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VELOCITY MEASUREMENT BY SCATTERING FROM INDEX OF REFRACTION FLUCTUATIONS INDUCED IN TURBULENT FLOWS

Lars Lading and Mark Saffman
Riso Research Center
Denmark

and

Robert Edwards
Case Western Reserve University
Cleveland, Ohio

# Induced Phase Screen Scattering

## What is it?

. Scatter light from from weak index of refraction fluctuations induced by turbulence.

## Basic Assumptions and Requirements

- **Quasi-elastic interaction**
- **Weak interaction, i.e. $\langle(\Delta n)^2\rangle << \langle n\rangle^2$**
- **The *probing scale* must be larger than the mean free path or the Debye length (plasma).**
- **Small scales are used to sample the dynamics of larger scales.**
- **The persistence time of the refractive index pattern must be longer than the residence time.**
- **The size of the measuring volume must be smaller than the scale to be measured.**

# Scattering and detection:

Scale requirements ⇒

forward scattering

far infrared light ($\lambda$ = 10 $\mu$m)

Weak interaction ⇒ infrared light (or shorter $\lambda$)

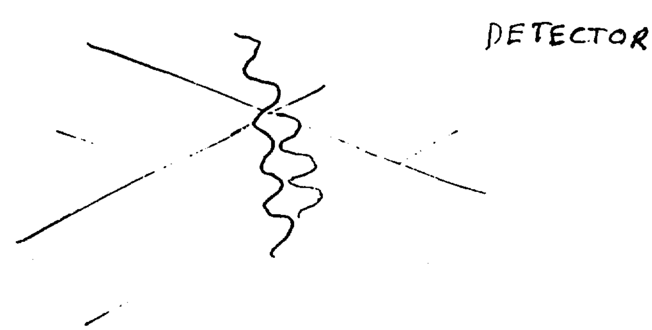
Coherent detection (phase fronts parallel over the detector area):

Reference beam detection because:

parametric amplification needed

simpler statistics

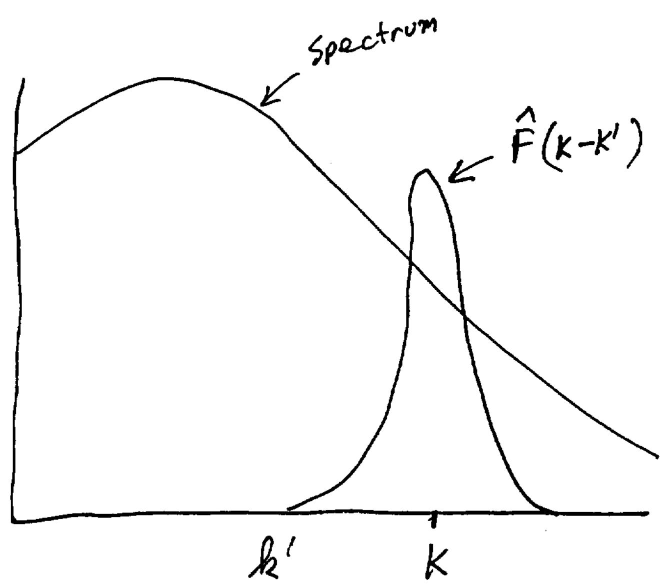
- Scale selected in the usual way by heterodyne scattering angle and laser wavelength.



$$i(t) = \int e^{-i\mathbf{k}\cdot\mathbf{r}} F(\mathbf{r}) S(\mathbf{r},t) d\mathbf{r}$$

$$i(t) = \int \hat{F}(\mathbf{k} - \mathbf{k}') \hat{S}(\mathbf{k}') d\mathbf{k}'$$

$\hat{S}(\mathbf{k}')$ is the Fourier Transform of the spatial distribution of the scattering power.



Here, fluctuations are induced by turbulence itself - e.g. Noise is a manifestation of pressure fluctuations.

Temperature fluctuations can be induced by turbulence in a gas flow

Local electron number density can be induced by turbulence in a plasma.

Divide fluctuations into two types:

1) Propagating - Sound Waves

2) Non-propagating - Temperature Fluctuations

$$dn = \beta_1 dT + \beta_2 dP$$

For a moving, compressible fluid

$$\rho\hat{C}_p\left(\frac{\partial (T - T_0)}{\partial t} + \mathbf{v} \bullet \nabla (T - T_0)\right) =$$

$$\alpha\rho\hat{C}_p\nabla^2 (T - T_0) - \left(\frac{\partial p}{\partial t} + \mathbf{v} \bullet \nabla p\right)$$

Take the spatial Fourier Transform

$$\frac{\partial \hat{\Theta}(\mathbf{k}, t + \tau)}{\partial \tau} - i\,\mathbf{k} \bullet \mathbf{v}\,\hat{\Theta}(\mathbf{k}, t + \tau) =$$

$$- k^2 \alpha \hat{\Theta}(\mathbf{k}, t + \tau) - \frac{p_0 \gamma \delta\omega}{2\rho_0 \hat{C}_p} e^{-i\,\mathbf{k} \bullet (\mathbf{v} + \mathbf{v}_s)(t + \tau)}$$

The correlation function for the temperature fluctuations

$$C_{TT}(\mathbf{k},\tau) \equiv \hat{\Theta}(\mathbf{k},t)\hat{\Theta}(\mathbf{k},t+\ \tau)$$

$$\frac{\partial C_{TT}(\mathbf{k},\tau)}{\partial \tau} - i\,\mathbf{k}\bullet\mathbf{v}C_{TT}(\mathbf{k},\tau)) = -\ k^2\,\alpha\, C_{TT}(\mathbf{k},\tau)$$

$$C_{TT}(\mathbf{k},\tau) = \ C_{TT}(\mathbf{k},0)\exp[\ i\,\mathbf{k}\bullet\mathbf{v}\ \tau]\exp[-\ k^2\ \alpha\ \tau]$$

If we assume sound waves do not attenuate over scale of measurement,

$$C_{PP}(\tau) = \ \exp[\ i\mathbf{k}\cdot(v \pm\ \mathbf{v}_s)\tau]$$

Overall

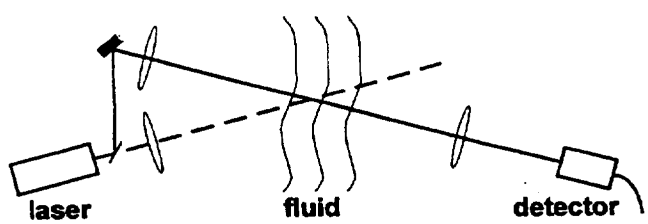
$$\hat{R}_{SS}(\mathbf{k},\tau) = \varepsilon_1^2\exp[-\ i\,\mathbf{k}\bullet\bar{\mathbf{v}}\ \tau]\exp[-\ k^2\ \alpha\tau] + \varepsilon_2^2\exp[-\ i\,\mathbf{k}\bullet(\bar{\mathbf{v}} \pm \mathbf{v}_s)\tau]$$

Note the thermal decay term.

For air, $\alpha \approx 0.1\ \mathrm{cm}^2\ \mathrm{sec}^{-1}$. If $k \approx 10^4\ \mathrm{cm}^{-1}$, the decay time is ca. 0.1 microseconds!

Means that practical system will need small scattering angle.

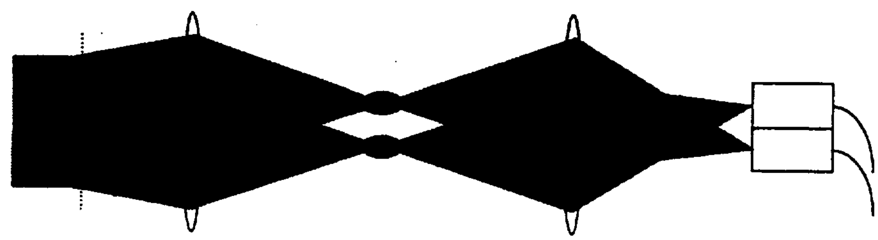
## Doppler Velocimeter




**The *code* of the system is a spatial wave packet.**

**The system has poor spatial resolution along the optical axis.**

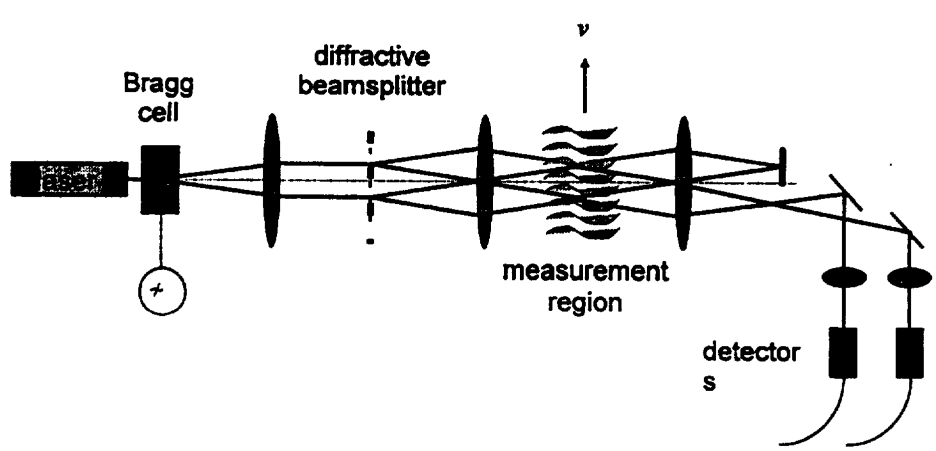
# A time-of-flight configuration

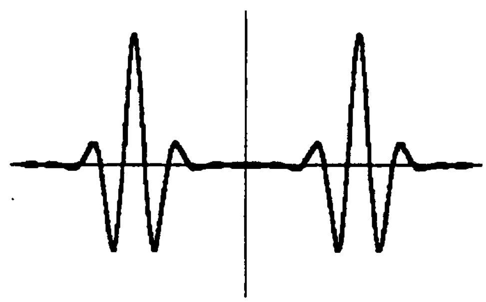


**The *code* is given by two displaced peaks.**

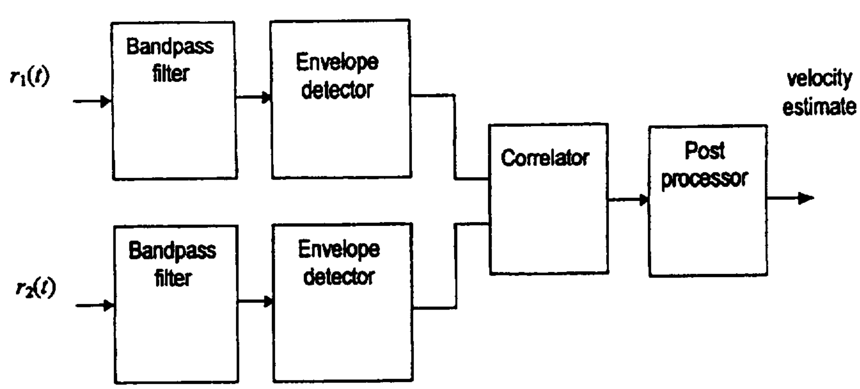
**Has very good resolution along the optical axis - but *does not work in the present case!***

# The hybrid system





The intensity distribution (deviation from the mean) as seen by the detectors of a hybrid laser anemometer, which defines the *code* of the system.



## Cross Correlation

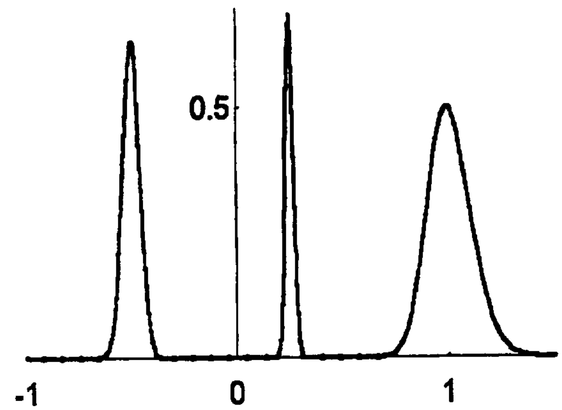
$$R_{12}(\tau) = \int\int \hat{F}(k-k')\hat{F}(k-k'')e^{-i\mathbf{k}''\cdot\mathbf{l}} < \hat{S}(k',t)\hat{S}(k'',t+\tau) > dk'dk''$$

## For turbulent flow

$$^{(2)}R_c(\tau) = \int_{\mathbf{all}\ v} {}^{(2)}R_{12}(\tau, v)\,\mathbf{d}P(v)$$

Envelope detection is used, so

$$R(\tau) = R_{12}^2(0) + 2\,R_{12}^2(\tau)$$




Correlation function assuming both propagating and nonpropagating fluctuations of the same initial power. The s.d. of the convection velocity is 0.1. The Mach # is 0.33.


Mach # of 4.

# Conclusion + current state

- Mean velocity + turbulence from peak + width of crosscorrelation if low turbulence intensity and well separated velocities
- Curve fitting is necessary in general
- Simplest if propagating fluctuations are negligible
- Optics of new system is operating and signals observed