NASA-CR-202250

COSMOLOGICAL IMPLICATIONS OF THE EFFECTS OF X-RAY CLUSTERS ON THE COSMIC MICROWAVE BACKGROUND

NASA GRANT NAGW-3104

Annual and Final Reports
For the Period 1 July 1992 through 30 June 1996

Principal Investigator
Dr. William R. Forman

September 1996

Prepared for:

National Aeronautics and Space Administration
Headquarters
Washington, DC 20546

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

The Smithsonian Astrophysical Observatory
is a member of the
Harvard-Smithsonian Center for Astrophysics

The NASA Technical Officer for this grant is Dr. Louis J. Kaluzienski, NASA Headquarters, Code SR, Building HQ Room 5F82, Washington, DC 20546-0001.

1 Introduction

We have been carrying forward a program to confront x-ray observations of clusters and their evolution as derived from X-ray observatories with observations of the cosmic microwave background radiation (CMBR). In addition to the material covered in our previous reports (including three published papers), most recently we have explored the effects of a cosmological constant on the predicted Sunyaev-Zel'dovich effect from the ensemble of clusters. In this report we summarize that work from which a paper will be prepared.

2 Cosmological Background

The z-dependence of the Hubble expansion parameter is given by

$$\frac{\dot{a}}{a} \equiv H \equiv H_0 E(z) = H_0 \left[\Omega(1+z)^3 + \Omega_R(1+z)^2 + \Omega_\Lambda\right]^{1/2}. \tag{1}$$

Here the redshift is $1+z = a_0/a(t)$, and Ω, Ω_R, and Ω_Λ are constants

$$\Omega = \frac{8\pi G\rho_0}{3H_0^2}, \qquad \Omega_R = \frac{1}{(H_0 a_0 R)^2}, \qquad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \tag{2}$$

and they satisfy the equation $\Omega + \Omega_R + \Omega_\Lambda = 1$.
The angular size distance (the ratio of the physical size and the angular size) is

$$d_A(z_e)(1+z_e) \equiv H_0 a_0 r(z) = H_0 a_0 R \sinh\left[\frac{1}{H_0 a_0 R}\int_0^{z_e} \frac{dz}{E(z)}\right] \tag{3}$$

for open models ($\Omega + \Omega_\Lambda \leq 1$, while for close models sinh should be replaced with sin. The luminosity distance, defined as $f = L/4\pi d_L^2$, where L and f are the bolometric luminosity and bolometric flux, respectively, is equal to

$$d_L(z) = (1+z)^2 d_A(z) \tag{4}$$

The number of objects per unit redshift is

$$\frac{d\mathcal{N}}{dz} = n_0 H_0^{-3} F_n(z), \qquad F_n(z) = \frac{[H_0 a_0 r(z)]^2}{E(z)}, \tag{5}$$

where n_0 is the comoving density.

3 Evolution in $\Omega = 1$, $\Lambda = 0$ Universe

In a flat universe (with cosmological density parameter $\Omega = 1$) there is no preferred timescale. Consequently, a power law (scale-free) mass spectrum of initial density fluctuations will evolve in a self-similar fashion.

Kaiser (1986) assumed self-similar evolution to derive scaling equations for the characteristic values of cluster parameters. If M is the total mass within a sphere of radius R, the spectrum of primordial fluctuations can be parametrized in terms of the rms density fluctuations in the sphere as $(\delta\rho/\rho)_{rms} \propto M^{-(\mathbf{n}+3)/6}$, where the index **n** is a free parameter. Since fluctuations grow as $(1+z)^{-1}$ for $\Omega = 1$, the mass scale of the density fluctuations which just becomes non-linear at a redshift z is given by $M_{NL} \propto (1+z)^{-6/(\mathbf{n}+3)}$. The requirement that M_{NL} increases with time along with the convergence of $(\delta\rho/\rho)_{rms}$ yield the constraint $-3 < \mathbf{n} < 1$. For cold dark matter (CDM) models, the local slope of the spectrum of mass fluctuations on the scale of clusters of galaxies is $\mathbf{n} \approx -1$ (Blumenthal *et al.* 1984); $\mathbf{n}= 1$ is the natural spectrum arising in the cosmic string model (Vilenkin 1985); and the spectral index $\mathbf{n}= 0$ corresponds to a Poisson distribution of initial fluctuations. The perturbation spectrum on cluster scales depends also on the fraction of so-called "hot" dark matter (van Dalen & Shaefer 1992). We only use n as a parameter for the evolutionary models, without any physical meaning. Kaiser's laws for characteristic density ρ^*, radius R^*, temperature (or velocity dispersion) T^*, and the comoving cluster number density N^* are:

$$\rho^* \propto (1+z)^3, \tag{6}$$
$$R^* \propto (1+z)^{-(5+n)/(3+n)}, \tag{7}$$
$$T^* \propto (1+z)^{(n-1)/(n+3)}, \tag{8}$$
$$N^* \propto (1+z)^{6/(n+3)}. \tag{9}$$

Equations (7) and (8) follow from the virialization condition at turn around, while (9) is a consequence of the self-similar scaling $M^*N^* = \text{const.}$

4 Evolution with $\Omega < 1$

Using the Press-Schechter (Press & Schechter 1974) formalism we can calculate the fraction of matter which is in gravitationally bound within systems with mass exceeding M as a fraction of space, where the *linearly evolved* density field, smoothed with the window function corresponding to mass M, exceeds some density threshold δ_c. For Gaussian initial perturbations this can be written as:

$$\frac{\Omega(>M)}{\Omega} = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(R)}\right), \tag{10}$$

where $\sigma(R)$ is the *rms* density variation on scale R, corresponding to mass M ($= 4\pi\rho_b R^3/3$).

Following the approach of Carroll, Press & Turner (1992), we can use the fractional perturbation growth rate, with respect to that in an $\Omega = 1$ universe:

$$g(\Omega) = \frac{5}{2}\Omega\left[1+\frac{\Omega}{2}+\Omega^{4/7}\right]^{-1} \tag{11}$$

for $\Lambda = 0$, and

$$g(\Omega) = \frac{5}{2}\Omega\left[\frac{1}{70}+\frac{209\Omega}{140}-\frac{\Omega^2}{140}+\Omega^{4/7}\right]^{-1} \tag{12}$$

for the flat Universe. In an $\Omega = 1$ universe, the growth rate is $\propto (1+z)^{-1}$, therefore

$$\sigma(R,z) = \sigma(R,0)\frac{g(\Omega(z))}{g(\Omega_o)}\frac{1}{1+z}, \tag{13}$$

where the z-dependence of the density parameter Ω is given by

$$\Omega(z) = \Omega_o\frac{1+z}{1+\Omega_o z} \tag{14}$$

for $\Lambda = 0$, and

$$\Omega(z) = \Omega_o\frac{(1+z)^3}{1-\Omega_o+\Omega_o(1+z)^3} \tag{15}$$

for $\Omega+\Omega_\Lambda = 1$. The cluster mass function is derived by differentiating eq (10) and multiplying the result by ρ_b/M:

$$n(M,z)dM = -\sqrt{\frac{2}{\pi}}\frac{\rho_b}{M}\frac{\delta_c}{\sigma^2(M,z)}\frac{d\sigma(M,z)}{dM}\exp\left[-\frac{\delta_c^2}{2\sigma^2(M,z)}\right]dM. \tag{16}$$

The characteristic feature of the mass function determined by eq (16) is the exponential cutoff at mass M^* defined by equation $\delta_c^2/2\sigma^2(M^*,z) = 1$. For the power law spectrum of primordial perturbations $\delta_k \propto k^n$ the *rms* density contrast on scales, corresponding to mass M, is also given by a power law:

$$\sigma(M) \propto M^{-\alpha}, \qquad \alpha = \frac{n+3}{6}, \tag{17}$$

which enables us to derive (using eq [13]) the scaling law for the characteristic mass M^*

$$M^* \propto \left(\frac{g(\Omega(z))}{g(\Omega_o)}\frac{1}{1+z}\right)^{\frac{1}{\alpha}} \tag{18}$$

For a flat critical density Universe ($\Omega(z) = 1$) the above equation reduces to the Kaiser's law $M^* \propto (1+z)^{-6/(n+3)}$.

For a very wide range of initial density contrast values δ, the expansion factor of a spherical perturbation at turn-around is $R_T/R_i \propto \delta^{-1}$, and the radius of the same perturbation after virialization is simply proportional to the turn-around radius (eg. Barrow & Saich 1993). This can be used to derive the scaling law for the characteristic density ρ^* (the density of an M^* cluster):

$$\rho^* \propto (\delta_i)^3 \propto (M^*)^{-3\alpha} \propto \left(\frac{g(\Omega(z))}{g(\Omega_o)}\frac{1}{1+z}\right)^{-3} \tag{19}$$

and the characteristic radius R^*

$$R^* \propto \left(R \sim \left(\frac{M}{\rho}\right)^{\frac{1}{3}}\right) = \left(\frac{g(\Omega(z))}{g(\Omega_o)}\frac{1}{1+z}\right)^{1+\frac{1}{3\alpha}}. \tag{20}$$

The virial temperature (or velocity dispersion) can be written as:

$$T^{*} \quad \propto \quad \left(T \sim \frac{M}{R}\right) \quad = \quad \left(\frac{g(\Omega(z))}{g(\Omega_{o})}\frac{1}{1+z}\right)^{-1+\frac{2}{3\alpha}}. \tag{21}$$

Finally, the comoving number density of M^{*} clusters is

$$N^{*} \quad \propto \quad \frac{1}{M^{*}} \quad \propto \quad \left(\frac{g(\Omega(z))}{g(\Omega_{o})}\frac{1}{1+z}\right)^{-\frac{1}{\alpha}} \tag{22}$$

so that $N^{*}M^{*} = \text{const.}$

5 Y-parameter evolution

The total microwave flux from a cluster is determined by the integrated Y-parameter (Markevitch et al. 1994):

$$Y_{1} \quad = \quad \int d\omega \int \frac{kT_{e}}{m_{e}c^{2}}\sigma_{T}n_{e}(l)\,dl \quad = \quad \sigma_{T}N_{e}\frac{kT_{e}}{m_{e}c^{2}}\frac{1}{d_{a}^{2}}, \tag{23}$$

where $T_{e}(\propto T)$ is the electron temperature and $N_{e}(\propto M)$ is the total number of electrons in the intracluster gas. Using this equation, we can write the scaling law for the the total y-parameter from the entire cluster population per unit redshift as:

$$\frac{dy}{dz} \quad \propto \quad Y_{1}^{*}N^{*}\frac{d\mathcal{N}}{dz} \quad \propto \quad M^{*}T^{*}N^{*}\frac{d\mathcal{N}}{dz}\frac{1}{d_{a}^{2}} \tag{24}$$

where Y_{1}^{*} is the integrated Y-parameter for M^{*} clusters, and $d\mathcal{N}/dz$ is the volume per unit redshift evolution. Using the relation (see previous sections)

$$N^{*}M^{*} = \text{const} \qquad \frac{d\mathcal{N}}{dz}\frac{1}{d_{a}^{2}} = \frac{(1+z)^{2}}{E(z)} \tag{25}$$

we finally write the y-parameter per unit redshift scaling as:

$$\frac{dy}{dz} \quad \propto \quad \frac{T^{*}(1+z)^{2}}{E(z)} \tag{26}$$

where $E(z)$ is defined by eq (1) and the T^{*} scaling law is defined by eq (21). The local $dy/dz|_{z=0}$ can be measured using various local cluster samples. The redshift dependence of dy/dz, calculated assuming $n = -1$ and normalized to its local value is shown in Fig 1.

As the figure shows, the integrated effects of the hot gas in clusters on the microwave background do not differ significantly for three popular cosmological models. The three models shown are for an open universe ($\Omega = 0.3$), a standard CDM universe ($\Omega = 1$) and a universe with a significant cosmological constant ($\Omega = 0.3$, $\Lambda = 0.7$). Thus, the introduction of a non-zero cosmological constant in an open universe model, will not produce a conflict with COBE limits on the spectral distortions in the cosmic microwave background radiation.

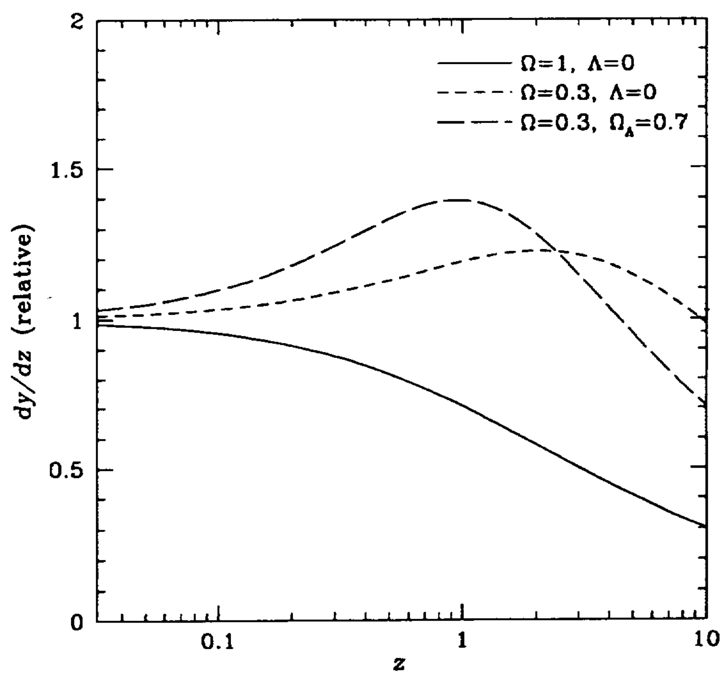

Figure 1: The redshift dependence of dy/dz for $n = -1$ and normalized to its local value is shown for three sets of popular cosmological models. The figure shows that the effect of clusters on the integrated S-Z effect for a cosmological model with $\Omega = 0.3$, $\Lambda = 0.7$ does not differ significantly from that in a model with $\Omega = 0.3$, $\Lambda = 0$.

6 References

Barrow, J. & Saich, P. 1993, 1993, MNRAS, 262, 717

Blumenthal, G.R., Faber, S.M., Primack, J.R., Rees, M.J. 1984: Nature, 311, 517

Carroll, S., Press, W. & Turner, E. 1992, ARA&A, 30, 499

Kaiser, N. 1986, MNRAS, 222, 323
Markevitch, M., Blumenthal, G. R., Forman, W., Jones, C. & Sunyaev, R. A. 1994, ApJ, 426, 1

Press, W.H. & Schechter, P. 1974, Ap.J., 187, 425

Van Dalen, A. & Shaefer, R. 1992, 1992, ApJ, 398, 33

Vilenkin, A. 1985, Phys. Rep., 121, 261