EFFECT OF SPACECRAFT ROTATION ON FLUID CONVECTION UNDER MICROGRAVITY

Valentin S. Yuferev, Elvira N. Kolesnikova, Yuri A. Polovko, and Alexander I. Zhmakin
Ioffe Physical-Technical Institute, St. Petersburg, 194021, Russia

ABSTRACT

The influence of the rotational effects on two-dimensional fluid convection in a rectangular enclosure with rigid walls during the orbital flight is considered. It is shown that the Coriolis force influence both on steady and oscillatory convection becomes significant at Ekman numbers which are quite attainable in the space orbital conditions. In the case of harmonic oscillations of the gravity force appearance of the resonance phenomena is demonstrated. Dependence of the height and shape of the resonance peak on aspect ratio of a rectangular domain and orientation of vectors of the gravity force and the angular rotation velocity is studied. Special attention is given to non-linear effects caused by convective terms of Navier-Stokes equations. The convection produced by variations of the angular rotation velocity of a spacecraft is also discussed. It is shown that in some cases the latter convection can be comparable with another kinds of convection.

INTRODUCTION

It is well-known that rotation of fluids can result in appearance of quite unusual flow patterns (ref. 1). It is natural therefore that rotational effects are to be considered when studying convection of fluids in the orbital flight conditions. Here first of all it is necessary to take into account the Coriolis force which is arisen when convective motion is considered in the coordinate system attached to the container with fluid. Strange as it may be seen but a very small attention has been paid so far to the Coriolis force effect on fluid convection in a microgravity environment (ref. 2). The most of researchers have restricted themselves to using the model of effective acceleration (ref. 3) that is equivalent to consideration of the case of harmonic gravity oscillation without any Coriolis force action. It is obvious that in such approach the peculiarities inherent to flow patterns in rotating fluids are lost.

The effect of the Coriolis force is determined by Ekman number $\text{Ek} = \frac{v}{(\Omega L^2)}$, where $L$ is the characteristic length, $v$ is the kinematic viscosity and $\Omega$ is the angular rotation velocity. In table 1 the basic characteristic of the fluid convection in the orbital experiments are compared with parameters of the atmosphere flows in the terrestrial conditions. One can see that Ekman numbers on a spacecraft board can be in principle as small as in the Earth atmosphere, where the influence of the Coriolis force is recognized. Another specific features of the convection during the orbital flight are manifested when angular rotation velocity of the spacecraft is varied. In particularly, in this case acceleration $r \times \frac{d\Omega}{dt}$ where $r$ is the radius-vector originating from some fixed point within the rotating container is appeared. Since this acceleration field is vortex it can not be included into the pressure gradient term and will result in convection motion even if the fluid is isothermal. It is just these rotational effects and their interaction with g-jitter that are studied in the present paper.

FORMULATION OF THE PROBLEM

Boussinesque approximation for the thermal convection of incompressible fluid is used. As the Ekman numbers are assumed to be moderate we use scaling adopted in the natural convection problems and take the enclosure height $h$, $\Delta T$, $\rho g_0 \beta h \Delta T$, $g_0$ and $\Omega_0$ as the scales for the length, temperature, pressure, microacceleration and angular velocity, respectively. At the same time the choice of the velocity and time scales were taken dependent on the convection intensity $\nu = \frac{U}{\Delta h} \sqrt{\frac{h}{g_0 \beta h \Delta T}}$, $[t] = h^2 \sqrt{\frac{\Delta h}{g_0 \beta h \Delta T}}$
where \( Gr = g_0 \beta \Delta T h^3 \nu^{-2} \) is the Grashof number and coefficient \( \alpha \) is equal to zero in the case of weak convection (Stokes approximation) and unity when convection is intensive and described by the full Navier-Stokes equation. The enclosure is assumed to be infinite in OY direction. The latter means that the present problem formulation describes the fluid flow in the middle part of the three-dimensional enclosure when its dimension in OY direction considerably greater than dimensions along OX and OZ axes. The microacceleration vector was written in the form \( \mathbf{g}(t) = \mathbf{g}(t) + \mathbf{r} \times \frac{d\mathbf{\Omega}}{dt} \) where the first term results in generation of Archimedian force while the second one describes the isothermal vortex acceleration field in the fluid. No restrictions are imposed on the directions of the vectors \( \mathbf{g} \) and \( \mathbf{\Omega} \). In solving the thermal equation the upper and lower walls were assumed to be isothermal while the side walls - to be adiabatic. The presented formulation generalizes the problem commonly used when considering flow in a rotating annulus heated from below or due to a horizontally applied temperature gradient (see, for example, ref. 4).

**CALCULATION RESULTS**

The effect of the Coriolis force on steady convection.— For constant gravity force this problem was studied by many authors and our only aim is to demonstrate shortly the Coriolis force influence on the magnitude of convection. For definiteness it is assumed that \( g_{1 \nu} = \mathbf{g} = 1; \ g_{1\gamma} = g_{1\zeta} = 0; \ \Omega_\gamma = 0, \ \Omega_\zeta = 1, \ T = z \) and aspect ratio \( d = 2 \). Fig. 1 shows dependence of the maximal velocity components on the Ekman number. With decrease of Ekman number the \( x \)- and \( z \)-components of the velocity diminishes while the \( y \)- component (thermal wind) at first increases quite quickly but then begins to decrease too. The Coriolis force influence on the structure of stationary flow is illustrated by fig. 2. One can see that the presence of the Coriolis force results in formation of the flow core with very low velocity and qualitative changes in the temperature field.

Weak convection under harmonic gravity oscillations.— It is known that if \( Gr < 100 \) or \( \omega >> 1 \) or Rossby number \( Ro = Ek \sqrt{Gr} << 1 \) then one can use the Stokes approximation. The bulk of attention in this section is given to the resonance phenomenon which arises when \( Ek << 1 \) and \( \omega >> 1 \). Here first of all it is necessary to note that point \( \omega = 2Ek^{-1} \) is singular. In this case near the walls of the enclosure the specific boundary layers with thickness proportional to \( \omega^{1/2} \) can be developed while in the central part of a cavity velocity distribution remains weakly dependent on the frequency. Besides, at \( \omega < 2Ek^{-1} \) differential operator of an inviscid equation changes its type transforming into hyperbolic operator. It is just of this change of the operator type that is the reason of the resonance appearance. As a characteristic of the intensity of oscillatory convection we used the root-mean-square (rms) velocity of the fluid (over a period of oscillation). Fig. 3 demonstrates the dependence of the maximum rms velocity on Ekman number and frequency \( \omega \) in cross-section \( x=0 \). One can see, that starting with some value of frequency this dependence has a distinct resonant character. The resonance appears when near the walls the Ekman boundary layers are developed while inside the enclosure an inviscid core is formed. The height and shape of the resonance peak strongly depends on aspect ratio of the enclosure and on orientation of the vectors of the microacceleration and angular rotation velocity. In real conditions of the orbital flight the resonance phenomenon will arise only under action of the extremely low frequency microaccelerations and can be observed provided that convection caused by these microaccelerations will be compared with the convection produced by the stronger high-frequency vibrations.

Nonlinear effects. — In the Stokes approximation the velocity field remains "monochromatic" under harmonically modulated gravity and contains only a single harmonic with the frequency \( \omega \) while the temperature is not changed in time at all. If we consider the full Navier-Stokes equations then additional harmonics of the flow field (including time-averaged component) are excited and time variation of the temperature nonuniform across a fluid region appears. The structure of the time-averaged flow in XOZ plane is shown in fig. 6 for three values of Ekman numbers, the middle of them being the value
corresponding to the resonance peak of the maximum of rms velocity. A rather complex pattern of the time-averaged flow is a result of non-linear effects. It is necessary to emphasize, that in the case under consideration the time-averaged flow is small relative to the basic pulsatile motion and therefore its study is of only theoretical interest. However, if fluid has a free boundary then intensity of the time-averaged flow can be quite significant and compared with that of the pulsatile one. Fig. 7 shows that in the case of the intensive convection \((Gr = 10^4)\) the resonance phenomena arises too, although Rossby number is of order of unity. It is seen that the consideration of the convective terms in governing equations results in decreasing the height of resonance peak and manifests itself mainly in vicinity of this peak.

The influence of variation of the angular rotation velocity.—Isothermal convection under harmonic oscillations of the angular rotation velocity \(\Omega = \Omega_0 \exp(j\omega t)\) is considered. Vector \(\vec{\Omega}\) is directed along OY axis and convection velocity is normalized to \(\Omega_0 h\). Fig. 8 shows that with increase of \(\omega\) maximal rms velocity quickly approaches to its maximal value. Thus, for example, if \(\omega, \Omega_0 = 0.001 \text{ s}^{-1}\) (free flight), \(v = 0.001 \text{ cm}^2 / \text{s}\) and \(h\) is of order 1-5 cm then convective velocity will be of order of \(10^{-5} \div 10^{-3} \text{ cm} / \text{s}\) that is compared with convection produced by Archimedian force at low level of g-jitter (\(\sim 1-10 \mu g\)). Moreover, co-action of variation of the angular velocity and j-gitter can in principle lead to appearance of the new quite unusual resonance depending on the phase difference between oscillations of gravity and angular velocity (ref. 5).

CONCLUDING REMARKS

1. The Coriolis force is an essential element of the microgravity environment. The influence of the Coriolis force can become significant when \(Ek<0.1\). These values of Ekman numbers are quite attainable in the space orbital conditions if, for example, the size of container with fluid is of order 10 cm for the water, 3-4 cm for the semiconductor melts and 1 cm for the liquid helium.

2. In design of a space fluid convection experiment it is necessary to control level of both microaccelerations and variations of angular rotation velocity.

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REFERENCES


Table 1

<table>
<thead>
<tr>
<th>Kinematic viscosity</th>
<th>Atmosphere flows</th>
<th>Convective motion in the orbital flight conditions</th>
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<tr>
<td>~ 1 cm²/s</td>
<td>10⁻² – 10⁻⁴ cm²/s</td>
<td>10⁻² – 10⁻⁴ cm²/s</td>
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<td>Length scale</td>
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<td>1 – 100 cm</td>
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<tr>
<td>Angular rotation</td>
<td>the Earth ~ 5·10⁻⁵ s⁻¹</td>
<td>spacecraft ~ 10⁻⁴ s⁻¹</td>
</tr>
<tr>
<td>Angular rotation</td>
<td>10⁻⁴ – 10⁻³</td>
<td>10⁻⁵</td>
</tr>
<tr>
<td>Ekman number</td>
<td></td>
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Fig. 1. The dependence of the maximal values of the velocity components and velocity module over cross-section XOZ on Ekman number. Pr=1.0; Gr=10⁴.

Fig. 2 The effect of the Coriolis force on the structure of the stationary flow. Gr = 10³; Pr = 1.0. right - Ek⁻¹ = 100; left - Ek⁻¹ = 0.
Fig. 3. The dependence of the maximal rms velocity on Ekman number and modulation frequency. Curves: 1 - Ek$^{-1}$ = 10; 2 - 20; 3 - 50; 4 - 100. Aspect ratio and orientation is the same as in Fig. 1.

Fig. 4. Effect of the aspect ratio on the maximal rms velocity. Ek = 0.01, curves: Aspect ratio: 1 - 6.0; 2 - 2.0; 3 - 1.0; 4 - 0.8; 5 - 0.6. Orientation is the same as in Fig. 1.

Fig. 5. The effect of the mutual orientation of the microacceleration and angular rotation vectors on the maximal rms velocity at the cross-section $x = 0$.

a) $\Omega_x = \Omega_z = 1; \ g_x = 1; \ g_y = g_z = 0; \ d = 1.0$;

b) $\Omega_x = 1; \ \Omega_z = 0; \ g_y = 1; \ g_x = g_z = 0; \ d = 2.0$.

Curves: 1 - $\omega = 20; 2 - 200; 3 - 400$. 
Fig. 6. Contour lines of the time-averaged flow: $y$ - component of the velocity (right), velocity vectors in XOZ plane (left) at $Gr=10^3$; $Pr=1$. $Ek^{-1}=50$ (upper); 110 (middle) and 200 (lower).

Fig. 7. The effect of the convection intensity on the maximal rms velocity. $Gr=10^4$, solid lines corresponds to solution of the full Navier-Stokes equations; dashed lines - Stokes approximation. Curves: 1 - $\omega = 1$; 2 - 2; 3 - 4.

Fig. 8. The dependence of the maximal rms velocity on frequency of the angular velocity variation (Stokes approximation).