Maximum Langmuir fields in planetary foreshocks determined from the electrostatic decay threshold

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Abstract. Maximum electric fields of Langmuir waves at planetary foreshocks are estimated from the threshold for electrostatic decay, assuming it saturates beam-driven growth, and incorporating heliospheric variation of plasma density and temperature. Comparison with spacecraft observations yields good quantitative agreement. Observations in type III radio sources are also in accord with this interpretation. A single mechanism can thus account for the highest fields of beam driven waves in both contexts.

1. INTRODUCTION

Electrostatic oscillations near the plasma frequency have been observed in the foreshocks of Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune [Gurnett et al., 1989; Grard et al., 1991; Cairns and Gurnett, 1992; Hospodarsky et al., 1994; and the references cited therein]. These oscillations have generally been interpreted as Langmuir waves. The observed field strengths systematically decrease with heliospheric distance [Cairns and Gurnett, 1992], as shown in Fig. 1. In Sec. 2 we briefly recapitulate the theory. A brief summary of the observations is given in Sec. 3, and it is shown that their upper bounds agree well with theory.

2. THEORY

The electrostatic decay process \( L \rightarrow L' + S \) has been studied by a number of authors, including Robinson et al. [1993a], who found the following threshold for the process to have a positive growth rate:

\[
E_{LO}^2 = \frac{3m_e^2}{2\pi \epsilon_0^2 \gamma} \frac{V^4}{\omega_p^2} \frac{\gamma L' \Delta v_b}{v_b},
\]

(1)

\[
E_{LO}^2 = \frac{3}{2\pi \epsilon_0^2 \gamma^2} \sqrt{\frac{m_i \gamma L' \Delta v_b}{m_e \omega_p}} \frac{V}{v_b} \frac{N k_B T_e}. \]

(2)

These equations give the threshold \( L \)-wave electric field \( E_{LO} \) in terms of the electron and ion masses \( m_e \) and \( m_i \), the electron and ion temperatures \( T_e \) and \( T_i \), the electron thermal speed \( V = (k_B T_e/m_e)^{1/2} \), the ion sound speed \( v_S = (\gamma k_B T_i/m_i)^{1/2} \), \( \gamma = 1 + 3T_i/T_e \), the beam speed \( v_b \) and its spread \( \Delta v_b \), the damping rate \( \gamma L' \) of the waves \( L' \), and the electron number density \( N \). They also assume that the product sound waves are relatively strongly damped. It should be noted that, although \( E_{LO} \) is the threshold for electrostatic decay to proceed, Langmuir fields can be driven slightly past this level before nonlinear damping is high enough to saturate linear growth. Robinson et al. [1993a] argued that fields of order \( 3E_{LO} \) could momentarily be attained in this way. Nonetheless, \( E_{LO} \) provides an estimate of the characteristic maximum fields attainable, subject to electrostatic
decay. Note that Robinson et al.'s [1993a] dynamical equations were derived assuming only a single decay, but the threshold (1) is independent of this assumption.

The variation of $E_{LO}$ with heliospheric distance $r$ is the main focus of this paper. By making some assumptions about the factors in (2), we can estimate the form of this variation. Specifically, our assumptions are (i) $v_b \propto V$ for the beams responsible for the strongest fields (i.e., not necessarily all beams), (ii) $\Delta v_b/v_b$ is constant for these beams, and (iii) $\gamma_{L'} \propto \omega_p$. These assumptions are all plausible if, indeed, there is a single mechanism behind the generation of foreshock Langmuir waves. Equivalently, one could simply assume that the electron velocity distribution has a fixed functional form, scaled only by the thermal speed. A third equivalent assumption would be that the electric energy density at threshold is a fixed fraction of the thermal energy density (cf., Gurnett et al. [1980]).

The variation of $N$ and $T_e$ in the solar wind has been studied by a number of authors. At large $r$, one has

$$N \propto r^{-2},$$

$$T_e \propto r^{-\zeta},$$

with significant ranges in $\zeta$ predicted theoretically (2/7 - 4/3) and obtained experimentally (0.24 - 0.70). Scudder and Olbert [1979] and Sittler and Scudder [1980] found that a theoretical value of $\zeta = 1/3$ fitted the observed values better than other alternatives between 0.45 and 4.76 AU; their empirical fit yielded $\zeta = 0.35 \pm 0.06$. Phillips et al. [1993] find $\zeta \sim 0.70$ from 1.2 to 3.8 AU and $\zeta \sim 0.24$ from 3.8 to 5.3 AU. Using the above assumptions, Eqs (2) to (4) yield

$$E_{LO} \approx 2.4 \left( \frac{\gamma_{L'}}{1 \text{ s}^{-1}} \right)^{1/2} \left( \frac{r}{1 \text{ AU}} \right)^{-1-\zeta/2} \text{ mV m}^{-1},$$

where we have normalized to typical parameters in the Earth's foreshock $T_e = 1.5 \times 10^5 \text{ K}$, $\gamma = 1.8$, $\omega_p = 1.5 \times 10^5 \text{ s}^{-1}$, $v_b = 5V$, and $\Delta v_b/v_b = 0.3$. We take $\gamma_{L'} \approx 10^{-3} \omega_p$ nominally, higher than the values of $\sim 10^{-4} \omega_p$ seen in the solar wind because the beams are slower here, leading to more damping by the background thermal plasma. Conversely, $\gamma_{L'}$ is lower than the $\sim 10^{-2} \omega_p$ found for beams deeper in the Earth's foreshock, which are slower still [Newman, 1985; Fitzenreiter et al., 1984].

3. COMPARISON WITH OBSERVATIONS

The observations shown by bars in Fig. 1 were made by the ISEE-1 and -2 and Voyager-1 and -2 plasma wave instruments. They show the ranges of rms Langmuir fields measured in the instrumental frequency channel nearest $\omega_p$ in each case. It should be noted that measured Langmuir fields tend to be underestimates for three reasons: First, the waves are narrowband, so if their frequency does not coincide with the center of one of the instrumental channels, the signals are reduced by filter roll-off effects [Robinson et al., 1993b]. Second, the logarithmic compressors used in the ISEE and Voyager [W. S. Kurth, personal communication, 1994] instruments have exponential time constants $\sim 50 \text{ ms}$, so shorter signals are underestimated. Third, the highest fields in the various foreshocks may not be among those actually sampled by the Voyager spacecraft in the short periods of observation (typically 1 hour) close to the tangent field line and upstream from the planet (i.e., before the beam loses considerable energy).

The sloping line in Fig. 1, given by (5) for $\zeta = 0.35$ (see below for further discussion) and $\gamma_{L'} = 10^{-3} \omega_p$, is consistent with the observed upper bounds. This supports our invocation of electrostatic decay as the saturation mechanism. The selection effects noted in the previous paragraph imply that the observed upper bounds underestimate the true ones, but the degree of underestimation is likely to be similar in all the cases represented by vertical bars, because very similar instruments were used throughout. Hence, the slope of the heliospheric variation in Fig. 1 should not be strongly affected by selection effects. Another source of uncertainty is the use of typical parameters at 1 AU to evaluate the numerical coefficient in (5); variation of this normalization by a factor of 3 - 5 is quite possible, depending on the solar wind density and temperatures and the value of $\gamma_{L'}$.

Phobos 2 observations in the Martian foreshock [Grard et al., 1991] yield maximum Langmuir fields of 10 mV m$^{-1}$, in good agreement with the sloping line in Fig. 1 (where upper bounds obtained other than by ISEE and Voyager are shown as dots). Maximum fields of $\sim 1 \text{ mV m}^{-1}$ were observed in the Venusian foreshock by

![Figure 1](A-695-109)

**Figure 1.** Electric fields of Langmuir waves observed in the foreshocks of Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune are shown vs. heliospheric distance. The theoretical upper bound (5) is shown as a sloping line for $\zeta = 0.35$. ISEE and Voyager observations are shown as solid bars, while upper bounds obtained by other spacecraft are shown as dots.
Galileo and the Pioneer Venus Orbiter [Hospodarsky et al., 1994, and references therein], much lower than (5)'s prediction of $\sim 40$ mV m$^{-1}$ for $\gamma_{L'} = 10^{-5} \omega_p$. Ulysses fast envelope sampler observations have identified large spikes (lasting only 1 - 2 ms or 1 - 2 samples) superposed on broader Langmuir wave packets in the Jovian foreshock [Thiessen and Kellogg, 1993]. The spikes are not related to the electrostatic decay process (see below). However, the broader wave packets have maximum levels of $\sim 5$ mV m$^{-1}$, consistent with the bound shown in Fig. 1. IMP 6 observations yielded a maximum field of 70 mV m$^{-1}$ in the Earth's foreshock [Filibert and Kellogg, 1979], above the predicted bound (for nominal parameters) but below $3E_{L0}$.

4. DISCUSSION

We have shown that the theoretical electrostatic decay threshold $E_{L0}$ is consistent with highest observed Langmuir fields in planetary foreshocks.

Several assumptions were made in Sec. 2 to obtain the scaling of $E_{L0}$ with heliospheric distance $r$, yielding a threshold energy density proportional to the plasma's thermal energy density. It could be argued that any process which yields a maximum field proportional to $N T_e$ would also be a candidate to explain the observed scaling with $r$. However, any alternative must also account for the quantitative values of the maximum fields.

One alternative mechanism is wave collapse, which yield maximum attainable fields (at the end of collapse)

$$E_{\text{max}} \approx \left(2N k_B T_e/\epsilon_0\right)^{1/2},$$

(e.g., Robinson and Newman [1989]). However, (6) yields an upper bound $\sim 60$ times higher than (5) for the same parameters and $\gamma_{L'} = 10^{-5} \omega_p$ — far above anything observed. Nonetheless, fields of this order are conceivable, provided they occur so rarely as to have escaped detection by the Voyager and Ulysses instruments. Robinson and Newman [1989] presented evidence that coherent wave packets do collapse occasionally in the Earth's electron foreshock, starting from a much lower background level of random-phase waves (with the collapse threshold near the rms wave level). It is consistent that the upper bound to the latter waves is set by the electrostatic decay threshold, with occasional wave collapse occurring. In this event, very few observations would be expected to detect extremely high fields between the collapse threshold and the boundary (6), because collapse is very fast in its latter stages when the fields are highest. Hence, the electrostatic decay threshold would still represent an effective cutoff except in very extensive sets of statistics. It is worth noting in this context that Cairns and Robinson [1992a,b] were unable to find any evidence for wave collapse in the Jovian foreshock; instead, they found strong evidence of electrostatic decay, as did Thiessen and Kellogg [1993]. This conclusion has since been complemented by similar results from independent studies of type III source regions [Robinson et al., 1993a; Gurnett et al., 1993; Cairns and Robinson, 1994]. Thiessen and Kellogg [1993] also showed that most of the spike events observed by Ulysses in the Jovian foreshock are quantitatively inconsistent with strong turbulence theory.

As in foreshocks, Langmuir waves in type III radio sources are generated by beams and it is worth asking whether the above scalings also apply to them. Significantly, earlier studies of type III sources implied a variation of the peak fields as $r^{-1.42 \pm 0.5}$ for $r < 1$ AU [Gurnett et al., 1980], consistent with the present work. Indeed, observations where $r < 1$ AU imply that $N T_e$ falls more rapidly than $r > 1$ AU, with $N T_e \sim r^{-1.3 \pm 0.1}$, so the agreement is even better than for $\zeta = 0.35$. Using (2), we find $E_{L0} = (2 - 8)$ mV m$^{-1}$ at 1 AU for typical type III parameters $T_e = (1 - 2) \times 10^5$ K, $e_0 = (10 - 30)V$, $\omega_p = (1 - 2) \times 10^5$ s$^{-1}$, $\Delta v_b/v_b = 0.3$, $\gamma = 1.3 - 2.0$, and $\gamma_{L'} = 10$ s$^{-1}$ [Robinson et al., 1993a,b]. The highest of these upper bounds agrees well with the maximum observed value of 7 mV m$^{-1}$ [Gurnett et al., 1980]. Saturation via electrostatic decay can thus explain the upper bounds of both foreshock and type-III Langmuir fields.

Theoretically, the exponent $\zeta$ in (4) for the radial variation of $T_e$ can be related to the electron ratio of specific heats $\gamma_e$ by $\zeta = 2(\gamma_e - 1)$ [Scudder and Olbert, 1979; Sittler and Scudder, 1980]. The latter authors fitted Mariner and Voyager-2 data from 0.45 to 4.76 AU and found very good agreement with (4) for $\zeta = 0.35 \pm 0.06$, although Phillips et al. [1993] obtained $\zeta = 0.70$ and 0.24 by analysing Ulysses data from 1.2 to 3.8 AU and 3.8 to 5.3 AU, respectively. Alternatively, fitting (5) to the maximum wave fields in Fig. 1, not including Venus, leads to the independent estimate $\zeta = 0.6 \pm 0.5$. This range is consistent with the electron observations but, except for ruling out adiabatic cooling with $\zeta = 4/3$, does not discriminate between the other theoretical and experimental values of $\zeta$ proposed previously.

In conclusion, from the above analysis and discussion of foreshock and type III Langmuir wave fields, and from the previous work mentioned in both contexts, the electrostatic decay is chiefly responsible for saturating the highest levels of beam-driven Langmuir growth in both cases.

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