stability of spacecraft orbiting close to the nucleus and planning for the Rosetta mission should consider this possibility. For example, the Rosetta spacecraft will have to use an adaptive strategy during its rendezvous with 46P/Wirtanen. The spacecraft may need to spiral down gradually as the comet's gravity field, rotational state, and moments of inertia become better known (with eventual retrograde orbiting in the case of a PA or near-PA rotation) in order to achieve the greatest stability (e.g., Chauvineau et al. 1993; Scheeres et al. 1995a; Scheeres et al. 1995b).

**Interpretation of Observed Non-Gravitational Forces**

The change in orbital period, $\Delta P$, due to non-gravitational forces is given by the following equation:

$$
\Delta P = \frac{3P^2}{2\pi a^3 \sqrt{1-e^2} M_0} \int [e \sin f \cdot F_R + (1+e \cos f) F_T] \, dt
$$

where $a$ is the semi-major axis of the orbit, $e$ is the eccentricity, $f$ is the true anomaly, and $M$ is the mass of the comet. $F_R$ and $F_T$ stand for the radial and
transverse non-gravitational forces, respectively. The first term in the equation represents the change in orbital period due to radial non-gravitational forces while the second term represents that due to transverse forces. The \( \sin f \) term associated with \( F_r \) and the fact that the peak water production occurs between \(-90^\circ < f > +90^\circ\), guarantees that the contribution to \( \Delta P \) from the radial non-gravitational force primarily comes from an asymmetry in the production rate subsequent to \( f \sim -90^\circ \) and prior to \( f \sim +90^\circ \). Production rate asymmetries that occur near perihelion, \( f \sim 0^\circ \), will have a much smaller effect. For 46P/Wirtanen, \( f = 90^\circ \) corresponds to about 114 days from perihelion.

The available lightcurve, which is based on visual observations from the 1986 and 1991 apparitions (Fig. 1 of Jorda and Rickman, 1995), covers from -50 days to +100 days and suggests a small shift towards the post-perihelion implying that there could be a small positive contribution to \( \Delta P \). However, the visual lightcurve for the 1991 apparition alone (Morris, 1994) suggests that, despite a small shift of the brightness peak to post perihelion, the fall off of the lightcurve is more rapid during the post-perihelion period. It is therefore unclear whether this contribution to \( \Delta P \) is necessarily positive. Jorda and Rickman (1995) concluded that the negative \( \Delta P \) observed for comet 46P/Wirtanen is mainly due to a large negative contribution from the transverse force. Based on this conclusion, they argue that the nucleus of the comet is in a retrograde rotational state presuming that the net transverse force is due to a thermal lag in outgassing. They determine this lag angle following the non-gravitational model of Festou et al. (1990). However, we note that the effective lag angle depends not only on the insolation patterns of individual active areas, but also on the geometry of the outgassing. In other words, the lag angle depends not only on thermally induced sublimation lags but also on any deviations in the slope of the local landscape of the nucleus from that of an ellipsoid, e.g., discrepancies between the local outward normal and the direction of gas outflow at the active areas (see also Sekanina 1993). For comet 1P/Halley the thermal lag effects are known to be small (Samarasinha and Belton, 1995, and references therein) and, in addition, our experience in modeling non-gravitational effects (Samarasinha and Belton, 1995) shows that complex rotational states also cause irregular insolation patterns. Therefore, the lag angle may not be directly connected with the sense of rotation. In other words, prograde rotational states may be capable of exhibiting both positive and negative transverse non-gravitational forces.

As a simple example, consider a cometary nucleus undergoing rapid prograde principal axis rotation with its spin axis aligned with the orbital axis. Assume that the outgassing from this nucleus is entirely due to an active area at the equator. Furthermore, due to the roughness of the surface, assume that the direction of outgassing is skewed by an angle \( \theta ( > 0) \) to the outward normal in the direction opposite to the rotation. In other words, the surface normal at the active area is oblique to the direction of momentum transfer. Since the heliocentric distance to the comet is nearly constant (i.e., constant true anomaly) during a rotation, the contribution to \( \Delta P \) from the transverse force during a rotation is proportional to the net transverse force over a rotation. This quantity is given by (Fig. 2):
\begin{equation}
\int_0^{P_{\text{spin}}} F_T \, dt = - \frac{\dot{m}}{V P_{\text{spin}}} \frac{\pi}{2} \int_{-\pi/2}^{\pi/2} \cos \lambda \sin(\lambda + \theta) \, d\lambda
\end{equation}

= - \frac{\dot{m} V P_{\text{spin}}}{4} \sin \theta < 0

where \( \dot{m} \) is the mass loss rate due to water production, \( V \) is the gas outflow velocity, \( \lambda \) is the sun-comet-outward normal angle, and \( P_{\text{spin}} = 2\pi/\omega_{\text{spin}} \) is the spin period. The \( \cos \lambda \) term in the equation describes the insolation term, while the \( -\sin(\lambda + \theta) \) term is due to the transverse reaction force. This example demonstrates that a negative transverse non-gravitational force can arise purely from a small tilt of the local landscape of the active area and/or the asymmetry between the morning and evening insolation patterns. Thus a negative \( \Delta P \) is possible without invoking a retrograde rotation and/or a thermally induced sublimation lag angle. We conclude that the available data on comet 46P/Wirtanen is not sufficient to determine whether the nucleus is in a prograde or a retrograde spin state.

**Observing During the 1997 Apparition**

Using the effective radius and albedo given by Jorda and Rickman (1995), we find that 46P/Wirtanen is at ~23 mag in 1995 and not favorably positioned for observing from the Northern hemisphere. In 1996, the brightness increases from magnitude 22 to 19 but it is still not well suited for the northern hemisphere observers. Assuming a level of activity similar to that seen during the last two apparitions, an activity turn-on can be expected around November, 1996, as the comet approaches 2 AU. Therefore, southern hemispheric observers should be able to observe the (nearly) bare nucleus at reasonable magnitudes (19th) in the second half of 1996 prior to the activity turn-on. Perihelion in 1997 is not very well observable from earth, because the comet will be only 45° from the sun. Post-perihelion observations are possible in 1998 with much better positioning for observing from the northern hemisphere but the nuclear magnitude will be fainter than 22 mag.

The best opportunity for observing the nucleus from the ground for the coming apparition will therefore be in the second half of 1996, pre-perihelion, from the southern hemisphere when the comet is near 19 mag. In order to determine whether or not our predictions of a possible complex rotational state are valid for a comet this faint, a substantial observing campaign, analogous to the extended scope (but aimed at a nuclear lightcurve) of the observing program mounted by Millis and Schleicher (1986) on comet 1P/Halley which yielded fundamental data on the primary periodicities in the spin state, is necessary. Ground-based programs that have successfully yielded information of complex motion (Millis and Schleicher, 1986, on 1P/Halley and Meech et al., 1993, on P/SW1 [29P/Schwassmann-Wachmann 1]) have in common an extended observing period relative to the underlying periodicity coupled with dense sampling of the lightcurve. As noted above there are weak suggestions in 46P/Wirtanen's
lightcurve for a periodicity with a time scale measured in days, as in 1P/Halley. Experience with 1P/Halley and P/SW1 suggests that one of the two fundamental periodicities that characterize complex rotation may have a period that is several (2 or 3) times longer than the shorter; also the amplitude of one periodicity will surely dominate the other if 46P/Wirtanen's nucleus has the elongated shape that seems to be characteristic of small cometary nuclei.

Summary
Based on the above comparative analysis, we expect comet 46P/Wirtanen to have an irregular, near-prolate, shape. A large active surface area relative to its mass, making it dynamically more akin to comet Halley than typical periodic comets, suggests that it is very likely to be in an excited rotational state and that rapid changes in the rotational state are possible. These possibilities may affect the design of the Rosetta mission since they bear on the stability of close nuclear orbits. The theoretical uncertainties are such that the available observational data are insufficient to determine whether the nucleus is in a retrograde rotational state as inferred by Jorda and Rickman (1995).

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References


ESA/SPC(94)37, Rosetta Science Management Plan, 14 October 1994.


**Figure Captions**

**Figure 1a.** *(Top)* The precession angle vs. orbit number for two cases corresponding to the orbit and outgassing rate of comet 46P/Wirtanen. The calculations were carried out for the peak water production rate of Jorda and Rickman (1995). The precession angle is the angle between the long axis of the nucleus and the rotational angular momentum vector. The open circles denote the evolution from a least energetic initial state where rotation is around the shortest axis *(i.e., long axis is 90° away from the rotational angular momentum vector)*. An initial period of 10 days is assumed. Solid circles denote the evolution when the initial period is 2 days. Notice that the nucleus is excited very rapidly from the initial state during the first orbit itself. In both cases, the nucleus gets dynamically excited and undergoes simultaneous rotational and precessional motions. For these simulations, the nucleus was assumed to be prolate with a 2:1:1 axial ratio *(i.e., 5.7 x 2.85 x 2.85 km)* and a density of 0.4 g cm$^{-3}$. The location of active areas are similar to Fig 12 of Samarasinha and Belton (1995) and dominated by a cometographic mid-latitude active area.

*(Bottom)* The total rotational kinetic energy of the nucleus per unit mass *(in cgs units)* vs. orbit number for the same two cases. Notice that the nucleus is spinning up in both cases.

**Figure 1b.** Same as Fig 1a, but for the water production rate of A’Hearn *et al.* (1995). Notice that similar to Fig 1a, the nucleus converts to a non-principal axis rotator.

**Figure 2.** An illustration showing how an equatorial active area on an irregular cometary nucleus can cause negative transverse non-gravitational forces. $\hat{n}$ denotes the outward normal at the active area and makes an angle $\lambda$ with the sun. R and T denote the radial and transverse directions with respect to the orbit. See the text for further details.
\[
\log_{10}(E/M)
\]

precession angle (deg)