Systems Engineering Metrics:
Organizational Complexity
and
Product Quality Modeling
(H-27289D)
Final Report

For

NASA-MSFC

by

OR Applications

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System Level 1

\[ y(t) = \sum_{i=1}^{n} c_i(t) \prod_{j=1}^{k} x_{ij}^{aq}(t) \]

System Level 2

\[ x_1(t) = \sum_{i=1}^{m} d_i(t) \prod_{j=1}^{l} w_{ij}^{hy}(t) \]
\[ x_2(t) = \sum_{i=1}^{p} g_i(t) \prod_{j=1}^{q} v_{ij}^{cq}(t) \]
Acknowledgments

The author wishes to thank Dr. Dale Thomas, EL01, for his innovation of combining complexity/fractals, stochastics, architecture, and organization in a metric environment, as well as his insightful comments and suggestions made while serving as COTR for this effort. I also wish to thank Dr. Thomas for allowing me to utilize communication concepts and wording that he developed for the recent INCOSE paper on this subject. Thanks are also due to Frank Fogle (EJ33), Melody Herrmann (PD21), Jim Parker (EL02), Glen Ritter (EL21), and Pat Vallely (ED11) for their peer review of this material.
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Executive Summary

Innovative organizational complexity and product quality models applicable to performance metrics for NASA-MSFC's Systems Analysis and Integration Laboratory (SAIL) missions and objectives are presented. An intensive research effort focuses on the synergistic combination of stochastic process modeling, nodal and spatial decomposition techniques, organizational and computational complexity, systems science and metrics, chaos, and proprietary statistical tools for accelerated risk assessment. This is followed by the development of a preliminary model, which is uniquely applicable and robust for quantitative purposes. Exercise of the preliminary model using a generic system hierarchy and the AXAF-I architectural hierarchy is provided. The Kendall test for positive dependence provides an initial verification and validation of the model. Finally, the research and development of the innovation is revisited, prior to peer review. This research and development effort results in near-term, measurable SAIL organizational and product quality methodologies, enhanced organizational risk assessment and evolutionary modeling results, and improved statistical quantification of SAIL productivity interests.

Major conclusions include:

1. The model provides programmatic risk as a complex, nonlinear function of relative architectural complexity, organizational complexity, budgetary weightings, technological maturities, and schedule duration.

2. The model form is scale-invariant (geometric), hierarchically recursive, and evolutionary, with a strong engineering foundation, and supported by current complexity and metamodel research.

3. Preliminary expansion/contraction rules have been developed for assessing the impact of budgetary and schedule changes on programmatic risk.

4. Preliminary sensitivity analyses indicate budget contractions are more damaging to risk than schedule contractions, while schedule expansions are more helpful than budgetary ones.

5. Major programmatic risk drivers for the model analysis of AXAF-I showed a positive dependence of 88% with that of AXAF-I program management.

6. Major model uncertainty drivers, in descending order, are basis function parameters, expansion/contraction rulebase, and organizational risk factors.
Key recommendations are:

1. Investigate the model validity, focusing on the uncertainty drivers of conclusion 6.

2. Utilize advanced statistical techniques to maximize validation information from limited data sources.

3. Incorporate a quality monitoring system for management control and feedback purposes.

4. Investigate model control mechanisms to achieve minimum risk for a given programmatic payoff.
1.0 Introduction

The field of Systems Science has enjoyed a resurgence as organizations have come to terms with shrinking budgets, aging resources, and increasing and more time critical requirements. The current strategic conditions are particularly distressing for larger, more bureaucratic organizations, such as public agencies, since the decentralized nature of these organizations, the very characteristic that provides the mechanism for superior technical performance, is a liability to the requirement of rapid change. Public agencies most susceptible to these conditions, currently, are large R&D organizations of a highly specialized nature. NASA-MSFC falls into this category, with its additional pressures of safety, schedule, and large complex projects. Large R&D organizations suffer from an additional difficulty when attempting to deal with these stressing requirements: how to measure productivity and quality in a meaningful way for R&D and intellectual products. NASA-MSFC's Systems Analysis and Integration (SAIL) Laboratory is responsible for providing some of the answers to this dilemma for MSFC, but also for its own organization.

As a consequence of the difficulty of measuring quality and productivity characteristics for intellectual and R&D products, most attempts at quantification are very sensitive to the method of quantification, as well as the assumptions required to carry out the analysis. This lack of robustness is particularly disturbing, since it results in the well-known phenomena of "tweaking the model parameters until the desired answer is achieved." This tendency is dangerous, in that it can result in inappropriately shoring of the status quo, or effecting rapid change that is damaging to the organizational missions.

A second problem with modeling organizational productivity and quality is the static nature of most attempts at quantification. Most modeling attempts do not account for the dynamic, evolutionary nature of organizations in times of rapid change. Consequently, the models either have to be frequently updated, or changed completely, an expensive proposition.

Lloyd [1996] has described a complex situation as one where despite perfect understanding of the subsystems, the integrated system behavior is not well understood. He has emphasized the question: "In general, how do complex, specific laws arise from simple, generic ones?"

A number of attempts have been made at developing robust, dynamical models of R&D organizational effectiveness. Streufert and Swezey [1986] have applied complexity theory to achieve measurements via a time-event matrix. Klir [1985] has also pursued complexity contributions combined with a metasystems approach to deliver robust results. Cameron and Whetten [1983] have addressed the demise of organizational effectiveness studies, and investigated the implications for effectiveness theory in the public sector.
Study Objectives

The overriding objective of this study is to produce an intensive research-based, peer review acceptable, robust, and quantitative systems meta-model for the SAIL organizational mission and products. Particular emphasis will be placed on providing a model which is relatively insensitive to tuning parameters and assumptions (robust), and evolutionary (dynamical) in nature. The three main questions to answer in this study are:

1. What is a mathematical relation of system architectural complexity to system size and the number of intersystem and intrasystem interfaces?

2. What is a mathematical relation of system complexity to system size, technological maturity, and the number of intersystem and intrasystem interfaces?

3. What is a mathematical relation of project complexity to system size, technological maturity, organizational dispersion, and the number of intersystem and intrasystem interfaces?

The answers to these three questions, as determined by this study are:

1. The architectural complexity of the system is proportional to an enumeration of the number of hierarchical levels and the number of interactions amongst systems and subsystems. The maximum number of interactions at a given subsystem level is equal to one-half of the product of the number of components and the number of components minus one. (See Figure 2.0-1.)

2. System complexity is proportional to an enumeration of the minimum number of parameters required to describe system size, technological maturity, and the number of interactions. System size is defined by the number of hierarchical levels and the number of subsystems at each level. The number of interactions is described above. Technological maturity is a function of time, as a minimum. (See Figure 2.0-1. and Section 4.0)

3. Project complexity is proportional to an enumeration of the minimum number of parameters required to describe system size, technological maturity, organizational dispersion, and the number of interactions. System size is defined by the number of hierarchical levels and the number of subsystems at each level. The number of interactions is described above. Technological maturity is a function of time, as a minimum. Organizational dispersion is a function of organization size, number of hierarchical levels, and number of interactions amongst individuals at each level. (See Figure 2.0-1. and Section 4.0)

2.0 Complexity Measures

(Schuster, 1996) explains how nature employs optimization during times of scarcity, creative innovation in times of abundance, and modular design to control uncertainty over time. While nature invariably uses increasing levels of complexity in successive stages of evolution, the manner in which complexity is employed is neither random nor linear, rather, nature employs complexity prudently.
Project success commonly has been defined as completion of an activity within time, cost, and performance constraints (Kerzner, 1989). To be relevant within the context of engineering systems development, a metric for system complexity must be comprehensible in terms of system technical performance, development cost, and schedule. Hence, the prudence of increased complexity in the engineering arena is determined on the basis of technical performance, development cost, and schedule. More specifically, since technical performance, development cost, and schedule are effectively joint stochastic variables prior to project completion, the prudence of increased complexity may be assessed in terms of the risk associated with each.

A system complexity metric could then be of value in analyzing the inherent difficulty associated with the project plan to build a system. It could be used in at least the following three manners:

- Identify key components within the system architecture, whose developmental problems will ripple widely in the system development process.

- Identify locales within the system architecture where higher technology readiness is needed and others where lower technology readiness can be tolerated.

- Identify organizational bottlenecks for a given organizational scheme to build the system.

In each of these three areas, the inherent difficulty is measured in terms of risk associated with the project technical performance, cost, and schedule.

We conjecture, intuitively, that the complexity of a system is a function of the technological maturity of its components, and the organizational and technological dispersion associated with developing the system. In turn, both the technological maturity and organizational/technological dispersion are functions of the size of the system and the development duration of the schedule. Casti [1994] states that "the complexity is directly proportional to the length of the shortest possible description of that object." Thus, system complexity is proportional to a count on the description parameters associated with the system size, technological maturity, organizational and technological dispersions, and development horizon. Finally, the maximum number of subsystem or component interactions in a two dimensional hierarchy is one-half the product of the system size times the system size - 1. These initial thoughts regarding complexity measures are provided in mathematical form in Figure 2.0-1 below.

Here, \( C \) is the system complexity measure, \( \lambda \) is the system technological maturity, \( \sigma \) is the combined organizational/technological dispersion, \( n \) is the system size (usually, the number of components or subsystems), \( t \) is the development time or duration, and \( M \) is the maximum number of subsystem interactions.
\[ C = f(\lambda, \sigma) \]
\[ \lambda = g(n, t) \]
\[ \sigma = h(n, t) \]
\[ \Rightarrow C = f[g(n, t), h(n, t)] \]
\[ C \propto Count(\lambda, \sigma, n, t) \]
\[ M = \frac{n(n-1)}{2} \]

Figure 2.0-1. Initial Mathematical Thoughts on Complexity

3.0 Preliminary Model Form

The preliminary complexity model form is shown in Figure 3.0-1. This form is a geometric or power form common to engineering analysis. According to Hastings and Sugihara [1995], the geometric form is the only form which is scale-invariant. This is important, because the model's utility is determined by the number of different systems (of different scales) to which it is applicable. This model form allows both independent and interacting subsystems in an evolutionary and recursive hierarchy. Handling both independent and interactive subsystems ensures a broader range of application, while recursiveness is desirable to address systems with differing numbers of layers in the hierarchy. The model form is also an efficient and commonly used data-fitting form, important for model validation. Finally, the model form allows for the use of neural net and global optimization solvers, an important consideration when the issue of system control is addressed. Due to these desirable attributes, this model form is found frequently in complexity, fractal, and metamodel literature.
4.0 Preliminary Complexity Model

The preliminary complexity model must be dependent on more than the size of the system. It should also take into account schedule and budgetary issues, technology choice and relative investment, the system architectural design, and relevant organizational considerations. The preliminary model, Complex Organizational Metric for Programmatic Risk Environments (COMPRÊ), is given by:

\[
\lambda(t) = \sum_{i=1}^{n} w_i(t)E[r_i(t)]
\]

\[
\sigma(t) = \left[ \sum_{i=1}^{n} w_i(t) \right]^{1/2}
\]

\[
C_{ij}(t) = \int_{0}^{t} \frac{\int_{0}^{t} \int_{0}^{t} \left[ r_i(t) - E[r_i(t)] \right] \left[ r_j(t) - E[r_j(t)] \right] dt}{\int_{0}^{t} dt}
\]

\[
E[r_i(t)] = \frac{1}{t} \int_{0}^{t} r_i(t) dt
\]

\[
\sum_{i=1}^{n} w_i(t) = 1, w_i(t) \geq 0
\]

Figure 3.0-1. Preliminary Complexity Model Form
where

\( \lambda(t) \) is a (time-dependent) technology development maturity function, representing a rate of payoff for investing in a certain level of technology,

\( n \) is the number of technology investments made,

\( w_i(t) \) is the relative investment weight for technology \( i \),

\( E[r_i(t)] \) is the expected return-on-investment for technology \( i \) over the development schedule represented by \( t \),

\( \sigma(t) \) represents total programmatic risk, and

\( C_{ij}(t) \) is the covariance between technologies \( i \) and \( j \) over the development duration.

The development maturity function, \( \lambda \), accounts for technology, schedule, and budgetary distribution decisions, as well as system size. The programmatic risk function, \( \sigma \), captures the same issues, as well as architectural design and organizational makeup. Preliminary results for the AXAF-I (NASA's Advanced X-ray Astrophysics Observatory) project, portfolio management problems, and computer system replacement decisions indicate that the ratio of these two functions may provide a useful complexity measure for decision-making and control purposes.

### 5.0 Standardized Complexity Methodology (COMPRÉ)

Figure 5.0-1 shows the standard methodology using COMPRÉ. Given a system architecture which includes technology readiness and organizational data as depicted in Figure 5.0-2, the profiles for the relative technology investments are captured in a component influence vector. Engineering basis functions (typically, linear, quadratic, and exponential) are utilized to develop the profile for a technology evolution vector. Once the development schedule is identified, the programmatic payoff can be determined as a function of the schedule. A technology covariance matrix is the intermediate step to achieving the programmatic risk profile. This matrix is adjusted for the presence, absence, and degree of architectural and organizational interactions.

The raw data required to run COMPRÉ are:

1. A project architecture/hierarchy, showing all systems, subsystems, components, and interactions at all levels.
2. The technological maturities for all components of the system at the lowest levels of the hierarchy.
3. Organizational responsibility for each of the systems, subsystems, and components of the hierarchy.
4. Relative budgetary investments in each of the systems, subsystems, and components of the hierarchy.

5. The schedule duration of the project development.

In addition to these five data requirements, COMPRI uses the following data, models, and assumptions:

1. Technology basis function models to describe technological maturity.

2. A rulebase for organizational dispersion factors (between organization dispersion).

3. A rulebase for budgetary/schedule contractions and expansions for conducting relative sensitivity analyses.

Appendix B provides numerical details, relating to these data and assumptions for the AXAF-I example of Section 7.0.

---

**Figure 5.0-1. Standardized COMPRI Methodology**

*Component Influence Vector*  
\[ \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_n(t) \end{bmatrix} \]

*Technology Evolution Vector*  
\[ \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_m(t) \end{bmatrix} \]

\[ \text{Time Frame} \quad [0,t] \]

\[ \downarrow \]

*Technology Covariance Matrix*  
\[ \begin{bmatrix} c_{11}(t) & c_{12}(t) & \cdots & c_{1n}(t) \\ c_{21}(t) & c_{22}(t) & \cdots & c_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1}(t) & c_{n2}(t) & \cdots & c_{nn}(t) \end{bmatrix} \]

\[ \Rightarrow \]

*Complexity Measure*

**Figure 5.0-2. System Architecture: A Single Layer Hierarchy**
The resulting complexity measure is then a function of cost, schedule, technology, architecture, and organization. When the measure is the ratio of programmatic risk to payoff, an understanding emerges concerning the efficacy with which the organization, architecture, technology, and resources (both time and dollars) are being utilized. Furthermore, this ratio may be used to identify those individual subsystems and architectural interactions that are contributing the most risk, relative to payoff, for the overall system being investigated.

6.0 Generic COMPRÉ Sensitivity Analysis

To gain a basic understanding of the performance of COMPRÉ, a generic sensitivity analysis is performed. The baseline assumptions for this sensitivity analysis are as follows:

1. This is a single layer hierarchy with three components, or subsystems, each interacting with each other.

2. A single organization is responsible for the development of all three components.

3. The budget is allocated uniformly across the three components.

4. The technology maturity classification is 3 (some new technology) for all three components.

5. The development duration is 5 time units.

Moreover, the following expansion/contraction rulebase is used to implement schedule and budgetary changes from the baseline:

A1. A budget change results in a single component technological maturity change, in addition to the reallocation of the relative component budgets.

A2. A schedule duration increase results in all components experiencing a maturity improvement, in addition to the change experienced by expansion of the program development.

A3. A schedule duration contraction results in all components experiencing a maturity degradation, and an increase in the number of organizations required to carry out the development, in addition to the change experienced by contraction of the program development.

Figure 6.0-1 shows the results of the generic sensitivity analysis. The total relative programmatic risk (risk divided by baseline risk) is plotted for 4 cases plus the baseline. This sensitivity shows that schedule duration increases are more effective than budgetary ones in reducing programmatic risk. However, budgetary reductions are slightly more damaging than schedule contractions in terms of risk. This is because shortening the schedule reduces the inherent volatility in program development, somewhat offsetting the increased risk of less mature technology.
7.0 AXAF-I Analysis Using COMPRÉ

In this section, a COMPRÉ analysis is performed for the AXAF-I program. Figure 7.0-1 shows the architectural hierarchy (nodal decomposition) for AXAF-I. AXAF-I is composed of three major subsystems: Spacecraft, Telescope, and ISIM. These three subsystems are further decomposed into their respective components. The numbers next to the subsystems and components represent the development maturities for the component technologies. The code for these maturities is found in the upper left-hand corner of the figure. Green lines connecting subsystems and components represent interfaces or interactions. Colors and shading represent organizational responsibilities via the organization codes located at the bottom right-hand corner of the figure. External interfaces are represented by magenta lines, but are not included in the analysis. Although development maturity codes are provided as estimates for the Spacecraft, Telescope, and ISIM subsystems, COMPRÉ derives these values itself.

Figure 7.0-2 shows the COMPRÉ plot for AXAF-I. This plot provides the coordinates for the expected technological maturity, \( \lambda \), and the expected programmatic risk, \( \sigma \), given by the preliminary (COMPRÉ) model for a duration of 5 years, ending in 1998. These values are based on the relative budget weights given in Table 7.0-1, below. Clearly, the Telescope and ISIM subsystems, packed with newer technology, pose the greatest risk to the program. The Spacecraft subsystem, meanwhile, carries much less programmatic risk. Finally, the properties of the AXAF-I system, as a whole, are properly weighted averages of the major subsystems.
**Development Maturity**

0 -- off the shelf flight HW
1 -- modified flight HW
2 -- new design proven technology
3 -- some new technology
4 -- significant new technology
5 -- new technology

---

**Figure 7.0-1. AXAF-I Architectural Hierarchy**

**AXAF-I Complexity Analysis (1998)**

*Programmatic = Organizational + Architectural + Technological
*Linear, Quadratic, Exponential Basis Functions

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**Figure 7.0-2. AXAF-I COMPRÉ Plot**
Table 7.0-1. AXAF-I Relative Budget Weights

<table>
<thead>
<tr>
<th>Component (%)</th>
<th>HRMA (68%)</th>
<th>SIM (28%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propulsion (13%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data (7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power (9%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pointing (32%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structure (16%)</td>
<td>Grate 2 (0%)</td>
<td></td>
</tr>
<tr>
<td>R/F (7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBA (18%)</td>
<td>HRC (22%)</td>
<td></td>
</tr>
<tr>
<td>SSA (10%)</td>
<td>ACIS (50%)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.0-3 shows the normalized COMPRÉ risk, $\sigma/\lambda$, for the three major AXAF-I subsystems. In both the normalized and absolute risk cases, the ISIM and Telescope subsystems dwarf the Spacecraft subsystem. However, the ISIM and Telescope have exchanged places in rank, indicating that for a constant expected technological payoff, the ISIM carries more risk than the Telescope. This result is consistent with the rankings provided by AXAF-I Program Management, who ranked ISIM problems as their top four out of five (see below).

Figure 7.0-4 shows the major risk sources, as provided by COMPRÉ. Two of the top four involved the HRMA, and the other two involved the ACIS. Figure 7.0-5 shows the major normalized risk sources, as provided by COMPRÉ. The top two sources involve the SIM interactions, ranked third and fourth by AXAF-I management.

Table 7.0-2 provides the rankings of the AXAF-I Program Management, and the correlation with COMPRÉ programmatic risk and normalized risk. Confidence values, given by Kendall’s measure of association are given. The confidence values should be interpreted as follows: there is an 88% chance that the rankings provided by AXAF-I Program Management and COMPRÉ’s programmatic risk are positively dependent. Similarly, there is an 59% chance that the rankings provided by AXAF-I Program Management and COMPRÉ’s normalized risk are positively dependent. It is possible that both of these results are better than expected at this stage of COMPRÉ development, given the fact that the COMPRÉ basis functions have not been baselined or validated. However, the difference in confidence values between the absolute and normalized risks should not be overemphasized at this juncture. Recall that at the subsystem level, COMPRÉ reversed the ISIM and Telescope rankings using normalized risk, correctly (with respect to AXAF-I Program Management) emphasizing the ISIM over the Telescope. The normalized risk underperformance at the component level could be a matter of COMPRÉ refinement.
AXAF-I Subprogram Analysis

*Programmatic = Organizational + Architectural + Technological
*Linear, Quadratic, Exponential Basis Functions

Figure 7.0-3. AXAF-I COMPRÉ Subsystem Analysis

AXAF-I Highest Risk Sources

*Programmatic = Organizational + Architectural + Technological
*Linear, Quadratic, Exponential Basis Functions

Figure 7.0-4. Major AXAF-I Programmatic Risk Sources
Table 7.0-2. Correlated Rankings: AXAF-I Program Management and COMPRÉ

<table>
<thead>
<tr>
<th>Component/Interaction</th>
<th>Program Management Ranking</th>
<th>COMPRÉ Programmatic Risk</th>
<th>COMPRÉ Normalized Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACIS</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>HRC</td>
<td>2</td>
<td>&gt;7</td>
<td>&gt;7</td>
</tr>
<tr>
<td>ACIS-SIM</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>HRC-SIM</td>
<td>4</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Propulsion</td>
<td>5</td>
<td>&gt;7 and &gt;HRC</td>
<td>&gt;7 and &gt;HRC</td>
</tr>
<tr>
<td>Confidence</td>
<td>----</td>
<td>88%</td>
<td>59%</td>
</tr>
</tbody>
</table>

Figure 7.0-5. Major AXAF-I Normalized Risk Sources
A sensitivity analysis was performed for AXAF-I budgetary and schedule expansions and contractions. The expansion contraction rulebase for budget changes is as follows:

B1. A +5% budget change allows for a single component technology maturity change from a 1 to 0 or 2 to 1, and forces a reallocation of the budget toward that component.

B2. A +10% budget change allows for a single component technology maturity change from a 4 to 3 or 3 to 2, and forces a reallocation of the budget toward that component.

B3. A +15% budget change allows for a single component technology maturity change from a 5 to 4 or removal of one interaction.

B4. A +20% budget change allows for a single component technology maturity upgrade of two levels or removal of 2 interactions or two upgrades of one level each.

Budget reductions work the same, but by reversing the direction from upgrade to downgrade, or by adding interactions.

The schedule change rulebase is given by the following:

C1. A +5% schedule expansion allows for a single component technology maturity change from a 1 to 0 or 2 to 1.

C2. A +10% schedule expansion allows for a single component technology maturity change from a 4 to 3 or 3 to 2.

C3. A +15% schedule expansion allows for a single component technology maturity change from a 5 to 4 or removal of one interaction.

C4. A +20% schedule expansion allows for a single component technology maturity upgrade of two levels or removal of 2 interactions or two upgrades of one level each.

Schedule contractions work the same, but by reversing the direction from upgrade to downgrade, or by adding interactions.

Figure 7.0-6 shows the sensitivity analysis for the schedule changes. The % baseline (100% budget) programmatic risk is shown versus the % baseline budget. It is interesting to note the lack of symmetry near the baseline budget (100%). Programmatic risk is more sensitive to budget decreases than increases near the baseline. This is because small increases in budget do not significantly alter the technological maturity of a single component, but they do overweight that component’s relative performance. Thus, little risk is initially reduced. On the other side, a small budget contraction can tilt a program’s delicate balance. Note that large changes, whether positive or negative in nature, do result in approximately symmetric sensitivities.

Figure 7.0-7 shows the AXAF-I/COMPRÉ surface for the relative impact of changes in budget and schedule on programmatic risk. While programmatic risk is generally monotonically decreasing with increasing budget allocations (applied judiciously, of course), the sensitivity of risk to changes in schedule duration is more complex. Above the baseline schedule duration,
reductions in technological risk more than compensate for the expected increases in system variability associated with a longer development timeframe. Thus, the overall programmatic risk is mostly reduced with increasing schedule durations. Below the baseline duration, however, the overall programmatic risk decreases with tightening schedules. This is because increasing technological risk is more than compensated for by decreasing system variability associated with tighter schedules. Thus, an AXAF-I risk containment strategy can actually be achieved by a tightening of the schedule.

![Figure 7.0-6. AXAF-I Budgetary/Contraction Sensitivity](image)

Figure 7.0-6. AXAF-I Budgetary/Contraction Sensitivity

![Figure 7.0-7. Representative COMPRÉ Risk Sensitivity Surface](image)

Figure 7.0-7. Representative COMPRÉ Risk Sensitivity Surface
8.0 Preliminary COMPRÉ Verification and Validation

Currently, COMPRÉ model validation is accomplished by performing statistical hypothesis tests on the degree of association between COMPRÉ's ranking of risk contributors and the ranking of "hot spots" as noted by program management. Typical association measures by Kendall and Spearman are easily understood and provide robust, nonparametric conclusions. COMPRÉ validation plans include the analysis of both governmental and commercial programs.

A method for validating the Type I and II error rates in COMPRÉ is currently planned. The Type I error probability is the probability that the COMPRÉ model is concluded to have underperformed its benchmark, when it actually outperformed it. The Type II error probability is the probability that the COMPRÉ model is concluded to have outperformed its benchmark, when it actually underperformed it. Acceptable benchmarks include Kendall confidence levels, as well as other statistical measures. Acceptable Type II error rates are usually smaller than acceptable Type I error rates, which are often in the 10-20% range for most types of risk assessments. OR Applications has developed a proprietary statistical acceleration technique for maximizing the statistical information for a limited amount of validation data. This technique is associated with the general field of statistical resampling, the method of calculation shown below.

\[
\hat{\alpha} = P\left(\frac{\hat{p}_* - \hat{p}}{\hat{\sigma}_*} > \frac{\hat{p} - p_0}{\hat{\sigma}}\right)
\]

\[
\hat{\beta} = P\left(\frac{\hat{p}_* - \hat{p}}{\hat{\sigma}_*} \leq \frac{\hat{p} - p_0}{\hat{\sigma}}\right)
\]

where
\(\hat{\alpha}\) = the resampled estimate for the probability of Type I error,
\(\hat{\beta}\) = the resampled estimate for the probability of Type II error,
\(\hat{p}_*\) = the resampled estimate for the performance measure,
\(\hat{p}\) = the data estimate for the performance measure,
\(p_0\) = the benchmark performance measure,
\(\hat{\sigma}_*\) = the resampled estimate for the standard deviation of the data,
\(\hat{\sigma}\) = the data estimate for the standard deviation of the data.

9.0 Quality Modeling Using COMPRÉ

A concept for monitoring and quality control of COMPRÉ is also being developed. This concept is particularly important as COMPRÉ moves from a predictive stage to a prescriptive one. Control of COMPRÉ takes many forms: schedule and budgetary contractions and expansions; technology choice; investment weighting decisions; architectural design and modification as the program unfolds; and organizational makeup and design for implementation purposes.
Each of these decisions permeates through COMPRÉ in complex, nonlinear ways. Control is not trivial, and “tweaking the model to get the desired results” frequently backfires. To compare recommended actions resulting from COMPRÉ use with actual results, accelerated quality control techniques for hypothesis testing are being used. The null hypothesis (provided below) is that the ratio of programmatic risk to payoff is less than a benchmark value.

\[
H_0: \frac{\sigma_s}{\lambda_s} \leq \frac{\sigma_B}{\lambda_B} \\
H_1: \frac{\sigma_s}{\lambda_s} > \frac{\sigma_B}{\lambda_B}
\]

A conclusion of the alternative hypothesis, that the normalized risk is greater than a prescribed standard, is a strong indicator for change. Improvement, as the development program evolves, is characterized by a reduction in this “normalized risk.” Benchmark values for this ratio are usually near 1.0. A value of 1.0 implies that the program management for the system development is using information (and resources) in such a way as to increase technological payoff (at the system level) by one unit for every additional unit of programmatic risk incurred.

A method for monitoring the performance is shown in Figure 9.0-1. This plot shows the maximum likelihood ratio versus time or sample number. The maximum likelihood ratio is the ratio of the likelihood that the alternative hypothesis is true (program management is underperforming the benchmark) to the likelihood that the null hypothesis is true (program management is outperforming the benchmark). The greater the ratio, the more likely that management is poorly performing. The sample number is merely the number of quality monitorings that has occurred. Monitorings should be done frequently enough to allow for course corrections, but not too frequently to be an unnecessary burden by overemphasizing short-term performance. In this figure, sufficient data has been collected to determine underperformance of management, with respect to its benchmark.
10.0 Final Assessment

Figure 7.0-7 gives us an indication of the possible utility of COMPRÉ analysis on programmatic considerations. Specifically, a 20% decrease in schedule duration results in a 25% decrease in programmatic risk. Moreover, since schedule contractions are often associated with reductions in budgetary risk (and absolute costs), the impact may be even more favorable than depicted. Since COMPRÉ plots such as this one can be updated over time as more accurate information becomes available, the COMPRÉ methodology provides a mechanism for program management to dynamically tailor their project characteristics to acceptable risks and technological requirements.

11.0 References


Thomas, D., Private Communication, August 12, 1996.
Appendix A: Basis Function Formulas and Covariance Structures

The general formulas for evaluating the technological maturities and programmatic risk are provided in Section 4.0, with the overall methodology provided in Section 5.0. This appendix provides the specific basis function manipulations to achieve system (and subsystem) level results.

The six basis functions, corresponding to the six technological maturities, are given by:

\[ r_0(t) = t \]
\[ r_1(t) = \frac{t}{2} + \frac{t^2}{2} \]
\[ r_2(t) = t^2 \]
\[ r_3(t) = \frac{3t^2}{4} + \frac{e^t}{4} \]
\[ r_4(t) = \frac{t^2}{2} + \frac{e^t}{2} \]
\[ r_5(t) = e^t \]

Note that the basis functions for technological maturities of 1, 3, and 4 are linear combinations of the basis functions for technological maturities of 0, 2, and 5. The expected technological payoff for the standard basis functions may be found by integration to be:

\[ E[r_0(t)] = \frac{t}{2} \]
\[ E[r_2(t)] = \frac{t^2}{3} \]
\[ E[r_5(t)] = \frac{e^t}{t} \]

The other basis function expectations may be calculated as linear functions of these expectations:

\[ E[r_1(t)] = \frac{2E[r_0(t)]}{2} + \frac{2E[r_2(t)]}{2} \]
\[ E[r_3(t)] = \frac{3E[r_2(t)]}{4} + \frac{E[r_5(t)]}{4} \]
\[ E[r_4(t)] = \frac{E[r_5(t)]}{2} + \frac{E[r_5(t)]}{2} \]
The standard basis covariances amongst themselves may be found through integration as:

\[
C_{00}(t) = \frac{t^2}{12}
\]

\[
C_{22}(t) = \frac{4t^4}{45}
\]

\[
C_{55}(t) = e^{2t} \left( \frac{1}{2t} - \frac{1}{t^2} \right)
\]

\[
C_{02}(t) = \frac{t^3}{12}
\]

\[
C_{05}(t) = e^{2t} \left( \frac{1}{2} - \frac{1}{t} \right)
\]

\[
C_{23}(t) = e^t \left( \frac{2t}{3} - 2 \frac{1}{t} \right)
\]

Finally, covariances amongst the other (nonstandard) basis functions are evaluated as linear combinations of the standard basis covariances, as given below.

\[
C_{01}(t) = \frac{C_{00}(t) + C_{22}(t)}{2}
\]

\[
C_{03}(t) = \frac{3C_{03}(t) + C_{05}(t)}{4}
\]

\[
C_{04}(t) = \frac{C_{02}(t) + C_{05}(t)}{2}
\]

\[
C_{11}(t) = \frac{C_{00}(t) + C_{22}(t) + C_{22}(t)}{4}
\]

\[
C_{12}(t) = \frac{C_{02}(t) + C_{22}(t)}{2}
\]

\[
C_{13}(t) = \frac{3C_{03}(t) + 3C_{22}(t) + C_{05}(t) + C_{23}(t)}{8}
\]

\[
C_{14}(t) = \frac{C_{02}(t) + C_{22}(t) + C_{05}(t) + C_{23}(t)}{4}
\]

\[
C_{15}(t) = \frac{C_{05}(t) + C_{23}(t)}{2}
\]

\[
C_{23}(t) = \frac{3C_{22}(t) + C_{23}(t)}{4}
\]

\[
C_{24}(t) = \frac{C_{22}(t) + C_{23}(t)}{2}
\]
\[ C_{33}(t) = \frac{9C_{22}(t)}{16} + \frac{3C_{25}(t)}{8} + \frac{C_{55}(t)}{16} \]
\[ C_{34}(t) = \frac{3C_{22}(t)}{8} + \frac{C_{25}(t)}{2} + \frac{C_{55}(t)}{8} \]
\[ C_{35}(t) = \frac{3C_{25}(t)}{4} + \frac{C_{55}(t)}{4} \]
\[ C_{44}(t) = \frac{C_{22}(t)}{4} + \frac{C_{25}(t)}{2} + \frac{C_{55}(t)}{4} \]
\[ C_{45}(t) = \frac{C_{25}(t)}{2} + \frac{C_{55}(t)}{2} \]

**Appendix B: Numerical Example Using AXAF-I Example Data**

As an example of the use of the equations provided in Appendix A, a portion of the AXAF-I analysis is included in this appendix. Specifically, the development of the complexity results for the ISIM subsystem is provided, because it includes examples of all possible COMPRE calculations. The reader should refer to Section 4.0 for the general equations, Appendix A for the specific equations, and Figure 7.0-1 for the raw input data for the AXAF-I example. The development schedule duration is 5 years \((t = 5)\).

The expected technological payoff for the SIM component (technological maturity = 3) is given in Appendix A as:

\[ E[r_{SIM}(5)] = E[r_{j}(5)] = \frac{3E[r_{2}(5)]}{4} + \frac{E[r_{5}(5)]}{4} = \frac{(3(5^2 / 3) + e^5 / 5)}{4} = 13.67 \]

Similarly, the expected technological payoffs for the HRC and ACIS components of the ISIM are given as:

\[ E[r_{HRC}(5)] = E[r_{4}(5)] = \frac{E[r_{2}(5)]}{2} + \frac{E[r_{5}(5)]}{2} = \frac{(5^2 / 3) + e^5 / 5}{2} = 19.01 \]
\[ E[r_{ACIS}(5)] = E[r_{5}(5)] = e^5 / 5 = 29.68 \]

Then, the composite development maturity for the ISIM system is given in Section 4.0 (also, see Table 7.0-1 for the relative budget weights) as:

\[ \lambda_{ISIM}(5) = 0.28(13.67) + 0.22(19.01) + 0.50(29.68) = 22.85 \]

Note, that the composite technological maturity of the ISIM subsystem corresponds to approximately a 4.4. The covariance between the SIM and HRC is given in Appendix A as:
\[ C_{SIM,HRC}(5) = C_{34}(5) = \frac{3C_{22}(5)}{8} + \frac{C_{25}(5)}{2} + \frac{C_{45}(5)}{8} = \frac{3(4)(5^4)}{8(45)} + \frac{e^\gamma(2(5) - 2 + \frac{2}{5})}{2} + \frac{e^{2(5)}(\frac{1}{2(5)} - \frac{1}{5^2})}{8} = 314.66 \]

An organizational dispersion factor of 2 is applied to account for the cross-organization interface between these two components. This brings the covariance up to 629.32. The covariance between the SIM and ACIS is handled similarly as:

\[ C_{SIM,ACIS}(5) = C_{35}(5) = \frac{3C_{25}(5)}{4} + \frac{C_{45}(5)}{4} = \frac{3e^\gamma(2(5) - 2 + \frac{2}{5})}{4} + \frac{e^{2(5)}(\frac{1}{2(5)} - \frac{1}{5^2})}{4} = 523.33 \]

An organizational dispersion factor of 2 is applied to account for the cross-organization interface between these two components. This brings the covariance up to 1046.66. Note that, due to a lack of interaction, the covariance between the HRC and ACIS components is 0. The covariances (variances) between the SIM and itself, HRC and itself, and ACIS and itself are 210.32, 486.80, and 1321.59, respectively. The overall programmatic risk for the ISIM subsystem is given in Section 4.0 as:

\[ \sigma_{ISIM}(5) = \{0.28[0.28(210.32) + 0.22(629.32) + 0.50(1046.66)] + 0.22[0.28(629.32) + 0.22(486.80) + 0.50(0)] + 0.50[0.28(1046.66) + 0.22(0) + 0.50(1321.59)]\}^{1/2} = 27.22 \]
**Title and Subtitle:** Systems Engineering Metrics: Organizational Complexity and Product Quality Modeling

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**Abstract:**
Innovative organizational complexity and product quality models applicable to performance metrics for NASA-MSFC’s Systems Analysis and Integration Laboratory (SAIL) missions and objectives are presented. An intensive research effort focuses on the synergistic combination of stochastic process modeling, nodal and spatial decomposition techniques, organizational and computational complexity, systems science and metrics, chaos, and proprietary statistical tools for accelerated risk assessment. This is followed by the development of a preliminary model, which is uniquely applicable and robust for quantitative purposes. Exercise of the preliminary model using a generic system hierarchy and the AXAF-I architectural hierarchy is provided. The Kendall test for positive dependence provides an initial verification and validation of the model. Finally, the research and development of the innovation is revisited, prior to peer review. This research and development effort results in near-term, measurable SAIL organizational and product quality methodologies, enhanced organizational risk assessment and evolutionary modeling results, and improved statistical quantification of SAIL productivity interests.

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