Measurement of Gear Tooth Dynamic Friction

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ABSTRACT

Measurements of dynamic friction forces at the gear tooth contact were undertaken using strain gages at the root fillets of two successive teeth. Results are presented from two gear sets over a range of speeds and loads. The results demonstrate that the friction coefficient does not appear to be significantly influenced by the sliding reversal at the pitch point, and that the friction coefficient values found are in accord with those in general use. The friction coefficient was found to increase at low sliding speeds. This agrees with the results of disc machine testing.

INTRODUCTION

Friction between sliding surfaces at the gear tooth contact is usually the major source of power loss in gear transmissions. The coefficient of friction is important for predicting scoring resistance and surface durability of gears, and it is a critical parameter in the design of traction drives.

The type of contact which exists in long-wearing gear systems is termed elastohydrodynamic lubrication, where a thin film of lubricant separates elastically deformed solids, and there is minimal surface asperity contact. The existence of this film is possible because of the very large increase in viscosity with pressure of the lubricant.

In the heavily loaded lubricated elastohydrodynamic contacts of gear teeth the lubricant can undergo a rapid rise of pressure from atmospheric to over one Giga Pascal (200 000 psi) in as little as 0.1 millisecond. At the same time the fluid undergoes shearing which leads to heat generation. Temperatures can reach several hundred degrees Celsius. In addition, there are rapid variations in sliding velocity and load as teeth pass along the line of contact. The very complex rheological behaviour of the fluid in these extreme conditions precludes the use of steady-state (static) measurements for the evaluation of fluid properties. Nearly all of the studies of this contact phenomenon have been based on disc machines, where most of the conditions existing at the tooth contact, other than the rapid variation of sliding speed and load, can be simulated by rolling discs against each other with a speed mismatch to simulate gear tooth sliding and rolling.

Comprehensive accounts of earlier experimental studies in elastohydrodynamic lubrication are given by Dowson (1967) and Dowson and Higginson (1966). Crook (1961) theoretically analysed the friction and temperatures in the oil film, and derived the friction versus sliding speed characteristic curve (Fig. 1). This curve shows the dependence of friction on sliding velocity. This analysis assumed that the oil film behaves as a Newtonian fluid with a viscosity dependent only on pressure and temperature. He assumed the viscosity variation with both temperature and pressure to be exponential with constant exponent coefficients.

The tests of Crook (1961) were carried out at comparatively low contact pressures (less than 0.59 GPa, 85 000 lbf/in²). It soon became apparent (Johnson and Cameron, 1967) that at high pressures and low speeds, the assumption of an exponential increase in viscosity predicted impossibly high tractions. Johnson and Cameron (1967) identified two critical features -- a large reduction in the rate of increase of viscosity with pressure above 0.7 GPa (100 000 lbf/in²), and a ceiling to the traction coefficient largely independent of contact pressure, rolling speed and disc temperature. They advanced a hypothesis of plastic shear when a critical stress was reached. Townsend (1968) summarised similar findings by other researchers, and stated that without such reductions in the viscosity coefficient, that the lubricant would become stronger than the bearing material.

Tevaarwerk (1985a) describes the development of a constitutive lubricant friction model for traction drives that incorporates a viscoelastic and plastic-like dissipative element. For conditions of high slide-roll ratios such as gear contacts this model was simplified by the omission of the elastic response of the fluid (Tevaarwerk, 1985b). Data from rig tests were used to determine the lubricant parameters.

The experimental measurement of friction has usually utilised disc machines, or in some instances, ball-testing. There have been several attempts to measure the friction coefficient through the mesh
cycle. Benedict and Kelly (1961) attempted to measure instantaneous gear tooth friction using a test rig in which one of the supports was strain-gaged (Fig. 2), but they encountered dynamic problems due to inertia and a low natural frequency of the assembly. As a result, they reverted to the use of a disc machine for friction measurement. Radzimovsky (1972) constructed a closed-loop gear test machine to measure the instantaneous coefficient of friction through recording the instantaneous torque required to rotate the gear set. However, the rig was operated at very slow speeds (6 rpm) to minimise dynamic effects due to system inertia. As a consequence, the contact conditions were not those where a hydrodynamic oil film could be developed, and therefore not applicable to elastohydrodynamic lubrication.

A number of measurements of overall losses due to friction have been carried out, for example Anderson and Loewenthal (1979), Krantz and Handschuh (1990) but these techniques cannot detect the variation in friction during the tooth engagement cycle.

An earlier series of tests by the authors (Rebbechi, et al., 1991, Oswald, et al., 1991) utilised in-situ calibration of an instrumented gear to separate the normal and frictional effects. These tests were successful in providing for accurate resolution of normal loads, but quantitative assessment of friction loads was not possible, as the calibrating friction force was just the limiting value of static friction attained as the gear pair were slowly rotated under load.

The aim of this report is to describe the design principles and operation of a calibration rig, to evaluate the dynamic normal and friction forces at tooth contact, and to present results from testing in the NASA gear noise rig. The data presented here include a comparison of measured friction values with theoretical predictions for a range of speeds and loads. The data used in this paper were from the same series of tests as Oswald, et al., (1996).

**APPARATUS**

Dynamic testing was carried out in the NASA gear noise rig as described in Oswald, et al., (1996). The rig includes a simple gearbox powered by a 150 kW (200 hp) variable speed electric motor, with an eddy current dynamometer to provide power absorption on the output. Test speeds ranged from 800 to 6000 rpm. The test gears were identical 28-tooth AGMA Class 15 gears (Table 1). Tests on two gear sets are described here, one set with fairly heavy profile modification (designated set D) and the other set unmodified (without tip relief, set A). The profiles for these gear sets are given by Oswald and Townsend (1995).

### Table 1.—Test Gear and Lubricant Parameters

<table>
<thead>
<tr>
<th>Gear tooth</th>
<th>Module, mm (diametral pitch)</th>
<th>Standard full depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers of teeth</td>
<td>28 and 28</td>
<td></td>
</tr>
<tr>
<td>Face width, mm (in.)</td>
<td>6.35 (0.25)</td>
<td></td>
</tr>
<tr>
<td>Pressure angle, deg</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Pitch circle diameter, mm (in.)</td>
<td>88.9 (3.5)</td>
<td></td>
</tr>
<tr>
<td>Contact ratio (nominal)</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td>100 percent torque, Nm (in.lbf)</td>
<td>71.7 (635)</td>
<td></td>
</tr>
<tr>
<td>Accuracy</td>
<td>AGMA 15</td>
<td></td>
</tr>
<tr>
<td>Lubricant</td>
<td>MIL-L-23699B</td>
<td></td>
</tr>
<tr>
<td>Viscosity, CP at 70 °C</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td>Pressure coefficient viscosity, Gpa^-1 (in.?lbf) at 54 °C</td>
<td>14.2 (0.000098)</td>
<td></td>
</tr>
<tr>
<td>Temperature coefficient viscosity, °C^-1 (°F^-1)</td>
<td>0.028 (0.016)</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity, W/(m·°C) (ft-lbf/(s·ft·°F))</td>
<td>0.14 (0.0175)</td>
<td></td>
</tr>
</tbody>
</table>
The lubricant used for the tests was synthetic turbine engine oil (MIL-L-23699B) which at the mean temperature used in these tests of 70 deg Celsius has an absolute viscosity of 8.7 cP.

**Static Calibration Rig**

A calibration rig was devised to enable independent application of the normal and tangential tooth forces (Figs. 3 and 4). In this rig one gear shaft, equipped with the instrumented test gear, is free to rotate only. The other shaft, which contains a single-tooth loading gear, is free to both rotate and slide. The sliding motion, which is accommodated by linear recirculating ball-bearings, is constrained so as to be perpendicular to the line of action—in other words, in the direction of friction. The arrangement is such that a normal force between the teeth can be applied without a friction force being present. Conversely, provided that there is sufficient normal force between the teeth to prevent them from sliding relative to one another, a tractive force (simulating friction) can be applied tangent to the tooth contact interface, independently of the normal force.

**Instrumentation**

Strain gages were installed on the root fillets of two successive teeth on the output gears, on both the tensile and compressive sides. The gage position was chosen to be at the 30 degree tangency position (Fig. 5). For static calibration wheatstone bridge circuits were used, and for dynamic measurements the gages were connected through a slip-ring assembly to constant current signal conditioners.

Data acquisition was achieved using a 12 bit data acquisition card installed in a personal computer. Sample rates ranged from 6.6 to 50 kHz for each of the five channels, being the four gages plus a once-per-revolution encoder signal which provided an angular position reference. These sample rates provided for approximately 500 samples per revolution for each channel.

**TEST PROCEDURE**

**Calibration**

The test gear was calibrated so as to enable subsequent evaluation of the dynamic normal and friction forces at tooth contact. This is possible because of the linear independence of the strain gage response to normal and tangential contact forces. Calibration was carried out using a single-tooth loading gear, so that load could be applied over the full range of the tooth engagement cycle, while
avoiding indeterminate load sharing from an adjacent tooth. The torque loading was applied in four increments; 0, 57, 85 and 113 percent of 71.7 Nm torque. This procedure was carried out at roll angle increments of 2 degrees from 10 to 32 degrees. One extra reading was taken at 21 degrees because this is approximately the pitch point of the gears. Once this "frictionless" calibration was complete, the procedure was repeated using a constant torque loading to prevent slip, and 2 traction (friction) loads of 100 and 190 N (22.3 and 42.4 lbf).

The data were used to generate a tooth force influence coefficient matrix as described in the following section. This procedure is similar to that described by Rebbechi, et al., 1991, but a significant improvement is now possible in that the calibration rig described here enables quantitative assessment of friction force in addition to normal force.

An inverse check of the calibration procedure was then carried out by engaging a conventional gear with the test gear, so that load sharing between adjacent teeth was present.

**Dynamic load measurement**

Dynamic strains were recorded for the set A and set D gears over 9 torque levels and four speeds (800, 2000, 4000, 6000 rpm). After acquisition, the data were digitally resampled using linear interpolation at 1000 samples per revolution, and then synchronously averaged to minimise non-synchronous components. The resample rate is greater than the acquisition rate to prevent the introduction of additional aliasing errors. The synchronously averaged strain data were used to compute dynamic tooth forces.

The direct measurement of tractive and normal force using these strain gages is expected to avoid the dynamic effects such as found by Benedict and Kelley (1961). The limiting factor here will be the natural frequency of the tooth itself in bending. A simple calculation shows this to be in excess of 10 kHz, well above the tooth engagement frequency of 2,800 Hertz at the maximum test speed, 6000 rpm. Another possible dynamic effect is the interesting feature remarked on by Johnson and Cameron, (1967) and Tevaarwerk (1985b) where the elastic compliance at the tooth contact in the direction of the tractive force, can result in tangential elastic compliance of similar order to that of the film itself. While this will modify the apparent lubricant viscosity, it is not expected to affect the measurement of friction force.

**Analytical Procedure**

**Calibration**

The analytical procedure is an extension of the procedure described in Rebbechi, et al., 1991. Measuring the strain outputs $S_c$ and $S_t$ of the gages mounted on the compressive and tensile sides respectively enables resolution of the normal ($F_n$) and tractive ($F_t$) tooth forces (Fig. 4), provided that the gage responses are linearly independent. Using as an example the situation where one tooth is loaded, the response of the compressive and tensile gages $S_c$ and $S_t$ can be written as:

\[ S_c = a_{11} F_n + a_{12} F_t \]  
\[ S_t = a_{21} F_n + a_{22} F_t \]

or alternatively as:

\[ \{S\} = [a] \{F\} \]  
\[ \{S\} = \begin{bmatrix} S_c \\ S_t \end{bmatrix} \]

\[ \{F\} = \begin{bmatrix} F_n \\ F_t \end{bmatrix} \]

where

\[ \{S\} = [a] \{F\} \]

and

\[ \{F\} = \begin{bmatrix} F_n \\ F_t \end{bmatrix} \]

the $a_{ij}$ are then the influence coefficients. For example, $a_{11}$ is the compressive strain due to a unit normal force $F_n$ and $a_{12}$ is the compressive strain due to a unit friction force $F_t$.

The strain influence coefficients are then evaluated by setting $F_n$ and $F_t$ in equations 1 and 2 alternately to zero. This is achieved in the calibration rig (Figs. 3 and 4) by either applying a torque in the absence of a tractive load ($F_t = 0$, Fig. 4), or by applying a constant torque, sufficient to prevent slip, and then a tractive load. In the latter case, it is assumed that the strain response of the tooth to the applied loads is linear, and the torque results in a constant offset. The strains due to this offset are subtracted from the incremental strains due to the tractive loading.

In the calibration rig the single-tooth gear was engaged with each instrumented tooth on the test gear, and strains from all four gages recorded. In this way the coefficients of a $4 \times 4$ matrix of coefficients can be constructed. By numerically simulating an additional instrumented tooth (Rebbechi, et al., 1991) the matrix becomes $6 \times 6$. The inclusion of effects from the adjacent tooth is an essential prerequisite of evaluating tooth loads where there is load-sharing. This is necessary because of the stress field in a gear, which is such that an applied load on one tooth will result in strains not only on that tooth, but also adjacent teeth. This effect will be more marked in the case of thin-rim gears.

Figure 6 shows the results for calibration at 114 percent torque with and without friction. Six-degree polynomials of the strain influence coefficients were computed to allow interpolation for any roll angle. Evaluation of the coefficients gives valid data anywhere where there is contact of tooth 1 or tooth 2 (Fig. 5). $F_n$ and $F_t$ are calculated by pre-multiplying by $[a]^{-1}$ so that

\[ \{F\} = [a]^{-1} \{S\} \]

**Theoretical Calculation of Friction**

Earlier work established that there are three distinct regions in the tractive force versus slip curve (Fig. 1) for heavily loaded elastohydrodynamic contacts, see for example Townsend (1968) and Tevaarwerk (1985a):

Region (A) - The linear low slip region. This is thought to be isothermal in nature, caused by the shearing of a linear viscous fluid (long transit time) or a linear elastic fluid, where the transit time of the oil is equal to or less than the relaxation time of the oil.

Region (B) - The non-linear region, still isothermal in nature but where the viscous element responds non-linearly. The experimentally noticed reduction in friction is greater than can be accounted for by the temperature rise alone. Non-linear and shear rate effects are thought to be important.
Region (C) - Thermal region. At high values of slip the traction decreases with increasing slip due to the heat generation at the high values of shearing, and the associated reduction in viscosity due to temperature rise in the film.

Theoretical calculations of friction force were made according to the procedures of Crook (1961). For these computations, the parameters as listed in Table 1 were taken for the gears and lubricant, with some modification to account for the temperature dependence of the viscosity coefficient, as evident from the pressure (Errichello, 1990). Crook's method assumed a constant pressure coefficient to evaluate the heat balance in the oil film and the resulting temperature rise. Friction force is evaluated by integrating over the Hertzian contact region.

The Hertzian contact width and contact pressure were calculated according to Bisson and Anderson (1964), for a line contact. The loads assumed for the computation were the dynamic tooth loads as measured during test. The theoretical friction coefficient was then computed according to the method of Crook (1961).

RESULTS AND DISCUSSION

Static Meshing

The accuracy of the gear load calibration procedure was tested by repeating the calibration procedure, but instead of meshing with the single-tooth gear, the test gear was meshed with its normal mating gear. This test provides for an inverse check of the calibration coefficients, and a test of the validity of the computations in the load-sharing mesh region.

The results of this static test procedure are shown in Figs. 7 and 8 for gear sets A and D. For these tests a normal force was applied, with no external friction force. The test gear was meshed with its normal mating gear. This test provides for an inverse check of the calibration coefficients, and a test of the validity of the computations in the load-sharing mesh region.

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The significant features of these results are several. Firstly, the test shows an excellent accuracy for the normal force, where the applied nominal force (torque/base radius) agrees within 3 percent. The regions of single-tooth contact where the normal force is constant are visible, and in the load-sharing regions the sum of the normal forces on each tooth equates closely to the constant total applied force. The marked difference between Figs. 7 and 8 is due to the unmodified profile of gear set A, versus the tip-modified gears of set D. The friction force in most instances is zero in the single tooth contact region. The static validation provides confidence in the reliability of the calibration procedure.

Dynamic Test

Dynamic strain data from the four strain gages were processed by the procedure described above to calculate the dynamic normal and frictional forces acting between the meshing gear teeth. A sample is shown in Fig. 9 for gear set D at 800 rpm and 141 percent torque. The gear tooth friction force and friction coefficient are plotted in...
appearance we may expect after viewing data from disc machine
friction force reverses direction. Whilst the data lack the smooth
positive Figs. 10 and 12 show it crossing the horizontal axis as the
direction at the pitch point. Although the friction coefficient is always
torques. The friction force (and
by dividing the friction force by the normal
residual DC-offset in the recorded tooth strain values.
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process of averaging is
sion process.
As was discussed by Rebbechi, et al., (1991), the friction measure-
values of compressive tooth loads, and is particularly sensitive to error
when the values are similar in value. The process of averaging is
expected to help, but cannot eliminate errors arising from synchro-
process. It is also likely that the measured strains are influ-
elastic factors, chiefly the low tooth loads resulting in small strains.
Accuracy of the strain data is likely to be compromised by sev-
several factors, chiefly the low tooth loads resulting in small strains.
As was discussed by Rebbechi, et al., (1991), the friction measure-
ment relies upon the difference between the magnitude of compres-
sion and tensile tooth strains, and is particularly sensitive to error
when the values are similar in value. The process of averaging is
expected to help, but cannot eliminate errors arising from synchro-
nous effects. It is also likely that the measured strains are influ-
enced by other loads such as gear blank vibration modes imposed on the gear. Finally, these errors are amplified by the matrix inver-
sion process.

The measured dynamic tooth friction forces are expected to be
most accurate in the single-tooth contact region, where the forces
are derived from the output of just two gages - tensile and compres-
sive on one tooth. The set D gears with their heavy tip relief have a
longer single contact zone, hence are better suited for this study. In
Figs. 10 and 11, the friction force has been adjusted vertically to lie
centrally around the x-axis. This adjustment was made because of a
small residual DC-offset in the recorded tooth strain values.

From the friction data, the friction coefficient can be evaluated by
dividing the friction force by the normal tooth load. The result-
friction coefficient is plotted in Figs. 10 and 11 for the higher
torques. The friction force (and thus its algebraic sign) reverses
direction at the pitch point. Although the friction coefficient is always
positive Figs. 10 and 12 show it crossing the horizontal axis as the
friction force reverses direction. Whilst the data lack the smooth
appearance we may expect after viewing data from disc machine
tests (for example Johnson and Cameron, 1967), a number of sig-
nificant observations can be made:

(a) There appear to be no discontinuities in the friction force due
to sliding direction reversal at the pitch-point.
(b) The coefficient of friction appears to decrease slightly with
increasing speed, but is largely insensitive to load, in the torque
values of 78 to 141 percent plotted here.
(c) The maximum friction coefficient is approximately 0.063, at
800 rpm.
(d) The friction coefficient at the highest speed of 6000 rpm
appears to be a maximum of 0.04.
(e) The friction values for gear set A (no relief) are similar to
those for gear set D (intermediate relief).

Overall, the friction measurements show that the features
observed in disc tests of highly loaded lubricated contacts are realised
throughout the gear tooth meshing cycle. Although the evaluation
of friction at very light loads was not reliable, the trend shows that
for loads in the normal operating range of these gears that friction
coefficient is largely independent of load. Finally, from observation
(b) above, the friction coefficient increases at low sliding speeds.
This is in accord with disk machine tests as reported in the references.

**Comparison with Theoretical Calculations**

The theoretical friction coefficient calculated according to Crook
(1961) is plotted in Fig. 12, for gear set D, 6000 rpm. The tooth
normal loads used in this computation were those experimentally
recorded at the nominal torques levels of 47 to 141 percent. From
these plots it is evident that in comparison with the measured data,
the theoretical calculation grossly overestimates the friction at low
speeds of sliding. At higher sliding speeds (away from the pitch
point) the theoretical friction coefficient merges for the different
loads, and numerically the results for theoretical calculation agree
more closely with the measured values.

At higher sliding speeds temperature effects become more
important, and the high viscosity due to pressure alone is modified
by the resulting high temperatures. The computed maximum tem-
perature rise of the lubricant, reached at the midpoint of the film, is
also plotted in Fig. 12. It can be seen that the temperature rise reaches
a peak value of 140 °C. Due to the reduced tooth load (from
load-sharing) at the larger roll angles, this peak is reached before
the extremes of sliding. At 141 percent torque, the computed val-
ues of maximum Hertzian pressure were 1.41 Gpa (204 600 lbf/
in²), the lubricant thickness 0.49 microns (17.5 micro-inches), and
the Hertzian half-width 0.19 mm (0.0074 inches). Computations
of theoretical friction at lower speeds resulted in unrealistically high
friction values, confirming further the limitations of a simple model
for the lubricant.

As a further comparison, the friction coefficient was calculated
according to Benedict and Kelley (1961). Their computation is
intended primarily for use in scoring failure predictions, and effec-
tively relates to the region C of Fig. 1, that is the thermal region.
Results using their equation are plotted in Fig. 13. The loads used
are those experimentally obtained. These results agree fairly well
with experimental data (Fig. 10) in the region away from the pitch-
point where the friction coefficient is approximately 0.04.
Figure 10.—Measured dynamic gear tooth friction loads and friction coefficient, gear set D.
Figure 11.—Measured dynamic friction force and friction coefficient for gear set A, 800 rpm.

Figure 12.—Theoretical friction coefficient and oil temperature for gear set D, 6000 rpm, using method of Crook (1961).

Figure 13.—Theoretical friction coefficient for gear set D at 6000 rpm, using method of Benedict & Kelley (1961).
SUMMARY AND CONCLUSIONS

Gear tooth normal and frictional forces were measured using strain gages mounted in the fillets of the gear teeth. The measured forces were used to compute the dynamic coefficient of friction existing between contacting teeth. The following conclusions were obtained:

1. The measured dynamic friction loads show friction coefficients of approximately 0.04 to 0.06. Friction coefficients increase at low sliding speeds. These results are in accord with disk machine tests as reported in the references.

2. The results show that the reversal of sliding which occurs at the pitch-point does not cause a discontinuity in the friction coefficient, which shows a smooth transition as the friction force reverses direction.

3. The technique described here offers the potential to study the variation in friction coefficient throughout the gear tooth meshing cycle, and examples of this variation for a range of loads and speeds are presented.

4. The measured data are more accurate at higher loads and in the single-tooth contact region.

REFERENCES


Measurements of dynamic friction forces at the gear tooth contact were undertaken using strain gages at the root fillets of two successive teeth. Results are presented from two gear sets over a range of speeds and loads. The results demonstrate that the friction coefficient does not appear to be significantly influenced by the sliding reversal at the pitch point, and that the friction coefficient values found are in accord with those in general use. The friction coefficient was found to increase at low sliding speeds. This agrees with the results of disc machine testing.