A HYBRID NONLINEAR CONTROL SCHEME
FOR ACTIVE MAGNETIC BEARINGS *

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SUMMARY

A nonlinear control scheme for active magnetic bearings is presented in this work. Magnet winding currents are chosen as control inputs for the electromechanical
dynamics, which are linearized using feedback linearization. Then, the desired magnet
currents are enforced by sliding mode control design of the electromagnetic dynamics.
The overall control scheme is described by a multiple loop block diagram; the approach
also falls in the class of nonlinear controls that are collectively known as the "integrator
backstepping" method. Control system hardware and new switching power electronics
for implementing the controller are described. Various experiments and simulation
results are presented to demonstrate the concepts' potentials.

INTRODUCTION

The dynamic model used for this work is described now, followed by a brief
discussion about several nonlinear control approaches.

System Modeling [6]

The dynamic model for one axis of an active magnetic bearing has several
components. A second-order, linear differential equation describes dynamics of the
rotor's mechanical position $x$ with respect to opposing forces ($F_1$ and $F_2$), which are
produced by the two windings of the bearing axis:

$$m\ddot{x} = F_1(i_1, x) - F_2(i_2, x) + \delta(t).$$  (1)

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In the equation above, the rotor mass is represented by \( m \), while the two winding currents are represented by \( i_1 \) and \( i_2 \). The variable \( \delta(t) \) represents external force disturbances. Force produced by each winding of the bearing is a nonlinear function of winding current and air gap dimension \( g(x) \), which is a function of the rotor position. Magnetic saturation is often ignored in fundamental electromagnetic analysis, yet the resulting relationship shows that force is proportional to the square of winding current, and inversely proportional to the square of air gap:

\[
F_n(i_n, x) = k \frac{i_n^2}{g^2(x)}, \quad n = 1, 2 .
\] (2)

Efforts to increase the force/mass ratio of bearings often result in operation under magnetically saturated conditions. In this case, the relationship between force and winding currents is further complicated. In this work, the relationship described in (2) is used. Hence, the electromechanical dynamics for an active magnetic bearing can be described as a second-order nonlinear system with two inputs (the two winding currents).

Electromagnetic properties for each winding are described by a first-order nonlinear differential equation relating winding current dynamics to the voltage \( e \) applied to the winding:

\[
e_n = R_i + \frac{\partial \lambda(i_n, g(x))}{\partial i} \frac{di}{dt} + \frac{\partial \lambda(i_n, g(x))}{\partial x} \frac{dx}{dt}, \quad n = 1, 2 .
\] (3)

In the above model, \( R \) represents the winding resistance, and the function \( \lambda \) represents the magnetic flux linkage of the winding. In the simplest analysis that does not account for magnetic saturation, the flux linkage is proportional to winding current and inversely proportional to air gap dimension. Therefore, the electromagnetic dynamics are nonlinear. As is true with the electromechanical dynamics, the nonlinearity in the electromagnetic equations becomes more complex in the presence of magnetic saturation.

In summary, the overall dynamics for one axis of an active magnetic bearing are described by a fourth-order, nonlinear model having two inputs. Nonlinearities in the magnetic bearing are differentiable with respect to position, velocity, and current.

Nonlinear Control Options

Advanced control methods for active magnetic bearings have been actively pursued in recent years, and research literature in several fields describe significant achievements using nonlinear methods. Techniques like feedback linearization [1], adaptive control [2], [3], variable structure or sliding mode control [4], and fuzzy logic control [5] have all been examined for magnetic levitation systems. In this work, a
marriage of two nonlinear control techniques is used. Specifically, a feedback linearization controller is used to linearize and control electromechanical dynamics, while a variable structure controller is used to achieve the desired electromagnetic dynamics. Motivation for this hybrid control approach stems from several considerations described in the next section.

CONTROL ALGORITHM CONSIDERATIONS

Each of the nonlinear controllers mentioned in the introduction has unique merits. But it is believed that a hybrid control approach may be able to take advantage of several algorithms’ benefits, and thus address the unique properties and needs of the various magnetic bearing subsystems. In addition, the overall control scheme would be divided into smaller subsystems, each of which may be easier to design than a full-state, nonlinear controller for the entire system dynamics.

Feedback Linearization Considerations

The feedback linearization control method is considered to help account for modeled nonlinearities in the system dynamics. This component shows great promise for compensating well-modeled smooth (differentiable) nonlinearities, yielding improved performance over classical PID control. However, the feedback linearization approach alone becomes very complex for active magnetic bearings when a design model that includes electromagnetic dynamics is used [1]. An intricate system of nonlinear coordinate transformation and nonlinear feedback is needed. In addition, a full-order feedback linearization type controller requires feedback of rotor position, velocity, and acceleration, as well as winding current. Despite these shortcomings the feedback linearization approach demonstrates significant improvement in dynamic response over more simple, linear control approaches [1], [6].

Variable Structure Considerations

Variable structure or sliding mode control is also a candidate to provide a measure of robust performance against disturbances and modeling errors that can be modeled by bounded, differentiable functions. The technique is especially attractive because the switching control signals are easily generated by modern switching amplifiers, whose superior efficiency is important for this application. Like the feedback linearization controller, however, the sliding mode controller is a full state feedback controller and requires position, velocity, and acceleration feedback in any design that accounts for electrical dynamics. Reduced feedback requirement is possible if electrical dynamics are ignored in the design model [4], but this is not an acceptable option for an active magnetic bearing, since the winding inductance is significant.

A Hybrid Nonlinear Control Approach.

To take advantage of merits from both types of controllers, a hybrid controller is
considered here. First, the feedback linearization method is used to design an improved controller for the second-order nonlinear electromechanical dynamics of the magnetic bearing. Application of feedback linearization to second-order dynamics doesn't require a coordinate transformation, uses a less complex nonlinear compensation component, and requires only rotor position and velocity feedback. The feedback linearization controller in this work treats the magnet winding currents as the control inputs. In reality, the winding currents are a state variable in any model that includes electrical dynamics. Thus, winding currents can be considered "pseudo" control signals. To enforce the desired winding currents, a variable structure or sliding mode controller is designed for the electromagnetic dynamics. The first-order electrical dynamics, though nonlinear, are easily and robustly controlled by the sliding mode control, which specifies a switching control signal. Control input for the electrical dynamics is a voltage, which is easily switched at high frequency by modern power electronics.

The hybrid nonlinear controller is illustrated by the multi-loop block diagram in Fig. 1. The outer-most feedback loop is a linear state feedback (PD or PID controllers are other options), used to stabilize the mechanical dynamics of the magnetic bearing. Outer loop controller design is based on a second-order linear state model with state variables being the bearing rotor position $x$ and velocity $\dot{x}$, and control input being "force" $v$. Since the actual electromechanical dynamics are nonlinear, an intermediate feedback loop is used to cancel the modeled electromechanical nonlinearity, so the mechanical dynamics are specified by the outer loop controller output $v$. Design of the nonlinear intermediate loop is based on the feedback linearization principle, and results in specifying two reference winding currents $i_{\text{ref}1,2}$ that cancel the force nonlinearities and also specify the desired linear mechanical dynamics.

![Fig. 1. Block diagram of the hybrid nonlinear controller.](image)

Each winding possesses significant electromagnetic dynamics, however, and current is developed under the influence of applied voltage. Therefore, designs of the outer and intermediate feedback loops based on a second-order model are not entirely correct (the
full design model is fourth order). As discussed in the Introduction, the electromagnetic dynamics are modeled by two first-order nonlinear differential equations (one equation for each winding of the axis) with continuous partial derivatives. By using a variable structure (sliding mode) control for the current, it is theoretically possible to eliminate the electrical dynamics - in practice the electrical time constant is made very small. The innermost feedback loop represents a current feedback controller that uses the variable structure control scheme. A very simple approximation of the variable structure controller is realized using a pulse-density modulated amplifier, which is described in the section to follow.

The multiloop control scheme described here is similar in structure to a technique that has been recently named by some researchers as the "integrator backstepping" or "interlacing" approach [7] for robust nonlinear system design, but it has historical roots in well-known classical servo-mechanism design methods [8].

CONTROL SYSTEM COMPONENTS

Implementation of the control system at Auburn University is accomplished using a digital controller and a new switching power electronics design. Highlights of these two subsystems are presented here.

Digital Controller

The digital controller is built around an Analog Devices ADSP21020 floating-point digital signal processor. Several variations of feedback sensing and control signal interface circuits have been designed and tested at Auburn University [9]. One system uses analog input/output (I/O) port chips, which greatly simplify the hardware packaging. To take advantage of the processor data bus width (32 bits wide for fixed point data), a second system has been developed to access several I/O channels in parallel. For example, 4 channels of 8-bit A/D converters are accessed in parallel. An interface that does not use analog output electronics has also been developed for the second system. Replacing the analog output circuits is a single 16-bit wide digital word that carries the information to directly control the power electronic switches of the four switching power amplifiers. Each amplifier has 4 switches that are directly commanded by the DSP.

Control programs for the system are written using the C programming language. A few basic service functions have been developed in assembly language.

Resonant Switching Amplifier

The power amplifier architecture also presents a technological advance for switching amplifiers. In the past, pulse width modulation has been most commonly used in switching amplifiers. The ratio of semiconductor device conduction time and cutoff time, known as the duty cycle, is varied in proportion to a control signal. While average
efficiency can be very good, the majority of losses can be traced to the switching instances, when the power devices are changing state. A particularly stressful period for a device is during the time that a state transition occurs and voltage is present across the device.

To reduce switching losses in this work, a resonant type of converter is being used in the power amplifier. The amplifier “front-end” is designed to produce a high frequency voltage pulse sequence. Switching of the power devices is synchronized to the time instances in which voltage level equals zero, thus minimizing device stresses and losses. The amplifier design includes an H-bridge to steer the voltage pulse to the load (see Fig. 2). By varying the number of positive and negative voltage pulses to the load, an “average” voltage can be achieved. This scheme is also known as a pulse-density modulation, and is related to the delta-modulation concept used in digital communication systems.

![Resonant-H-Bridged Link](image)

Fig. 2. Resonant-link, pulse-density modulated amplifier

With a very high frequency pulse train, the pulse density modulation scheme is a good approximation of the type of signals often used in sliding mode control designs. In this work, three control voltages can be achieved with this amplifier: “positive,” “negative,” and “zero” voltage. Controller design for this amplifier is also described in the reference [10].

**SOME EXPERIMENTAL RESULTS**

Various experiments are conducted to demonstrate the validity of the control
approach. The feedback linearization concepts is first tested using linear amplifiers. The variable structure controller for the current amplifier is tested in a separate experiment.

Feedback Linearization Component

The feedback linearization controller and a PD controller are compared by examining step responses and sinusoidal responses. Both controllers work well if the deviation from the design state is small. When the perturbation from the design state is large, however, the nonlinear controller exhibits a greater stability margin, as can be seen in the experimental data plotted in Fig. 3a. The PD controlled system does not respond favorably, as can be seen in Fig. 3b.

Variable Structure Component

Shown in Fig. 4 are the recorded oscilloscope traces of the current amplifier responses to step changes in current reference. The current reference is the lowermost trace. Voltage pulses being applied to the winding are shown in the center trace. The current response is the uppermost trace.

SOME SIMULATION RESULTS

The simulated response of the hybrid controller is shown in Fig. 5. The upper left plot shows the position response from an initial error of 1 mm. The velocity response is shown in the upper right plot. The two magnet winding currents are shown in the lower plots. These results show good promise for the hybrid control scheme.

CONCLUSIONS

A hybrid nonlinear control scheme and electronic hardware have been described. The hybrid controller consists of two components: a feedback linearization of the electromechanical dynamics, and a variable structure or sliding mode control of the electromagnetic dynamics. The design of each control component is based on a reduced order model of the subsystem under consideration, so the designs are less complicated than a full-state design of a single controller. In the feedback linearization design, the rotor position and velocity information is needed, while the sliding mode controller uses only current feedback. This hybrid control scheme can be interpreted as a type of "backstepping" or "interlacing" design. Experimental results of the various control system components have been presented, and simulation results for the complete system are encouraging.
References


Fig. 3. Step responses of feedback linearization and PD controllers.
(a) Feedback linearization
(b) PD controller
Reference change exceeds stability limits of PD controller.
Fig. 4. Response of the resonant current amplifier.
Upper trace: load current
Middle trace: load voltage
Lower trace: reference current

Fig. 5. Simulated response of the hybrid nonlinear controller.
Upper plots: position and velocity response.
Lower plots: magnet currents