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# MAGNETOTHERMAL CONVECTION IN NONCONDUCTING DIAMAGNETIC AND PARAMAGNETIC FLUIDS

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## ABSTRACT

Nonuniform magnetic fields exert a magnetic body force on electrically nonconducting classical fluids. These include paramagnetic fluids such as gaseous and liquid oxygen and diamagnetic fluids such as helium. Recent experiments show that this force can overwhelm the force of gravity even at the surface of the earth; it can levitate liquids and gases, quench candle flames, block gas flows, and suppress heat transport. Thermal gradients render the magnetic force nonuniform through the temperature-dependent magnetic susceptibility. These thermal gradients can therefore drive magnetic convection analogous to buoyancy-driven convection. This magnetothermal convection can overwhelm convection driven by gravitational buoyancy in terrestrial experiments.

The objectives of the proposed ground-based theoretical study are (a) to supply the magnetothermohydrodynamic theory necessary to understand these recent experiments and (b) to explore the consequences of nonuniform magnetic fields in microgravity. Even the linear theory for the onset of magnetothermal convection is lacking in the literature. We intend to supply the linear and nonlinear theory based on the thermohydrodynamic equations supplemented by the magnetic body force. We intend to investigate the effect of magnetic fields on gas blockage and heat transport in microgravity. Since magnetic fields provide a means of creating arbitrary, controllable body force distributions, we intend to investigate the possibility of using magnetic fields to position and control fluids in microgravity. We also intend to investigate the possibility of creating stationary terrestrial microgravity environments by using the magnetic force to effectively cancel gravity. These investigations may aid in the design of space-based heat-transfer, combustion, and human-life-support equipment.

## INTRODUCTION

Elementary electromagnetism shows that a nonuniform magnetic field exerts a body force on permeable materials. The familiar potential energy  $U = -\mathbf{m} \cdot \mathbf{B}$  favors alignment of a magnetic dipole  $\mathbf{m}$  with a static external magnetic induction  $\mathbf{B} = \mu\mathbf{H}$  (in SI units). When  $\mathbf{B}$  is nonuniform, this potential produces a force  $\mathbf{F}_m = -\nabla U = \nabla(\mathbf{m} \cdot \mathbf{B}) = (\mathbf{m} \cdot \nabla)\mathbf{B}$  on the dipole. The corresponding force per unit volume on a permeable magnetic material of magnetization (magnetic moment per unit volume)  $\mathbf{M} = \chi\mathbf{H}$  is

$$f_m = (\mathbf{M} \cdot \nabla)\mathbf{B} = \frac{1}{2}\chi\mu_0\nabla H^2, \tag{1}$$

where the small typical volumetric susceptibilities  $-10^{-5} < \chi < 10^{-3}$  have allowed us to set  $\mu = \mu_0(1 + \chi) \approx \mu_0$ . This magnetic body force is not the pondermotive force of magnetohydrodynamics, which requires a conducting fluid, but arises simply from the force of a nonuniform magnetic field on the molecular dipoles. More detailed treatments (ref. 1) confirm the presence of this magnetic body force.

Apart from exotic ferrofluids (colloidal suspensions of monodomain ferromagnetic particles which do not occur naturally), all electrically insulating fluids are either diamagnetic ( $\chi < 0$ ;  $\chi \approx -10^{-5}$ ) or paramagnetic ( $\chi > 0$ ;  $\chi \approx 10^{-3}$ ). Magnetic properties result from the orbital angular

momentum and spin of electrons, which combine to yield a net intrinsic magnetic moment of the molecule. In addition, an applied magnetic field leads to a small induced magnetic moment whose magnetic flux opposes the applied magnetic flux. In paramagnetic materials such as oxygen, unfilled electronic shells produce a net intrinsic magnetic moment which overwhelms the induced moment. In these materials, the magnetic moments tend to align in the direction of the field so that  $\chi > 0$ , thereby enhancing the field. Diamagnetic materials such as helium, argon, and water have ground-state electronic shells that are completely filled, yielding no net intrinsic magnetic moment. In contrast with the mislabel “nonmagnetic” often given to these materials, their small induced magnetic moments tend to align in a direction opposite the field, yielding small negative values of  $\chi$  and thereby reducing the magnetic field. Consequently, the magnetic body force, Eq. (1), tends to move diamagnetic materials into regions of lower field, whereas it tends to move paramagnetic materials into regions of higher field. This force can balance gravity in both diamagnetic and paramagnetic materials. All ferromagnetic materials are paramagnetic in the molten state.

Paramagnetic fluids have volumetric susceptibilities that are typically proportional to the ratio of mass density  $\rho$  to temperature  $T$  according to Curie’s law (refs. 2, 3);

$$\chi = C \frac{\rho}{T}, \quad (2)$$

where  $C$  is a constant characteristic of the fluid.

The proposed study will utilize the equations governing magnetothermal convection in electrically nonconducting incompressible fluids,

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P + \rho \mathbf{g} + \eta \nabla^2 \mathbf{v} + \mathbf{f}_m \quad (3a)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = D \nabla^2 T \quad (3b)$$

$$\nabla \cdot \mathbf{v} = 0. \quad (3c)$$

Here,  $\mathbf{f}_m$  is the magnetic body force discussed above,  $\eta$  is the viscosity,  $\mathbf{v}$  is the fluid velocity,  $D$  is the thermal diffusivity, and  $\mathbf{g}$  is the acceleration of gravity. General flows require an additional term in this force proportional to the gradient of a scalar (ref. 1). This additional term has been subsumed into the pressure term above, so that the pressure  $P$  contains both thermal and magnetic contributions. The magnetic contribution vanishes identically when the susceptibility is directly proportional to the density, such as for Eq. (2).

An upward magnetic body force  $\mathbf{f}_m$  can easily balance the downward gravitational body force  $\mathbf{f}_g = \rho \mathbf{g}$ , thereby levitating liquids and gases on earth. This balance requires a magnetic induction gradient  $G = |B \nabla B| = \mu_0 g / |\chi_m|$ , where  $\chi_m = \chi / \rho$  is the specific susceptibility. A number of ordinary diamagnetic fluids including water, ethanol, and hydrogen have recently been levitated with gradients up to  $G = 3000 \text{ T}^2/\text{m}$  and fields up to 30 T (refs. 4, 5). Field gradients necessary to balance earth’s gravity for paramagnetic fluids are reduced by a factor  $10^{-2} - 10^{-3}$  owing to their larger susceptibilities. In space, the necessary field gradients are reduced by a factor  $10^{-4} - 10^{-6}$  owing to the reduction in the gravitational acceleration, making levitation possible using small, even hand-held, permanent magnets or electromagnets. Thus, it is feasible to control and position classical fluids in space using the magnetic body force.

## MAGNETOTHERMAL CONVECTION

Buoyancy-driven convection in a stationary closed container requires gradients in the gravitational body force, often achieved by temperature-induced density gradients. Correspondingly,

gradients in the magnetic body force can also drive convection analogous to buoyancy-driven convection. For paramagnetic fluids, gradients in the temperature and the density can supply such gradients through Eq. (2).

The first clear observation of such convection, called the Glenda effect, was reported by Carruthers and Wolfe in 1968 (ref. 3), who experimented with the effect of a 1.55 T electromagnet on gaseous oxygen near room temperature and pressure. They used a rectangular convection chamber in which one wall was heated, the opposite wall was cooled, and the remaining walls were insulated. In the absence of applied magnetic fields, gravitational buoyancy formed a circulation with upflow along the heated vertical wall and downflow along the cooled vertical wall. Placing the chamber in a horizontal field with a vertical gradient (due to fringing of field lines) produced a temperature-dependent magnetic body force opposite to gravity, which reversed the direction of circulation compared to the nonmagnetic case. Thus Carruthers and Wolfe demonstrated that a nonuniform magnetic field can reverse gravitational convection in oxygen.

With the heated wall at the floor of the chamber, the horizontal magnetic field augmented or reduced the Rayleigh-Benard convection observed with no field, depending on the direction of the field gradient. Most startling of all was the observation of vigorous magnetically-driven convection when heating from above, a situation which is stable in the absence of a magnetic field.

The same phenomena were seen in oxygen at reduced pressures and in air, although the magnitudes were smaller. In nitrogen, which is diamagnetic, there were no observable magnetic effects. Carruthers and Wolfe attributed their observations to the fact that oxygen is paramagnetic and that its susceptibility depends on temperature. Although this explanation is likely to be correct, Carruthers and Wolfe did not derive the equations of motion or present any solutions.

In 1977, Clark and Honeywell (ref. 6) rediscovered the Glenda effect the hard way. Beginning about 1970, Honeywell's group began a series of tests to measure the Senftleben-Beenakker effect, a reduction in the thermal conductivity of oxygen gas at low temperatures and pressures caused by a uniform magnetic field. They performed tests using an annular convection chamber in which the inner cylinder was heated. The axis of the chamber was vertical and the magnetic field was horizontal. Because the expected reduction in thermal conductivity was only about 1%, the experiment was conducted with great precision.

The investigators were shocked when the data indicated a 20 to 50% increase in thermal conductivity. After years of rechecking the experiments and the analysis, they finally traced this increase to the small (0.2%) inhomogeneities in their magnetic field. Thus, the Glenda effect can appear where it is unexpected, and can be surprisingly large. It is likely that other experimentalists have seen the Glenda effect without realizing it. Indeed, Carruthers and Wolfe had earlier pointed out the potential of the Glenda effect to distort precision electrical measurements.

Clark and Honeywell (ref. 6) presented a rather cursory derivation of the governing equations and obtained a solution for fully developed natural convection of an ideal gas between infinite, parallel vertical plates, including both gravitational and Glenda forces. Although the solution was two dimensional and the experiment axisymmetric, the theoretical predictions were close enough to the data to convince the authors that the Glenda effect dominated the flow. Their analysis ignores horizontal magnetic gradients, which may play an important role.

A series of recent experiments (refs 7, 8) reveals convection in a paramagnetic solution of gadolinium nitrate. These experiments identify precise convective thresholds for vertical thermal and magnetic-field gradients. For example, they found that stabilizing vertical magnetic-field gradients exceeding  $G = 15 \text{ T}^2/\text{m}$  completely suppress convection when the layer is heated from below, and that destabilizing magnetic-field gradients exceeding  $G = 5 \text{ T}^2/\text{m}$  produce convection when the layer is heated from above. These thresholds, and the accompanying convection, are of prime interest in the proposed study.

Finlayson (ref. 9) considered the theoretical convective instability of ferrofluids under vertical magnetic fields which are *spatially uniform* in the absence of a thermal gradient. Further experimental and theoretical work on such ferrofluids is summarized in Stiles and Kagan (ref. 10).

Theoretical analysis of the experiments of Carruthers and Wolfe (ref. 3) and of Braithwaite *et al.* (ref. 7) can be carried out generally. Both experiments involve a temperature gradient in the fluid imposed by two parallel bounding conducting plates at temperatures  $T = T_0 \pm \Delta T/2$ . These plates are horizontal for Braithwaite *et al.* and are both horizontal and vertical for Carruthers and Wolfe. To understand the various convection patterns seen in these experiments, it is convenient to write the equations of motion in terms of the deviations from a static thermal conduction state with a uniform thermal gradient  $\nabla T = (\Delta T/d)\hat{n}$ , where  $\hat{n}$  is the unit vector normal to this gradient, and  $d$  is the separation between these plates. Dimensionless deviations about this state satisfy

$$\mathcal{P}^{-1} \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \mathbf{R}\theta + \nabla^2 \mathbf{u} ,$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta + \hat{n} \cdot \mathbf{u} = \nabla^2 \theta$$

$$\nabla \cdot \mathbf{u} = 0 .$$

Here,  $\mathbf{u}$ ,  $p$ , and  $\theta$  are respectively the dimensionless velocity, pressure, and temperature; length, time, temperature, and pressure are measured respectively in units of  $d$ ,  $d^2/D$ ,  $\Delta T$ , and  $D\eta/d^2$ . Dimensionless parameters include the Prandtl number  $\mathcal{P} = \nu/D$  and a vector Rayleigh number

$$\mathbf{R} = \frac{g\alpha\Delta T d^3}{\nu D} \left( -\hat{g} + \frac{\mu_0}{2\alpha g \rho_0 d} \frac{d\chi}{dT} \Big|_{T_0} \nabla H^2 \right) \quad (4)$$

which includes magnetic effects. The applied magnetic field  $\mathbf{H}$  is measured in amperes per meter, but is considered here as a function of dimensionless position  $\mathbf{x}$ . In arriving at these equations, we have made use of first-order Taylor expansions about  $T_0$  in the temperature-dependent density and susceptibility. We have also employed the usual Boussinesq approximation, that is, we have retained density changes only in the large gravity term. An important task in the project is to justify this approximation rigorously.

The vector Rayleigh number involves the thermal expansion coefficient  $\alpha$ , the density  $\rho_0$  at temperature  $T_0$ , the kinematic viscosity  $\nu = \eta/\rho_0$ , and the unit vector  $\hat{g}$  in the direction of gravity. The prefactor in the Rayleigh number applies to Rayleigh-Bénard convection for a fluid layer heated from below in the absence of any magnetic fields. For a Curie's law paramagnetic material [Eq. (2)], the magnetic term involves

$$\frac{d\chi}{dT} \Big|_{T_0} = -\chi_0 \alpha \left( 1 + \frac{1}{\alpha T} \right) .$$

The first and second terms in this expression follow respectively from the implicit temperature dependence through  $\rho$  and from the explicit temperature dependence in Eq. (2). Setting  $\hat{z} = \hat{g}$  and  $\mathbf{H} = H(z)\hat{z}$  yields  $\mathbf{R} = -R_m\hat{z}$ , with a scalar 'magnetic' Rayleigh number  $R_m$  agreeing with Braithwaite *et al.* The form  $\mathbf{H} = H(z)\hat{z}$  gives an unphysical nonzero divergence  $\nabla \cdot \mathbf{B}$  of the magnetic induction  $\mathbf{B} = \mu\mathbf{H}$ . In contrast, our vector Rayleigh number is completely general, allowing for both horizontal and vertical components of  $\mathbf{H}$  satisfying both  $\nabla \cdot \mathbf{B} = 0$  and  $\nabla \times \mathbf{H} = 0$ . Thus, the appropriate Rayleigh number for the problem is a vector which is generally nonuniform in both magnitude and direction owing to nonuniformities in  $\nabla H^2$ . One of the objectives of the project is to study the effects of these nonuniformities to determine when magnetic effects [the second term in Eq. (4)] can be used to cancel buoyancy effects [the first term]. Such studies will be carried out both analytically and numerically.

## POTENTIAL SIGNIFICANCE

The Glenda effect allows the creation of a virtually arbitrary, controllable body force distribution in paramagnetic fluids such as oxygen, nitric oxide, aqueous ferric perchlorate, and numerous transition metal molten salts. Similar, although weaker, forces can be produced in diamagnetic fluids. The potential applications of this unique and largely unexplored phenomenon are likely far broader than we can presently envision, but several attractive possibilities have been suggested.

In considering potential uses, it is important to recognize that the magnetic body force is independent of gravity. While strong enough to be important in normal Earth gravity for fluids such as oxygen and air, its greatest utility will surely be in microgravity environments where it may be the only significant body force.

Magnetothermal forces should occur whenever a wire carrying electricity heats the surrounding air. Undoubtedly the effect is usually negligible, but there may be some anomalies for which it is the unrecognized cause. It would be particularly important to account for or eliminate the Glenda effect in very precise measurements of fluid properties.

In principle, one could inhibit or enhance the effects of gravitational convection on Earth or create artificial "natural" convection in an orbiting spacecraft, thus enhancing or suppressing mixing and heat transfer as may be opportune. These abilities may be of interest for both engineering and scientific applications.

A remarkable apparatus called the Geophysical Fluid Flow Cell has used electrical permittivity gradient forces to simulate a radial gravity field in the gap between rotating concentric hemispherical shells for planetary and stellar atmospheric circulation experiments. The working fluid in these tests was a liquid with a Prandtl number of 8.4 (ref. 11), which is at least an order of magnitude higher than the prototype value in most cases. If an experiment of this type could be designed using the Glenda effect, oxygen, with its Prandtl number of 0.7, could be used to achieve closer similarity with the prototype. The simulated gravity field in the Geophysical Fluid Flow Cell varied with the inverse fifth power of the radius, rather than the inverse square dependence of true gravity. It might be possible to approach the correct dependence more closely using Glenda forces.

Measurements have confirmed that the gravitational environment on an orbiting spacecraft is characterized by an effective gravitational acceleration which varies randomly in direction and in time with a frequency range of 0.1 to 10 Hz. The amplitude of this g-jitter is on the order of 0.01% of the earth normal value (ref. 12). Given a suitable control system, it might be possible to cancel much of the g-jitter by using the Glenda effect. It would very likely be possible to generate controlled g-jitter without using a shake table, either in orbit or on the Earth.

The Glenda effect could be used to pump paramagnetic fluids such as liquid oxygen or to deflect fluid currents without using any moving parts, if that should be desirable.

Many potential applications lie in the field of materials processing, particularly in the microgravity environment. In some instances it is important to prevent contact between a melt and the walls of the container. Electromagnetic forces, acoustic standing waves, and gas streams have been used to achieve this goal (ref. 12). The Glenda effect may have advantages for insulating paramagnetic fluids. The possible importance of the Glenda effect in crystal growth was pointed out by Carruthers and Wolfe (ref. 3) who used it to explain observations on the growth of ferrous ammonium sulfate crystals in a magnetic field. The Glenda effect might be particularly useful in connection with the growth of crystals from insulating paramagnetic melts.

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