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INFLUENCE OF THERMOCAPILLARY FLOW ON CAPILLARY STABILITY: LONG FLOAT-ZONES IN LOW GRAVITY

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ABSTRACT

A model problem is posed to study the influence of flow on the interfacial stability of a nearly cylindrical liquid bridge for lengths near its circumference (the Plateau-Rayleigh limit). The flow is generated by a shear stress imposed on the deformable interface. The symmetry of the imposed shear stress mimics the thermocapillary stress induced on a float-zone by a ring heater (i.e. a full zone). Principal assumptions are i) zero gravity ii) creeping flow and iii) that the imposed coupling at the free surface between flow and temperature fields is the only such coupling. A numerical solution, complemented by a bifurcation analysis, shows that bridges substantially longer than the Plateau-Rayleigh limit are possible. An interaction of the first two capillary instabilities through the stress-induced flow is responsible. Time-periodic standing waves are also predicted in certain parameter ranges. Motivation comes from extra-long float-zones observed in MEPHISTO space lab experiments (June 1994).

INTRODUCTION

Observation of several float-zones (MEPHISTO, 1994) shows lengths much longer than their average circumference (up to 50 % longer)[1]. Since knowledge of precise experimental conditions is lacking, there is room for a variety of explanations. The results of the simple model problem posed below suggest one possible mechanism.

In the absence of flow ($c = 0$), the pinned cylindrical interface exhibits, with increasing length, a sequence of shape instabilities due to surface tension. Destabilizing shapes that are antisymmetric (eigenvalues p_n) are interlaced with those that are symmetric about the mid-plane (eigenvalues q_n, where $n = 1, 2, ...$). The instability at the Plateau-Rayleigh limit is antisymmetric and corresponds to eigenvalue p_1.

If the quiescent bridge is perturbed by a flow that is symmetric about the midplane ($c > 0$), the symmetry of the bifurcations corresponding to p_n are preserved giving a steady-streaming-like effect. In particular, that effect on p_1 gives stabilization (figure 1a). At the same time, bifurcations q_n are broken (figure 1b). Symmetry plays a key role in the analysis. Previous studies, including those by Ribicki & Floryan[2], Chen & Shen & Lee[3] and Dijkstra[4], have largely been numerical simulations.

A brief sketch of the model problem is given first. A concise formulation is then set down and symmetry issues are discussed followed by an identification of the underlying mathematical structure (the universal unfolding).

MODEL SKETCH

Figure 2 shows the schematic. Fluid surface tension is assumed to be a linear function of surface temperature $\sigma = \overline{\sigma} + \frac{\partial \sigma}{\partial T}(T - \overline{T})$ and is nowhere zero along the fluid surface. The aspect ratio

$A \equiv (r_o - r_i)/r_o$ models a possibly solidified core and provides a potential unfolding parameter in the analysis. The temperature field of the ambient is approximated by a sinusoidal function $T_a = \Delta T \sin(\pi z)$ to simulate the effect of external heating. Normal stress induced by thermocapillary flow is calculated by the lubrication approximation. Figure 3 plots a typical bifurcation diagram (schematically) obtained by a numerical continuation method [5]. The two state variables, $\varepsilon_1 \equiv < u, \phi_1 >$ and $\varepsilon_2 \equiv < u, \psi_1 >$, are used to measure the solutions in an appropriate function space (see FORMULATION for symbol definitions). The dimensionless parameter c controls the thermocapillary strength, $c \equiv -\frac{\partial \sigma}{\partial T}\Delta T/\overline{\sigma}$.

The bifurcation structure near the singular points p_1 and q_1 is predicted by the symmetry issues alluded to above and discussed in detail in the next section. The first bifurcation point is shifted from the classical limit (2π) by an amount proportional to c — the $\mathbf{Z}_2$-invariant branch may be stable even for bridge length $\ell > 2\pi$. By further increasing c it is possible to turn the pitchfork bifurcation over — i.e. to make it supercritical. The trivially-invariant branch may be stabilized.

FORMULATION AND SYMMETRY ISSUES

The model assumes zero Bond, small Reynolds and small Peclet numbers. The imposed tangential stress at the interface drives the flow. The normal stress balance, however, determines the shape stability and is our focus. Incorporating appropriate boundary conditions, the normal stress balance is set-up as a map

$$\mathcal{F}(u,\ell,c) \equiv \sigma\, 2\mathcal{H}(u,\ell) - P_s + \mathcal{N}(u,\ell,c). \tag{1}$$

The normal stress induced by thermocapillary flow $\mathcal{N}(u,\ell,c)$ is balanced by the mean curvature $2\mathcal{H}(u,\ell)$, with surface deflection function $u(z)$, and static pressure P_s.

Note that at $c = 0$, $\mathcal{N}(u,\ell,0) \equiv 0$ and the classical Young-Laplace equation is recovered. The linear map $\partial_u \mathcal{F}(0,\ell,0)$ has two distinct categories of null space ($n = 1, 2...$)[6]:

$$(I)\ \phi_n = \sin(p_n z), \quad p_n = 2n\pi. \tag{2}$$

$$(II)\ \psi_n = \frac{2}{q_n}(\cos(q_n z) + \frac{q_n}{2}\sin(q_n z) - 1), \quad \tan(\frac{q_n}{2}) = \frac{q_n}{2}. \tag{3}$$

The branching solutions tangent to class (I) form a subcritical pitchfork bifurcation at $\ell = p_n$; the solutions branching from the class (II) form a transcritical bifurcation at $\ell = q_n$. The bridge loses its stability at $\ell = 2\pi$. We shall demonstrate that, when $c \neq 0$, the nonlinear interaction of $\{\phi_1\}(p_1 = 2\pi)$ and $\{\psi_1\}(q_1 = 8.98)$ modes may stabilize an otherwise unstable bridge.

The map $\mathcal{F}$ has reflection symmetry ($\mathbf{Z}_2$) inherited from heating at the midplane

$$\mathcal{F}(\gamma u,\ell,c) = \gamma\mathcal{F}(u,\ell,c), \tag{4}$$

where the flip transformation $\gamma \in \mathbf{Z}_2$ is defined as $\gamma u(z) = u(1-z)$. Here, z is normalized by ℓ. Two remarks are in order:

(i) By the equivariant property (4) one can show that the projection $g \equiv < \mathcal{F}(\varepsilon\phi + w, \lambda, c), \phi^* >$ preserves the symmetry, where w is in the complement of span$\{\phi\}$, ie.

$$g(-\varepsilon,\ell,c) = -g(\varepsilon,\ell,c), \quad \forall n. \tag{5}$$

Generically, (5) shows a pitchfork bifurcation (c.f. figure 1(a)).

(ii) Since $< \partial_c \mathcal{N}(u,\ell,0), \psi^* > \neq 0$, there exists a unique branch of solutions $c = \hat{c}(\varepsilon,\ell)$ near $\ell = q_n$ by the implicit function theorem. The transcritical breaks to a limit point bifurcation[7] (c.f. figure 1(b)).

UNIVERSAL UNFOLDING

The analytical structure and stability of the bifurcation diagram is now considered. Solutions of the nonlinear problem (1) are obtained by the Lyapunov-Schmidt method near the singular points. The bifurcation equation is obtained by the projection of F onto ϕ_1^* and ψ_1^*[8,9]:

$$\mathbf{g} = \left(-\frac{1}{2(8.98)}\varepsilon_2^2 - \frac{1}{4}\lambda_2\varepsilon_2 - \delta_1 c,\ (-\frac{3}{32} + \xi_3 c)\varepsilon_1^3 + (-\frac{1}{4}\lambda_1 + \xi_1 c)\varepsilon_1\right), \tag{6}$$

where $\lambda_1 = 1 - (2\pi/\ell)^2$, $\lambda_2 = 1 - (8.98/\ell)^2$ measure the deviation of the bridge length from the classical bifurcation points, and the coefficients $\xi_1(A)$, $\xi_3(A)$ and $\delta_1(A)$ are obtained through the reduction. A straightforward evaluation using (6) gives the positions of the two bifurcation points (the pitchfork and the limit point), and criteria that turn the trivially-invariant solutions supercritical.

DISCUSSION

Results of numerical continuation and the reduction equations may be summarized. The problem falls into the class of a bifurcation problem in two state variables with $\mathbf{Z}_2$ symmetry $(x, y) \mapsto (x, -y)$. By appropriate coordinate transformations the bifurcation equations may be fitted into a normal form with the topology of a $\mathbf{Z}_2$-codimension 2 bifurcation:

$$\mathbf{g}(x, y) = (x^2 + y^2 - \lambda,\ (\mu x^2 - \beta x + y^2 - \alpha)y), \tag{7}$$

$\mu > 0$. The normal form renders stability of the branching solutions within the limitation of negligible Reynolds and Peclet numbers. The unfolding diagram is given in [9, p.441]. A somewhat surprising result, as seen from that diagram, is that the two-mode interaction may lead to a Hopf bifurcation. A standing wave solution is possible. Predicting the stability of such solution is beyond the present model.

Good estimates from the space experiment, of the strength of the perturbation from zero gravity (relative to the thermocapillary perturbation) and the amplitude of the interface deformation, are most important in assessing the relevance of this model.

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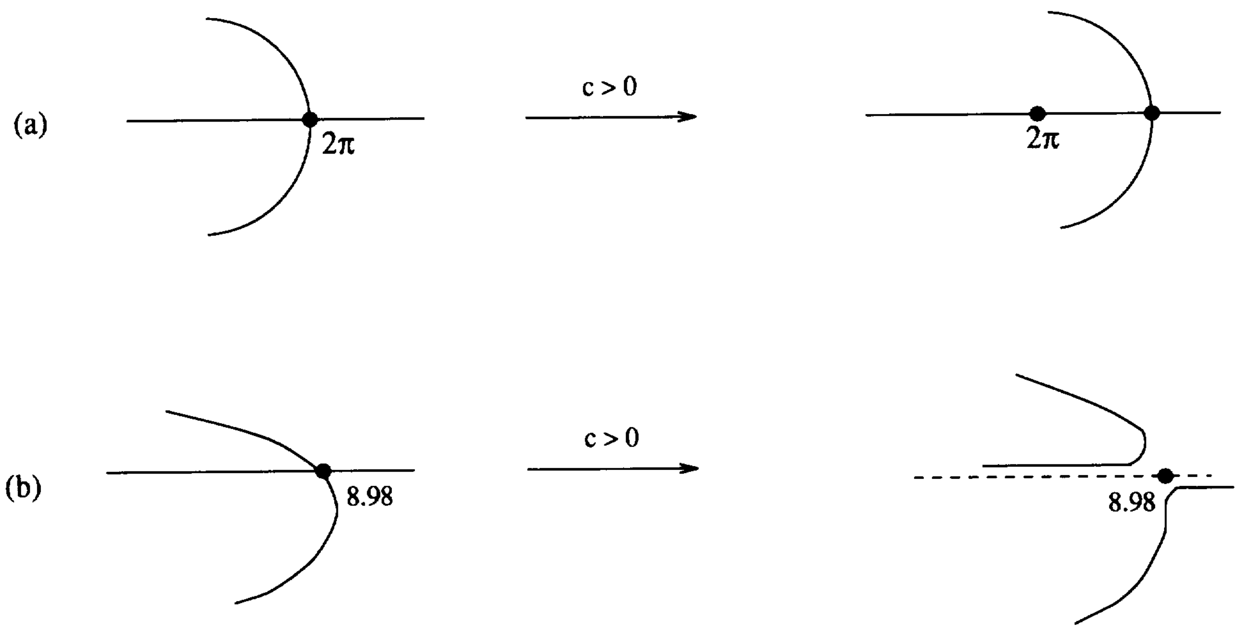
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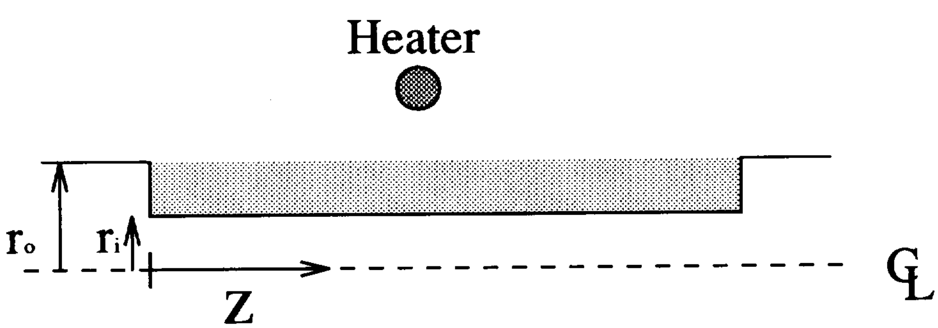
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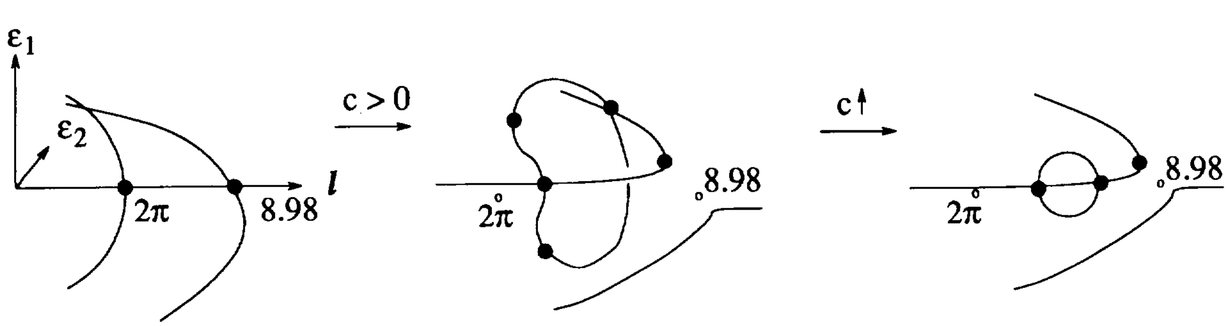
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