

# VIBROACOUSTIC RESPONSE OF PAD STRUCTURES TO SPACE SHUTTLE LAUNCH ACOUSTIC LOADS

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**Abstract** This paper presents a deterministic theory for the random vibration problem for predicting the response of structures in the low-frequency range (0 to 20 hertz) of launch transients. Also presented are some innovative ways to characterize noise and highlights of ongoing test-analysis correlation efforts titled the Verification Test Article (VETA) project.

## 1. Introduction

During a Shuttle launch, structures in the proximity of the launch pad are subjected to acoustic pressure loads generated by rocket exhausts. The design of some structures, particularly those having a large area-to-mass ratio, is governed by vibroacoustics, an environment induced by high-intensity acoustic noise (~180 dB). It manifests itself to payload and pad structures in the form of transmitted acoustic excitation and as structure-borne random vibration. The purpose of ongoing research at the Kennedy Space Center (KSC) are threefold: (1) to measure, process, and characterize launch acoustic loads; (2) to develop methods to predict random vibration response; and (3) test-analysis correlation of response analysis methods used.

## 2. Acoustic Loads

Transition from analog to digital methods and use of personal computers in the last decade or so have led to new developments at KSC — starting from controlling data acquisition and processing to innovative formulation of functions to characterize noise. In the frequency domain, acoustic loads are modeled by power spectral densities (PSD's), cross-power spectral densities (CPS's), coherence (COH), pressure response spectra (PRS), pressure correlation lengths (PCL's), correlated pressure distribution (CPD's), and normalized CPS's (NCPS's). The last four functions are unique to the vibration community, yet are often standard input to dynamic response analysis. Moreover, these functions also mark the beginning of a systematic effort to bridge the gap between measurements and analysis in an effort to characterize random, nonstationary, non-Gaussian acoustic loads (see figure 1).

## 3. Response Analysis

Two unique approaches to the response analysis were considered. A probabilistic approach, based on the classical solution of the random vibration theory [1,2], was dropped since it assumes a stationary input response relation and is suited for responses above 50 hertz. The

second deterministic approach favored low-frequencies (0 to 20 hertz), where fundamental resonances of most pad structures lie.

### 3.1 Probabilistic Approach

The exact solution for a steady-state response of a linear structure in a random stationary acoustic field, which may or may not be homogeneous, written in terms of PSD and CPS of modal coordinates is:

$$\Phi_{qq} = H_m \cdot b^T \cdot dA \cdot \Theta_{pp} \cdot dA \cdot b \cdot H_m^*$$

*kxk    kxk    kxn    nxn    nxn    nxn    nxk    kxk* (1)

where  $\Phi_{qq}$  is the solution matrix of  $k$  modal coordinates for  $k$  normal modes.  $H_m$  is the matrix of modal frequency response functions (FRF's).  $\Theta_{pp}$ ,  $b$ , and  $dA$  are the acoustic load matrix, the matrix of  $n$  modal displacements at  $n$  loaded points due to  $k$  normal modes, and the diagonal matrix of areas at  $n$  nodal points. The definition of acoustic load matrix using simplified models (white noise PSD and white noise decay CPS) is to simulate random launch transient led to errors. Recent work [3,4] at KSC yielded a more accurate formulation of launch data as follows:

$$\Theta_{pp} = S_p(f) \cdot Nc$$

*nxn            nxn* (2)

where  $S_p(f)$  is a scalar multiplier with a physical meaning of acoustic pressure PSD, and  $Nc$  is the NCPS (see figure 2). Combining equations 1 and 2, the PSD of a generalized modal load (GML) is:

$$[ b^T \cdot dA \cdot Nc \cdot dA \cdot b ] \cdot S_p(f)$$

*kxn    nxn    nxn    nxn    nxk* (3)

### 3.2 Deterministic Approach

The basic premise behind the concept of response spectra is that a total structural response consists of uncoupled responses in individual structural vibration modes. Therefore, the response in a mode can be obtained by integrating the equation of motion for that mode in the time domain. If the time history of a GML is known, the integration is possible even if the load is a random transient. In this integration process, the input is treated as a deterministic pressure time history (measured launch data); however, the definition of a corresponding GML contains the elements of a random response analysis and is uniquely related to the PSD of the generalized modal load in equation 1.

Thus, a given transient pressure time history  $p(t)$ , having a PSD  $S_p(f)$  that is derivable, does not require assumptions of stationary random process. Also, NCPS's derived from short/long data processing intervals were found to be time invariant even if  $p(t)$  were a transient. Thus, functions derived from NCPS's, such as PCL's and CPD's, are also time invariant.

Consequently, for a structural vibration mode, the product of normal mode displacements and a CPD is also time invariant. This product, when integrated over the area of the structure, defines a GML for a constant and a unit  $p(t)$ . Thus, for a time variable  $p(t)$ , the generalized modal load for a  $j$ -th mode proportional to  $p(t)$  is given by:

$$GL_j(t) = AJ_j \cdot p(t) \quad (4)$$

where  $AJ_j$ , the above product, is called the vibroacoustic coupling and is independent of time. The PSD of a GML is given by:

$$S_{gl_j}(f) = (AJ_j)^2 \cdot S_p(f) \quad (5)$$

The equation of motion for a  $j$ -th mode (omitting the subscripts for brevity) in terms of the GML,  $q$ , is:

$$\ddot{q} + 2\xi\Omega\dot{q} + \Omega^2q = (AJ/M) \cdot p(t) \quad (6)$$

where  $M$ ,  $\Omega$ , and  $\xi$  are parameters of the  $j$ -th mode, generalized modal mass, circular resonance frequency, and modal damping. Equation 6 is then integrated for a variable  $q/(AJ/M)$ , with zero initial conditions and an array of frequencies,  $f=\Omega/2\pi$ , so as to include resonances of all modes of interest. Similar to well known shock spectra, only the peak (maximax) values are retained in the solution. Since a solution corresponds to each of the assumed frequencies,  $q=q(f)$ , for presentation purposes, the plotted variable on the response spectra plots is:

$$Y(f) = q(f) / (AJ/M/\omega^2) \quad (7)$$

Utilization of response spectra  $Y(f)$  in computations of maximax value of modal coordinate  $q(f)$  defined by equation 7 requires knowledge of  $AJ(f)$  for each normal mode of a vibrating structure. By combining equations 3 and 5 and canceling  $S_p(f)$  on each side, the required theoretical relation is obtained as follows:

$$[AJ_j(f=f_j)]^2 = \text{DIAG}_{jj} [b^T \cdot dA \cdot Nc \cdot dA \cdot b]_{@ f=f_j} \quad (8)$$

Practical computations, however, of  $AJ(f)$  must be made from the definition of a GML by means of a CPD, which results in equation 4. The CPD is a function of PCL, and PCL is a function of frequency. PCL's together with CPD's provide a measure of correlation in an acoustic field generated during a launch. PCL is obtained from a set of two measurements at a given distance  $D_s$ , using either NCPS or COH and phase from CPS, since they define correlation between pressures in an acoustic field.

$$PCL(f) = 2 \cdot \pi \cdot D_s / \text{ARCCOS}(2 \cdot (A_p^{0.5}) \cdot [(\text{COH-NL})^{0.25}] \cdot \text{COS}(\text{Phase}/2) - 1) \quad (9)$$

where  $A_p=1.0$  for a homogeneous field and lower than 1.0 for a nonhomogeneous field on PCL(f), NL defines COH noise level equal to squared noise level in  $\text{Mag}(\text{NCPS})$ , since  $\text{Mag}(\text{NCPS})=\text{SQRT}(\text{COH})$ . PCL(f) is frequency dependent since both COH and phase are also. Only discrete values of  $f$  corresponding to modal resonances are of importance. Lastly, along a given direction of a PCL(f), the corresponding CPD is assumed to vary as a cosine function:

$$CPD(f,x)=(1+COS[2 \cdot \pi \cdot x/PCL(f)])/2 \quad (10)$$

A similar definition of  $CPD(f,y)$  may be obtained for the y-direction.

### 3.3 Equivalent Loads Concept

One interesting feature of PRS involves the plotted quantity of equation 7. Consider the case of a single span structure with a fundamental mode resembling a deformed shape of the same structure under a uniform static load and with the PCL greater than three times the span. Here, the numerical value of Y (in units of psi and at the frequency of the fundamental mode) is nearly equal to an equivalent static load that induces a peak deflection equal to the peak dynamic amplitude of the fundamental mode, when amplitude is computed from PRS. Such equivalency applies only to directly loaded structural parts (single-span beams, plates, etc.) and to their connections only and not to the underlying structure having a different fundamental resonance. Figure 3 shows an equivalent static design load for  $f = 10$  hertz.

### 4. VETA Project

Unlike vehicles and payloads, launch support structures cannot be tested and verified prior to a launch because the valid loads can only be generated by the launch of a full-scale vehicle. Thus, any test-analysis correlation effort to validate the KSC-developed theory (deterministic approach) necessitates test data acquisition during actual Shuttle launches. VETA involves a fully instrumented steel cantilever beam for simultaneous measurement of acoustic loads (via microphones) and structural response (via accelerometers and strain gages) in the low-frequency range (0 to 20 hertz). The first four flexural modes of VETA have frequencies of 9, 54, 145, and 302 hertz, respectively. During the past year, VETA tests have yielded valuable insight into Space Shuttle liftoff and plume impingement effects on acoustic loads and related frequency composition. Also, a new data acquisition system to remotely acquire and monitor launch data has been developed and implemented. Post-processing of data is in progress for several near-field and far-field launches.

### 5. Concluding Remarks

The experience gained from Space Shuttle launches has led to significant developments in data acquisition and analysis methodology. Efforts to characterize rocket noise resulted in newly developed functions (NCPS's, PCL's, etc.) as main descriptors defining an acoustic field, while CPD's have provided a graphic illustration of vibroacoustic coupling. Since the existing database of special functions is small, theoretical developments supported by measurements will continue. Effort to assess nonlinear and nonstationary vibrations in the context of the VETA project are planned. Methods for passive/active control of these nonlinear and nonstationary vibrations will be researched.

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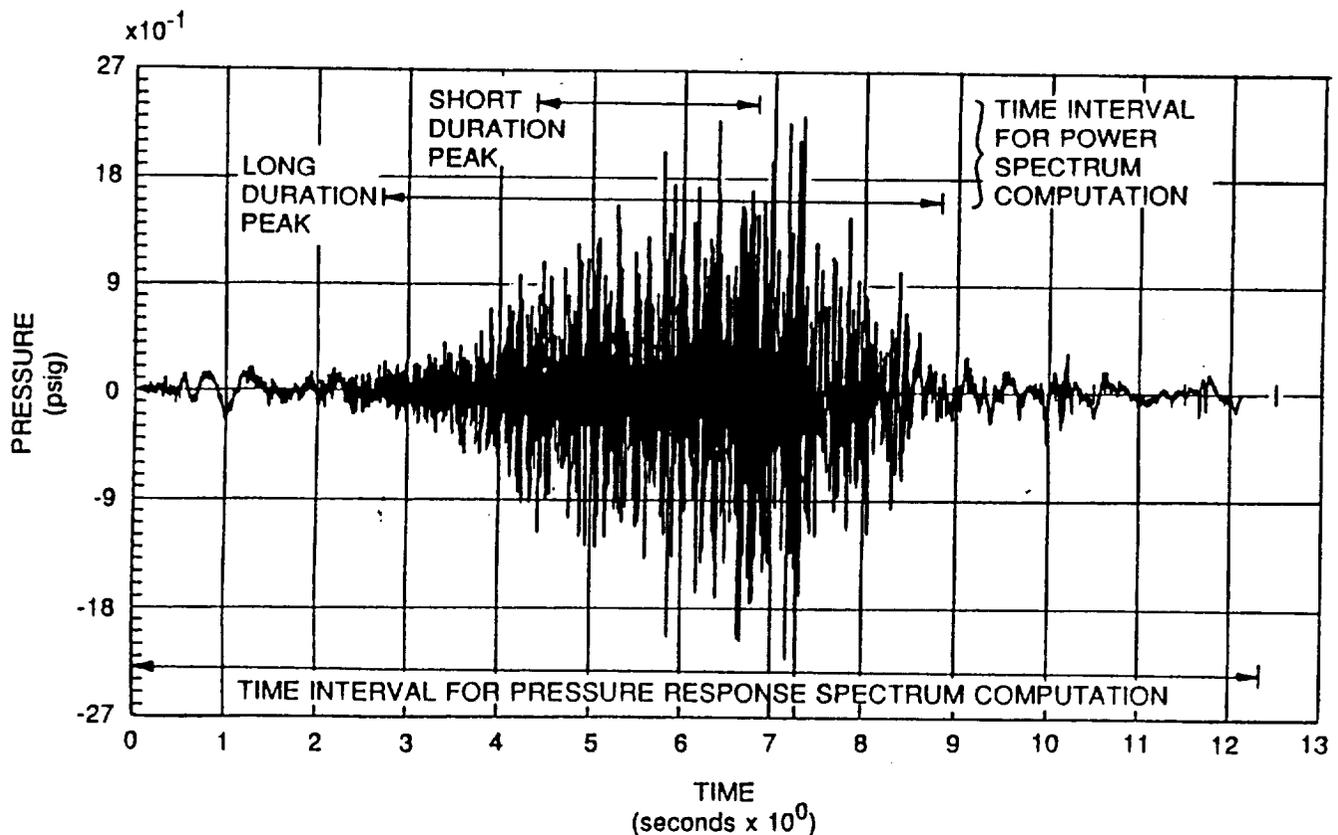


Figure 1. Shuttle Data: Short Transient With Random Pressure Amplitudes

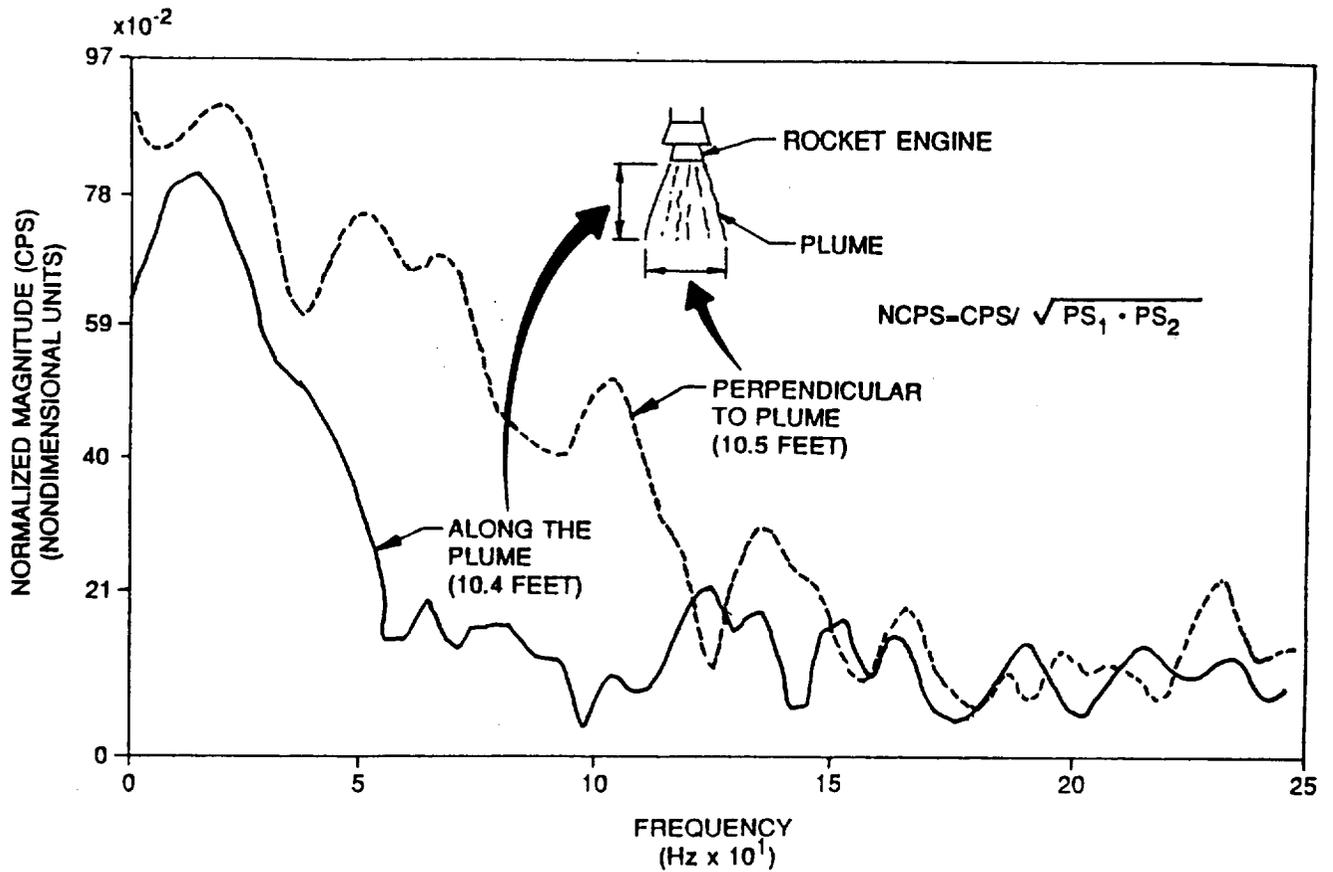


Figure 2. Variation of Normalized Cross-Power Spectrum Along the Rocket Plume Axis

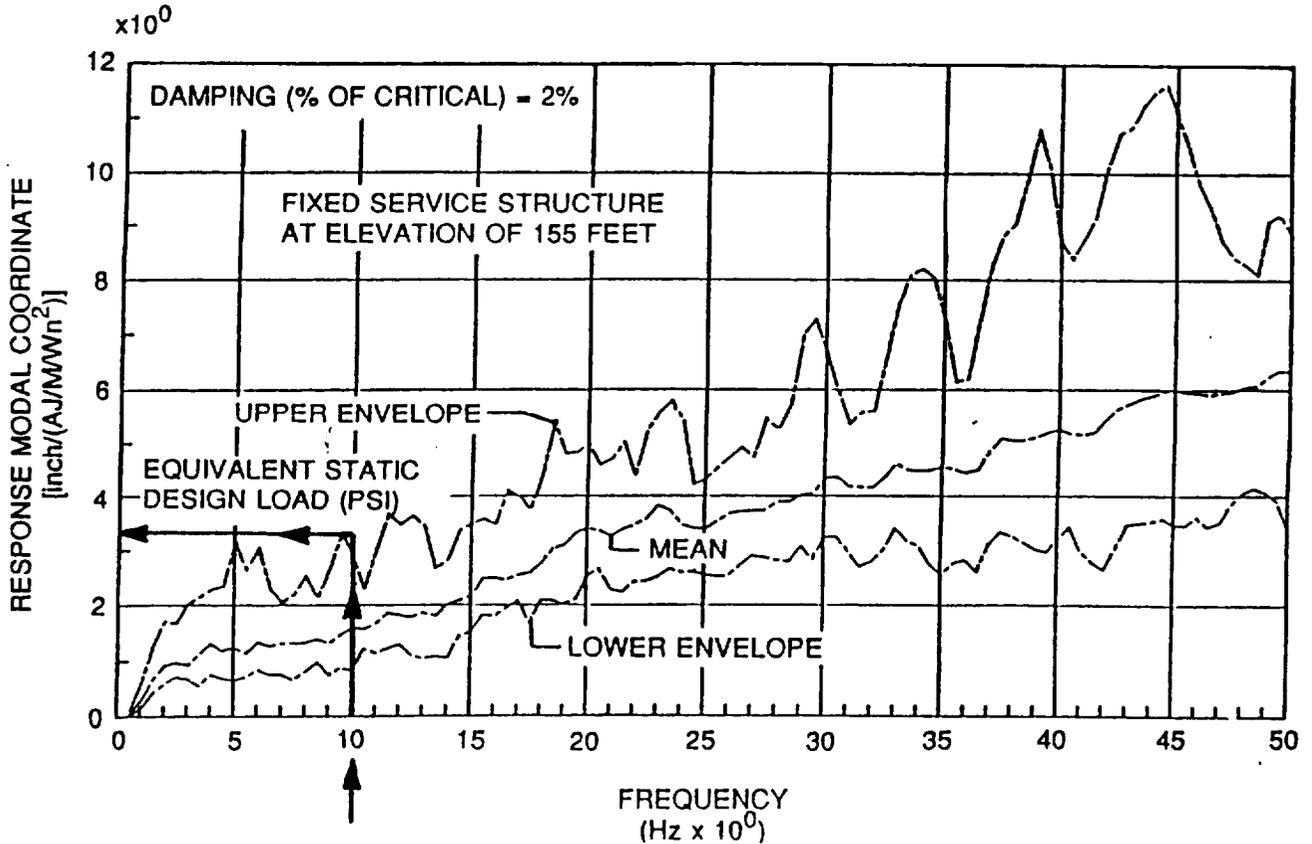


Figure 3. Liftoff Pressure Response Spectrum to Rocket Noise