FINAL REPORT

NASA ASTROPHYSICAL THEORY GRANT NAG5-2882

FORMATION OF STRUCTURE IN THE UNIVERSE

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Summary of Research Progress

This grant supported research by the investigators through summer salary support for Strauss and Weinberg, support for graduate students at Princeton University and Ohio State University, and travel, visitor, and publication support for the investigators. The grant originally had a duration of 1 year, and it was extended (without additional funding) for an additional year. The impact of the grant was considerable given its relatively modest duration and funding level, in part because it provided “seed” funding to get Strauss and Weinberg started at new institutions, and in part because it was combined with support from subsequent grants. Here we summarize progress in the three general areas described in the grant proposal.

Lya absorbers and the intergalactic medium

Miralda-Escudé, Weinberg, and their collaborators used hydrodynamic cosmological simulations to make predictions for Lya absorption in the spectra of high-redshift quasars. These simulations appear to reproduce properties of observed quasar spectra remarkably well. The low column-density absorption that makes up the “Lya forest” arises mainly in “small scale structure”: sheets and filaments of warm, photoionized gas that are the high-redshift analog of today’s galaxy superclusters. Lyman limit and damped Lya absorption arises in the radiatively cooled gas associated with forming galaxies.

The simulations provide a compelling theoretical picture for the origin of the Lya forest. While this picture incorporates ingredients from many of the more ad hoc theoretical models advanced in the 25 years since the forest was first discovered, the simulations provide a more cohesive theoretical framework that ties the Lya forest into a broader picture of structure formation in the universe. Starting with cosmological models that are motivated by observations of galaxy clustering and the cosmic microwave background, one performs simulations with the relevant physical processes, and the forest falls out “for free,” with no special tweaks.

In the view that emerges from the simulations, the Lya forest is produced by fluctuating absorption in a diffuse intergalactic medium. The physical processes that determine the spatial distribution and thermal state of this medium are relatively simple — amenable to numerical study and, to some degree, to analytic modeling. Starting from this point of view, Weinberg, Miralda-Escudé, and collaborators have developed new statistical approaches for characterizing QSO absorption spectra and derived new constraints on the average density of baryons in the universe. The results of these analyses imply that most of the baryons in the high-redshift universe reside in the diffuse intergalactic medium.

Weinberg and collaborators have also used the cosmological simulations to interpret recent observations of HeII absorption by HST and the Hopkins Ultraviolet Telescope (HUT); this absorption also arises in the fluctuating intergalactic medium, but more of it occurs in low density gas that is virtually undetectable by other observational means. The simulations of HeII absorption are now being used to help guide the design of a potential successor to HUT.
Galaxy formation

Weinberg and collaborators completed a long paper describing the methods used for their hydrodynamic cosmological simulations (the basis for much of the work mentioned above). In addition to detailing the numerical methods and the physical treatment of photoionization and star formation, this paper presents illustrative applications to galaxy formation in the cold dark matter model. Confirming and strengthening earlier work by the same group, these illustrations show that gas cooling and star formation within dark matter halos leads to the formation of plausible galaxies and galaxy groups.

One main goal of the work on galaxy formation has been to understand the influence of photoionization. Weinberg and IAS postdoc Anne Thoul completed a paper studying the suppression of low mass galaxy formation by photoionization, using Thoul's 1-dimensional, spherically symmetric, gravity/hydrodynamics code. They find that ionization by the ultraviolet radiation background prevents any collapse and cooling of gas in objects with circular velocities smaller than $v_c \sim 35$ km s$^{-1}$ but has little effect for systems with $v_c > 75$ km s$^{-1}$. Weinberg and collaborators also used the SPH code to study photoionization effects in large volume, 3-dimensional simulations, reaching conclusions consistent with those of the 1-dimensional approach, while also illustrating interactions between numerical resolution and microphysics that can affect these sorts of simulations.

Closely connected to the problem of galaxy formation is the study of damped Lyα absorption, which arises in the dense, neutral gas of young galaxies. Weinberg, Miralda-Escudé, and collaborators have used a combination of numerical and semi-analytic methods to examine the nature of damped Lyα absorbers in the cold dark matter scenario and to see what constraints on cosmological parameters can be derived from the observed abundance of these absorbers.

The technique used by Weinberg and collaborators is a 3-dimensional, Lagrangian method known as "smoothed particle hydrodynamics," or SPH. Ohio State graduate student Michael Owen, supported in part by this grant, has developed a cosmological code employing a new form of SPH that uses anisotropic smoothing kernels in order to adapt to the local collapse geometry. He and his collaborators have completed a paper describing this adaptive SPH code. Owen has used the code to study the influence of a finite gas temperature (and corresponding Jeans mass) on the formation of low mass objects, an issue fundamental to understanding the implications of hydrodynamic cosmological simulations. In collaboration with Weinberg and others, Owen has completed a study of hydrodynamic simulations that start from scale-free cosmological initial conditions; with such initial conditions, one can test the extent to which numerical simulations are able to reproduce analytically predicted, self-similar scaling laws.

Large scale structure

Strauss and collaborators completed observational work on the Optical Redshift Survey (ORS) and have begun to analyze galaxy clustering in the sample. This survey covers a volume similar to that of the 1.2-Jy survey of IRAS-selected galaxies, so a comparison of the derived density fields, and of the density fields of different morphological subsets of
the ORS, will yield direct measures of the relative clustering of different types of galaxies. With these will come clues to the relation between the distribution of galaxies and the underlying distribution of mass. With other collaborators, Strauss has analyzed a sample of galaxies detected in a 21cm survey of the Bootes void, yielding information on the clustering of galaxies in a region where the background density defined on a very large scale is well below the cosmic mean.

Strauss, in collaboration with Princeton graduate student Jeremy Kepner and Princeton postdoc Frank Summers, used cosmological N-body simulations to show that the relation between local galaxy density and velocity dispersion can be used to constrain the cosmological density parameter \( \Omega \). This technique, an extension of the Cosmic Virial Theorem, should have good discriminatory power when applied to volume limited subsets of the Sloan Digital Sky Survey. In collaboration with Princeton undergraduate Daniel Koranyi, Strauss showed that the galaxy distribution in the 1.2-Jy IRAS survey is consistent with large scale homogeneity and Hubble's law, refuting an earlier claim by Segal that this sample is better fit by a quadratic redshift-distance relation.

Strauss, in collaboration with Princeton graduate student Rita Kim, examined the counts-in-cells distribution of galaxies in the 1.2-Jy IRAS survey. They developed and applied a new technique based on the Edgeworth expansion to derive maximum-likelihood estimates of the skewness and kurtosis parameters of the probability distribution function of galaxy densities, obtaining more reliable values with smaller statistical uncertainties than those from previous studies. Results from the 1.2-Jy survey support the scale-invariance of skewness and kurtosis parameters predicted by cosmological models with Gaussian primordial density fluctuations.

Fisher and Weinberg, in collaboration with Shaun Cole, measured the anisotropy of the redshift-space power spectrum in the 1.2-Jy survey and used this to derive constraints on \( \Omega \) and on the bias parameter, \( b \), which describes the relative clustering of galaxies and mass. Fisher carried out two other theoretical investigations of redshift-space clustering, explicating the relation between the two most widely used frameworks for describing the clustering distortions caused by peculiar velocities, and using the Zel'dovich approximation to compute the expected departures from the distortions predicted by linear theory. Weinberg and Ohio State graduate student Piotr Popowski are examining the expected anisotropy of the quasar correlation function. Here the main source of distortion is non-Euclidean geometry rather than peculiar motions, and measurement of the effect for a large quasar sample should yield significant constraints on the cosmological constant, \( \Lambda \). Weinberg and Ohio State graduate student Vijay Narayanan are working on improved methods for recovering primordial density fluctuations from the observed galaxy distribution, by incorporating dynamical corrections via the Zel'dovich approximation. This technique can ultimately be combined with the anisotropy analysis to yield tighter constraints on \( \Omega \) and \( b \) from existing redshift samples.

Weinberg and collaborators used a series of large N-body simulations to study the formation of large-scale structure in cosmological models that were constrained to match the amplitude of cosmic microwave background fluctuations observed by the COBE satel-
lite. They considered models dominated by cold dark matter with various values of $\Omega$, one sequence assuming an open universe and one sequence assuming a flat universe with a cosmological constant. By comparing the predicted masses of rich galaxy clusters to observed masses derived from galaxy velocity dispersions and X-ray observations, they show that $\Omega$ must be larger than 0.4 in the open models and larger than 0.25 in the flat models. All of the models require a non-linear relation between galaxy density and mass density in order to match observed galaxy clustering.
Publications Supported By This Grant


Paper Abstracts

THE LYMAN-ALPHA FOREST IN THE COLD DARK MATTER MODEL  
Lars Hernquist, Neal Katz, David H. Weinberg, and Jordi Miralda-Escudé  
The Astrophysical Journal (Letters), 457, L51

Cosmological simulations with gas provide a detailed description of the intergalactic medium, making possible predictions of neutral hydrogen absorption in the spectra of background QSOs. We present results from a high-resolution calculation of an $\Omega = 1$ cold dark matter model. Our simulation reproduces many of the observed properties of the Lyα forest surprisingly well. The distribution of HI column densities agrees with existing data to within a factor of $\sim$ two over most of the range from $10^{14}$ cm$^{-2}$ to $10^{22}$ cm$^{-2}$; i.e., from unsaturated Lyα forest lines to damped Lyα systems. The equivalent width distribution matches the observed exponential form with a characteristic width $W_0 \approx 0.3$ Å. The distribution of $b$-parameters appears consistent with that derived from QSO spectra. Most of the low column density absorption arises in large, flattened structures of moderate or even relatively low overdensity, so there is no sharp distinction between the Lyα forest and the "Gunn-Peterson" absorption produced by the smooth intergalactic medium. Our results demonstrate that a Lyα forest like that observed develops naturally in a hierarchical clustering scenario with a photoionizing background. Comparison between simulations and high-resolution QSO spectra should open a new regime for testing theories of cosmic structure formation.

SMALL SCALE STRUCTURE AND HIGH REDSHIFT HI  
David H. Weinberg, Neal Katz, Lars Hernquist, and Jordi Miralda-Escudé  
in Cold Gas at High Redshift, eds. M. Bremer, H. Rottgering, C. Carilli  
and P. van de Werf, Kluwer, Dordrecht (1996), 93

Cosmological simulations with gas dynamics suggest that the Lyman-alpha forest is produced mainly by "small scale structure" — filaments and sheets that are the high redshift analog of today's galaxy superclusters. There is no sharp distinction between Lyman-alpha clouds and "Gunn-Peterson" absorption produced by the fluctuating IGM — the Lyman-alpha forest is the Gunn-Peterson effect. Lyman limit and damped Lyman-alpha absorption arises in the radiatively cooled gas of forming galaxies. At $z \sim 2 - 3$, most of the gas is in the photoionized, diffuse medium associated with the Lyman-alpha forest, but most of the neutral gas is in damped Lyman-alpha systems. We discuss generic evolution of cosmic gas in a hierarchical scenario of structure formation, with particular attention to the prospects for detecting 21cm emission from high redshift HI. A scaling argument based on the present-day cluster mass function suggests that objects with $M_{HI} \gtrsim 5 \times 10^{11}h^{-1}M_\odot$ should be extremely rare at $z \sim 3$, so detections with existing instruments will be difficult. An instrument like the proposed Square Kilometer Array could detect individual damped Lyman-alpha systems at high redshift, making it possible to map structure in the high redshift universe in much the same way that today's galaxy redshift surveys map the local large scale structure.
THE LYMAN ALPHA FOREST FROM GRAVITATIONAL COLLAPSE IN THE CDM+LAMBDIA MODEL

Jordi Miralda-Escudé, Renyue Cen, Jeremiah P. Ostriker, and Michael Rauch

The Astrophysical Journal, 471, 582

We use an Eulerian hydrodynamic cosmological simulation to model the Lyα forest in a spatially flat, COBE normalized, cold dark matter model with $\Omega = 0.4$, and find that the intergalactic, photoionized gas is predicted to collapse into sheet-like and filamentary structures which give rise to absorption lines having similar characteristics as the observed Lyα forest. A typical filament is $\sim 1h^{-1}$Mpc long with thickness $\sim 50 - 100h^{-1}$kpc (in proper units), and baryonic mass $\sim 10^{10}h^{-1}M_\odot$. In comparison our nominal numerical resolution is $(2.5, 9)h^{-1}$kpc (in the two simulations we perform) with true resolution, perhaps a factor of 2.5 worse than this; the nominal mass resolution is $(10^4.2, 10^5.8)M_\odot$ in gas for the two simulations. The gas temperature increases with time as structures with larger velocities collapse gravitationally. We show that the predicted distributions of column densities, $b$-parameters and equivalent widths of the Lyα forest clouds agree with the observed ones, and that their evolution is consistent with our model, if the ionizing background has (as expected from our simulations) an approximately constant intensity between $z = 2$ and $z = 4$. The new method of identifying lines as contiguous regions in the spectrum below a fixed flux threshold is suggested to analyze the absorption lines, given that the Lyα spectra arise from a continuous density field of neutral hydrogen rather than discrete clouds. We also predict the distribution of transmitted flux and its correlation along a spectrum and on parallel spectra, and the HeII flux decrement as a function of redshift. We predict a correlation length of $\sim 80h^{-1}$kpc perpendicular to the line of sight for features in the Lyman alpha forest. In order to reproduce the observed number of lines and average flux transmission, the baryon content of the clouds may need to be significantly higher than in previous models because of the low densities and large volume-filling factors we predict. If the background intensity $J_{\text{HI}}$ is at least that predicted from the observed quasars, $\Omega_b$ needs to be as high as $\sim 0.025h^{-2}$, higher than expected by light element nucleosynthesis; the model also predicts that most of the baryons at $z > 2$ are in Lyα clouds, and that the rate at which the baryons move to more overdense regions is slow. A large fraction of the baryons which are not observed at present in galaxies might be intergalactic gas in the currently collapsing structures, with $T \sim 10^5 - 10^6$K.

VOIGT-PROFILE ANALYSIS OF THE LYMAN-ALPHA FOREST IN A COLD DARK MATTER UNIVERSE

Romeel Davé, Lars Hernquist, David H. Weinberg, and Neal Katz

The Astrophysical Journal, 477, 21

We use an automated Voigt-profile fitting procedure to extract statistical properties of the Lyα forest in a numerical simulation of an $\Omega = 1$, cold dark matter (CDM) universe. Our analysis method is similar to that used in most observational studies of the forest, and we compare the simulations to recently published results derived from Keck HIRES spectra. With the Voigt-profile decomposition analysis, the simulation reproduces the large number of weak lines ($N_{\text{HI}} \leq 10^{13}\text{cm}^{-2}$) found in the HIRES spectra. The column density
distribution evolves significantly between $z = 3$ and $z = 2$, with the number of lines at fixed column density dropping by a factor $\sim 1.6$ in the range where line blending is not severe. At $z = 3$, the $b$-parameter distribution has a median of $35$ km s$^{-1}$ and a dispersion of $20$ km s$^{-1}$, in reasonable agreement with the observed values. The comparison between our new analysis and recent data strengthens earlier claims that the Ly$\alpha$ forest arises naturally in hierarchical structure formation as photoionized gas falls into dark matter potential wells. However, there are two statistically significant discrepancies between the simulated forest and the HIRES results: the model produces too many lines at $z = 3$ by a factor $\sim 1.5 - 2$, and it produces more narrow lines ($b < 20$ km s$^{-1}$) than are seen in the data. The first result is sensitive to our adopted normalization of the mean Ly$\alpha$ optical depth, and the second is sensitive to our assumption that helium reionization has not significantly raised gas temperatures at $z = 3$. It is therefore too early to say whether these discrepancies indicate a fundamental problem with the high-redshift structure of the $\Omega = 1$ CDM model or reflect errors of detail in our modeling of the gas distribution or the observational procedures.

INTERGALACTIC HELIUM ABSORPTION IN COLD DARK MATTER MODELS
Rupert A. C. Croft, David H. Weinberg, Neal Katz, and Lars Hernquist
The Astrophysical Journal, in press

Observations from the Hubble Space Telescope and the Hopkins Ultraviolet Telescope have recently detected HeII absorption along the lines of sight to two high redshift quasars. We use cosmological simulations with gas dynamics to investigate HeII absorption in the cold dark matter (CDM) theory of structure formation. We consider two $\Omega = 1$ CDM models with different normalizations and one open universe ($\Omega_0 = 0.4$) CDM model. The simulations incorporate the photoionizing UV background spectrum computed by Haardt & Madau (1996), which is based on the output of observed quasars and reprocessing by the Ly$\alpha$ forest. The simulated gas distribution, combined with the Haardt & Madau spectral shape, accounts for the relative observed values of $\tau_{HI}$ and $\tau_{HeII}$, the effective mean optical depths for HI and HeII absorption. If the background intensity is as high as Haardt & Madau predict, then matching the absolute observed values of $\tau_{HI}$ and $\tau_{HeII}$ requires a baryon abundance larger (by factors between 1.5 and 3 for the various CDM models) than our assumed value of $\Omega_b h^2 = 0.0125$. The simulations reproduce the evolution of $\tau_{HeII}$ over the observed redshift range, $2.2 \leq z \leq 3.3$, if the HeII photoionization rate remains roughly constant.

HeII absorption in the CDM simulations is produced by a diffuse, fluctuating, intergalactic medium, which also gives rise to the HI Ly$\alpha$ forest. Much of the HeII opacity arises in underdense regions where the HI optical depth is very low. We compute statistical properties of the HeII and HI absorption that can be used to test the CDM models and distinguish them from an alternative scenario in which the HeII absorption is caused by discrete, compact clouds. The CDM scenario predicts that a substantial amount of baryonic material resides in underdense regions at high redshift. HeII absorption is the only sensitive observational probe of such extremely diffuse, intergalactic gas, so it can provide a vital test of this fundamental prediction.
THE OPACITY OF THE LYMAN-ALPHA FOREST AND IMPLICATIONS FOR $\Omega_{\text{baryon}}$ AND THE IONIZING BACKGROUND
Michael Rauch, Jordi Miralda-Escudé, Wallace L.W. Sargent, Tom A. Barlow
David H. Weinberg, Lars Hernquist, Neal Katz, Renyue Cen, Jeremiah P. Ostriker
The Astrophysical Journal, in press

We have measured the distribution function of the flux decrement $D = e^{-\tau}$ caused by Ly$\alpha$ forest absorption from intervening gas in the lines of sight to high redshift QSOs from a sample of seven high resolution QSO spectra obtained with the Keck telescope. The observed flux decrement distribution function (FDDF) is compared to the FDDF from two simulations of the Ly$\alpha$ forest: a $\Lambda$CDM model (with $\Omega = 0.4$, $\Lambda = 0.6$) computed with the Eulerian code of Cen & Ostriker, and a standard CDM model (SCDM, with $\Omega = 1$) computed with the SPH code of Hernquist, Katz, & Weinberg. Good agreement is obtained between the shapes of the simulated and observed FDDFs for both simulations after fitting only one free parameter, which controls the mean flux decrement. The difference between the predicted FDDFs from the two simulations is small, and we show that it arises mostly from a different temperature in the low-density gas (caused by different assumptions that were made about the reionization history in the two simulations), rather than differences between the two cosmological models per se, or numerical effects in the two codes which use very different computational methods.

A measurement of the parameter $\mu \propto \Omega_b^2 h^3 / \Gamma$ (where $\Gamma$ is the HI ionization rate due to the ionizing background) is obtained by requiring the mean flux decrement in the simulations to agree with the observed one. Estimating the lower limit $\Gamma > 7 \times 10^{-13}$ s$^{-1}$ from the abundance of known QSOs, we derive a lower limit on the baryonic matter density, $\Omega_b h^2 > 0.021(0.017)$ for the $\Lambda$CDM (SCDM) model. The difference between the lower limit inferred from the two models is again due to different temperatures in the low-density gas. We give general analytical arguments for why this lower limit is unlikely to be reduced for any other models of structure formation by gravitational collapse that can explain the observed Ly$\alpha$ forest. The large $\Omega_b$ we infer is inconsistent with some recent D/H determinations (Rugers & Hogan 1996a,b), favoring a low deuterium abundance as reported by Tytler, Fan & Burles (1996). Adopting a fixed $\Omega_b$, the measurement of $\mu(z)$ allows a determination of the evolution of the ionizing radiation field with redshift. Our models predict an intensity that is approximately constant with redshift, which is in agreement with the assumption that the ionizing background is produced by known quasars for $z < 3$, but requires additional sources of ionizing photons at higher redshift given the observed rapid decline of the quasar abundance.

A LOWER BOUND ON THE COSMIC BARYON DENSITY
David H. Weinberg, Jordi Miralda-Escudé, Lars Hernquist, Neal Katz
The Astrophysical Journal, submitted

We derive lower bounds on the cosmic baryon density from the requirement that the high-redshift intergalactic medium (IGM) contain enough neutral hydrogen to produce the observed Ly$\alpha$ absorption in quasar spectra. These analytic bounds follow from a key theoretical assumption — that absorbing structures are on average no more extended in
redshift space than in real space — which is likely to hold in the gravitational instability picture of the Ly\(\alpha\) forest, independently of the details of the cosmological model. The other ingredients that enter these bounds are an estimate of (or lower limit to) the intensity of the photoionizing UV background from quasars, a temperature \(T \sim 10^4\) K for the “warm” photoionized IGM that produces most of the Ly\(\alpha\) absorption, a value of the Hubble constant, and observational estimates of the mean Ly\(\alpha\) flux decrement \(\overline{D}\) or, for a more restrictive bound, the distribution function \(P(\tau)\) of Ly\(\alpha\) optical depths. With plausible estimates of the quasar UV background and \(\overline{D}\), the mean decrement bound implies a baryon density parameter \(\Omega_b \geq 0.0125h^{-2}\), where \(h \equiv H_0/100\ \text{km s}^{-1}\ \text{Mpc}^{-1}\). A recent observational determination of \(P(\tau)\) implies \(\Omega_b \geq 0.0125h^{-2}\) even for a conservative estimate of the quasar UV background, and \(\Omega_b \geq 0.018h^{-2}\) for a more reasonable estimate. These bounds are consistent with recent low estimates of the primordial deuterium-to-hydrogen ratio \((D/H)_p\), which imply \(\Omega_b \approx 0.025h^{-2}\) when combined with standard big bang nucleosynthesis. Since the bounds account only for baryons in the warm IGM, their combination with the nucleosynthesis constraint implies that most of the baryons in the universe at \(z \sim 2 - 4\) were distributed in diffuse intergalactic gas rather than in stars or compact dark objects. The \(P(\tau)\) bound on \(\Omega_b\) is incompatible with some recent high estimates of \((D/H)_p\), unless one drops the assumptions of standard big bang nucleosynthesis or abandons the idea that Ly\(\alpha\) forest lines originate in the smooth, large-scale structures of photoionized gas that arise in gravitational instability theories.

**IMAGING THE FOREST OF LYMAN-LIMIT SYSTEMS**

Andrew Gould and David H. Weinberg

The Astrophysical Journal, 468, 462

We show that it is now possible to image optically thick Ly\(\alpha\) clouds in fluorescent Ly\(\alpha\) emission with a relatively long (\(\sim 20\) hr) integration on a large (\(\sim 10\) m) telescope. For a broad range of column densities \((N \geq 10^{18.5}\ \text{cm}^{-2})\), the flux of Ly\(\alpha\) photons from recombination cascades is equal to \(\sim 0.6\) times the flux of ionizing photons, independent of the geometry of the cloud. Additional Ly\(\alpha\) photons are produced by collisional excitations when these are the cloud’s primary cooling mechanism. For typical physical conditions expected in optically thick clouds, these mechanisms together lead to a Ly\(\alpha\) emission flux that is \(\sim (2/3)\langle\nu\rangle/\nu_0\) times the flux of ionizing photons, where \(\langle\nu\rangle\) is the mean frequency of ionizing background photons and \(\nu_0\) is the Lyman limit frequency. Hence, measurement of the surface brightness from an optically thick cloud (known to exist, e.g., from a quasar absorption line) gives a direct measure of the energy in the ionizing radiation background. Moreover, in the same long slit spectrum one could hope to detect emission from \(\sim 200\) other Ly\(\alpha\) systems. Such detections would allow one to make a 2-dimensional map of the distribution of Ly\(\alpha\) clouds. By taking a series of such spectra, one could map the clouds in three dimensions, revealing structure in the high-redshift universe.
COSMOLOGICAL SIMULATIONS WITH TreeSPH
Neal Katz, David H. Weinberg, and Lars Hernquist
Astrophysical Journal Supplements, 105, 19

We describe numerical methods for incorporating gas dynamics into cosmological simulations and present illustrative applications to the cold dark matter (CDM) scenario. Our evolution code, a version of TreeSPH (Hernquist & Katz 1989) generalized to handle comoving coordinates and periodic boundary conditions, combines smoothed-particle hydrodynamics (SPH) with the hierarchical tree method for computing gravitational forces. The Lagrangian hydrodynamics approach and individual time steps for gas particles give the algorithm a large dynamic range, which is essential for studies of galaxy formation in a cosmological context. The code incorporates radiative cooling for an optically thin, primordial composition gas in ionization equilibrium with a user-specified ultraviolet background. We adopt a phenomenological prescription for star formation that gradually turns cold, dense, Jeans-unstable gas into collisionless stars, returning supernova feedback energy to the surrounding medium. In CDM simulations, some of the baryons that fall into dark matter potential wells dissipate their acquired thermal energy and condense into clumps with roughly galactic masses. The resulting galaxy population is insensitive to assumptions about star formation; we obtain similar baryonic mass functions and galaxy correlation functions from simulations with star formation and from simulations without star formation in which we identify galaxies directly from the cold, dense gas.

HYDRODYNAMIC SIMULATIONS OF GALAXY FORMATION
II. DISSIPATION AND THE MAXIMUM MASS OF GALAXIES
Anne A. Thoul and David H. Weinberg
The Astrophysical Journal, 465, 608

Photoionization by the high-redshift ultraviolet radiation background heats low density gas before it falls into dark matter potential wells, and it eliminates the neutral hydrogen and singly ionized helium that dominate cooling of primordial gas at temperatures of \(10^4 - 10^5\)K. We investigate the influence of photoionization on galaxy formation using high-resolution simulations with a 1-dimensional, spherically symmetric, Lagrangian hydrodynamics/gravity code. We find that the presence of a photoionizing background suppresses the formation of galaxies with circular velocities \(v_{\text{circ}} < 30 \text{ km s}^{-1}\) and substantially reduces the mass of cooled baryons in systems with circular velocities up to \(v_{\text{circ}} \sim 50 \text{ km s}^{-1}\). Above \(v_{\text{circ}} \sim 75 \text{ km s}^{-1}\), photoionization has no significant effect. Photoionization exerts its influence primarily by heating gas before collapse; the elimination of line cooling processes is less important. We discuss the implications of these results for hierarchical theories of galaxy formation.

PHOTOIONIZATION, NUMERICAL RESOLUTION, AND GALAXY FORMATION
David H. Weinberg, Lars Hernquist, and Neal Katz
The Astrophysical Journal, 477, 8

Using cosmological simulations that incorporate gas dynamics and gravitational forces, we investigate the influence of photoionization by an ultraviolet radiation background on
the formation of galaxies. In our highest resolution simulations, we find that photoionization has essentially no effect on the baryonic mass function of galaxies at $z = 2$, down to our resolution limit of $\sim 5 \times 10^9 M_\odot$. We do, however, find a strong interplay between the mass resolution of a simulation and the microphysics included in the computation of heating and cooling rates. At low resolution, a photoionizing background can appear to suppress the formation of even relatively massive galaxies. However, when the same initial conditions are evolved with a factor of eight improvement in mass resolution, this effect disappears. Our results demonstrate the need for care in interpreting the results of cosmological simulations that incorporate hydrodynamics and radiation physics. For example, we conclude that a simulation with limited resolution may yield more accurate results if it ignores some relevant physical processes, such as photoionization. At higher resolution, the simulated population of massive galaxies is insensitive to the treatment of photoionization or the inclusion of star formation in the simulations, but it does depend significantly on the amplitude of the initial density fluctuations. By $z = 2$, an $\Omega = 1$ cold dark matter model normalized to produce the observed masses of present-day clusters has already formed galaxies with baryon masses exceeding $10^{11} M_\odot$.

DAMPED LYMAN-ALPHA AND LYMAN LIMIT ABSORBERS IN THE COLD DARK MATTER MODEL
Neal Katz, David H. Weinberg, Lars Hernquist, and Jordi Miralda-Escudé
The Astrophysical Journal (Letters), 457, L57

We study the formation of damped Ly\textalpha and Lyman limit absorbers in a hierarchical clustering scenario using a gas dynamical simulation of an $\Omega = 1$ cold dark matter universe. In the simulation, these high column density systems are associated with forming galaxies. Damped Ly\textalpha absorption, $N_{\text{HI}} \gtrsim 10^{20.2} \text{ cm}^{-2}$, arises along lines of sight that pass near the centers of relatively massive, dense protogalaxies. Lyman limit absorption, $10^{17} \text{ cm}^{-2} \lesssim N_{\text{HI}} \lesssim 10^{20.2} \text{ cm}^{-2}$, develops on lines of sight that pass through the outer parts of such objects or near the centers of smaller protogalaxies. The number of Lyman limit systems is less than observed, while the number of damped Ly\textalpha systems is quite close to the observed abundance. Damped absorbers are typically $\sim 10 \text{ kpc}$ in radius, but the population has a large total cross section because the systems are much more numerous than present day $L_*$ galaxies. Our results demonstrate that high column density systems like those observed arise naturally in a hierarchical theory of galaxy formation and that it is now possible to study these absorbers directly from numerical simulations.

THE POPULATION OF DAMPED LYMAN-ALPHA AND LYMAN LIMIT SYSTEMS IN THE COLD DARK MATTER MODEL
Jeffrey P. Gardner, Neal Katz, Lars Hernquist, David H. Weinberg
The Astrophysical Journal, in press

Lyman limit and damped Ly\textalpha absorption systems probe the distribution of collapsed, cold gas at high redshift. Numerical simulations that incorporate gravity and gas dynamics can predict the abundance of such absorbers in cosmological models. We develop a semi-analytical method to correct the numerical predictions for the contribution of unresolved low mass halos, and we apply this method to the Katz et al. (1996) simulation of
the standard cold dark matter model ($\Omega = 1$, $h = 0.5$, $\Omega_b = 0.05$, $\sigma_8 = 0.7$). Using this simulation and higher resolution simulations of individual low mass systems, we determine the relation between a halo's circular velocity $v_c$ and its cross section for producing Lyman limit or damped Ly\(\alpha\) absorption. We combine this relation with the Press-Schechter formula for the abundance of halos — itself calibrated against the simulated halo population — to compute the number of absorbers per unit redshift. The resolution correction increases the predicted abundances by about a factor of two at $z = 2, 3, \text{and } 4$, bringing the predicted number of damped Ly\(\alpha\) absorbers into quite good agreement with observations. Roughly half of these systems reside in halos with circular velocities $v_c \geq 100 \text{ km s}^{-1}$ and half in halos with $35 \text{ km s}^{-1} \leq v_c \leq 100 \text{ km s}^{-1}$. Halos with $v_c > 150 \text{ km s}^{-1}$ typically harbor two or more systems capable of producing damped absorption. Even with the resolution correction, the predicted abundance of Lyman limit systems is a factor of three below observational estimates, signifying either a failure of the standard cold dark matter model or a failure of these simulations to resolve most of the systems responsible for Lyman limit absorption. By comparing simulations with and without star formation, we find that depletion of the gas supply by star formation affects absorption line statistics at $z \geq 2$ only for column densities exceeding $N_{\text{HI}} = 10^{22} \text{ cm}^{-2}$, even though half of the cold, collapsed gas has been converted to stars by $z = 2$.

DAMPED LYMAN-ALPHA SYSTEMS IN COSMOLOGICAL SIMULATIONS
Chung-Pei Ma, Edmund Bertschinger, Lars Hernquist, David H. Weinberg, and Neal Katz
The Astrophysical Journal, in press

Any viable cosmological model must produce enough structure at early epochs to explain the amount of gas associated with high-redshift damped Ly\(\alpha\) (DLA) systems. We study the evolution of DLA systems in cold dark matter (CDM) and cold+hot dark matter (CDM+HDM) models using both $N$-body and hydrodynamic simulations. By comparing dissipationless simulations to ones that include gas, we find a shallower slope for the column density distribution $f(N)$ and a smaller fraction of neutral dense gas $\Omega_g$ in DLA systems than previously inferred from the dark matter under the assumption that the neutral gas fraction in galactic halos is the same as the global baryonic fraction. These differences are driven by ionization of hydrogen in the outskirts of galactic halos and by gaseous dissipation near the halo centers, and they tend to exacerbate the problem of late galaxy formation in CDM+HDM models reported in earlier studies. If the current observational limits inferred from a small number of absorbers are robust, the amount of gas in DLA systems at high redshifts in the $\Omega_{\nu} = 0.2$ CDM+HDM model falls well below the observations.

TESTING COSMOLOGICAL MODELS AGAINST THE ABUNDANCE OF DAMPED LYMAN-ALPHA ABSORBERS
Jeffrey P. Gardner, Neal Katz, David H., Weinberg, Lars Hernquist
The Astrophysical Journal, in press

We calculate the number of damped Ly\(\alpha\) absorbers as a function of redshift expected in various popular cosmological models and compare our predictions with observed abundances. The Press-Schechter formalism is used to obtain the distribution of halos with circular velocity in different cosmologies, and we calibrate the relation between circular
velocity and absorption cross-section using detailed gas dynamical simulations of a "standard" cold dark matter (CDM) model. Because of this calibration, our approach makes more realistic assumptions about the absorption properties of collapsed objects than previous, analytic calculations of the damped Lyα abundance. CDM models with $\Omega_0 = 1$, $H_0 = 50 \text{ km s}^{-1}\text{Mpc}^{-1}$, baryon density $\Omega_b = 0.05$, and scale-invariant primeval fluctuations reproduce the observed incidence and redshift evolution of damped Lyα absorption to within observational uncertainty, for both COBE normalization ($\sigma_8 = 1.2$) and a lower normalization ($\sigma_8 = 0.7$) that better matches the observed cluster abundance at $z = 0$. A tilted ($n = 0.8$, $\sigma_8 = 0.7$) CDM model tends to underproduce absorption, especially at $z = 4$. With COBE normalization, a CDM model with $\Omega_0 = 0.4$, $\Omega_\Lambda = 0.6$ gives an acceptable fit to the observed absorption; an open CDM model is marginally acceptable if $\Omega_0 \geq 0.4$ and strongly inconsistent with the $z = 4$ data if $\Omega_0 = 0.3$. Mixed dark matter models tend to underproduce absorption, being roughly comparable to tilted CDM models if $\Omega_\nu = 0.2$ and failing drastically if $\Omega_\nu = 0.3$.

ADAPTIVE SMOOTHED PARTICLE HYDRODYNAMICS: METHODOLOGY II
J. Michael Owen, Jens V. Villumsen, Paul R. Shapiro, and Hugo Martel
The Astrophysical Journal Supplements, submitted

We describe an alternative formulation and present further tests of Adaptive Smoothed Particle Hydrodynamics (ASPH), the new version of Smoothed Particle Hydrodynamics (SPH) described in Shapiro et al. (1996; Paper I). ASPH generalizes the isotropic smoothing algorithm of SPH with a form of anisotropic smoothing, allowing resolution scales to adapt to locally anisotropic density evolution and thereby maximize the potential resolution of a simulation. This is accomplished by allowing each ASPH node to sample in locally ellipsoidal, rather than spherical, interpolation volumes. By deforming and rotating these ellipsoidal interpolation volumes so as to follow the anisotropy of volume changes local to each particle, ASPH strives to optimally adapt its local spatial resolution scale so that each particle samples an equal number of particles in all directions at all times. This can significantly improve the spatial resolving power of ASPH over SPH (for a fixed number of particles per simulation) in modeling physical systems which develop strong, inherent anisotropies.

In this paper we describe a tensor derivation of ASPH in a dimension free formalism, present the 2D and 3D cases, and describe a series of 2D and selected 3D test problems. The test cases include the problem of a cosmological pancake collapse, the Riemann shocktube, cylindrical and spherical Sedov blast waves, the collision of two strong shocks, and problems involving shearing disks intended to test the angular momentum conservation properties of the method. The goal of this work is to utilize ASPH in combination with standard N-body gravitational techniques in order to study large-scale cosmological structure formation, accounting for a collisionless (dark matter) species coupled gravitationally with a collisional (baryonic) species. ASPH's improved dynamic range in spatial resolution relative to SPH promises to be most useful for such studies, as anisotropic evolution is a generic feature of gravitational collapse scenarios and limited dynamical range remains one of the outstanding computational challenges faced by cosmological investigations.
We investigate the properties of hybrid gravitational/hydrodynamical simulations, examining both the numerics and the general physical properties of gravitationally driven, hierarchical collapse in a mixed baryonic/dark matter fluid. We demonstrate that, under certain restrictions, such simulations converge with increasing resolution to a consistent solution. The dark matter achieves convergence provided that the relevant scales dominating nonlinear collapse are resolved. If the gas has a minimum temperature (as expected, for example, when intergalactic gas is heated by photoionization due to the ultraviolet background) and the corresponding Jeans mass is resolved, then the baryons also converge. However, if there is no minimum baryonic collapse mass or if this scale is not resolved, then the baryon results err in a systematic fashion. In such a case, as resolution is increased the baryon distribution tends toward a higher density, more tightly bound state. We attribute this to the fact that under hierarchical structure formation on all scales there is always an earlier generation of smaller scale collapses, causing shocks which irreversibly alter the state of the baryon gas. In a simulation with finite resolution we therefore always miss such earlier generation collapses, unless a physical scale is introduced below which such structure formation is suppressed in the baryons. We also find that the baryon/dark matter ratio follows a characteristic pattern, such that collapsed structures possess a baryon enriched core (enriched by factors $\sim 2$ or more over the universal average) which is embedded within a dark matter halo, even without accounting for radiative cooling of the gas. The dark matter is unaffected by changing the baryon distribution (at least in the dark matter dominated case investigated here), allowing hydrodynamics to alter the distribution of visible material in the universe from that of the underlying mass.

COSMOLOGICAL SIMULATIONS WITH SCALE FREE INITIAL CONDITIONS
I: ADIABATIC HYDRODYNAMICS
J. Michael Owen, David H. Weinberg, August E. Evrard, Lars Hernquist, and Neal Katz
The Astrophysical Journal, submitted

We analyze hierarchical structure formation based on scale-free initial conditions in an Einstein-de Sitter universe, including a baryonic component with $\Omega_{\text{bary}} = 0.05$. We present three independent, smoothed particle hydrodynamics (SPH) simulations, performed at two resolutions ($32^3$ and $64^3$ dark matter and baryonic particles), and with two different SPH codes (TreeSPH and P3MSPH). Each simulation is based upon identical initial conditions, which consist of Gaussian distributed initial density fluctuations that have a power-spectrum $P(k) \propto k^{-1}$. The baryonic material is modeled as an ideal gas subject only to shock heating and adiabatic heating and cooling. Radiative cooling and photoionization heating are not included. The evolution is expected to be self-similar in time, and under certain restrictions we identify the expected scalings for many properties of the distribution of collapsed objects in all three realizations. The distributions of dark matter masses, baryon masses, and mass and emission weighted temperatures scale quite reliably.
However, the density estimates in the central regions of these structures are determined by the degree of numerical resolution. As a result, mean gas densities and Bremsstrahlung luminosities obey the expected scalings only when calculated within a limited dynamic range in density contrast. The temperatures and luminosities of the groups show tight correlations with the baryon masses, which we find can be well-represented by power-laws. The Press-Schechter (PS) approximation predicts the distribution of group dark matter and baryon masses fairly well, though it tends to overestimate the baryon masses. Combining the PS mass distribution with the measured relations for $T(M)$ and $L(M)$ predicts the temperature and luminosity distributions fairly accurately, though there are some discrepancies at high temperatures/luminosities. In general the three simulations agree well for the properties of resolved groups, where a group is considered resolved if it contains more than 32 particles.

THE OPTICAL REDSHIFT SURVEY II: DERIVATION OF THE LUMINOSITY AND DIAMETER FUNCTIONS AND OF THE DENSITY FIELD
Basilio X. Santiago, Michael A. Strauss, Ofer Lahav, Marc Davis, Alan Dressler, and John P. Huchra
The Astrophysical Journal, 461, 38

We quantify the effects of Galactic extinction on the derived luminosity and diameter functions and on the density field of a redshift sample. Galaxy magnitudes are more affected by extinction than are diameters, although the effect on the latter is more variable from galaxy to galaxy, making it more difficult to quantify. We develop a maximum-likelihood approach to correct the luminosity function, the diameter function and the density field for extinction effects. The effects of random and systematic photometric errors are also investigated. The derived density field is robust to both random and systematic magnitude errors as long as these are uncorrelated with position on the sky, since biases in the derived selection function and number counts tend to cancel one another. Extinction-corrected luminosity and diameter functions are derived for several subsamples of the Optical Redshift Survey (ORS). Extinction corrections for the diameter-limited subsamples are found to be unreliable, possibly due to the superposition of random and systematic errors. The ORS subsamples are combined using overall density scaling factors from a full-sky redshift survey of IRAS galaxies, allowing the reconstruction of the optical galaxy density field over most of the sky to 8000 km/s.

AN HI SURVEY OF THE BOOTES VOID II: THE ANALYSIS
Arpad Szomoru, Jacqueline H. van Gorkom, Michael Gregg, and Michael Strauss
The Astronomical Journal, 111, 2150

We discuss the results of a VLA HI survey of the Bootes void and compare the distribution and HI properties of the void galaxies to those of galaxies found in a survey of regions of mean cosmic density. The Bootes survey covers 1100 Mpc$^3$, or \~{}1% of the volume of the void, and consists of 24 cubes of typically 2 Mpc x 2 Mpc x 1280 km/s, centered on optically known galaxies. 16 targets were detected in HI; 18 previously uncataloged objects were discovered directly in HI. The control sample consist of 12 cubes centered on IRAS selected galaxies with FIR luminosities similar to those of the Bootes
targets and located in regions of 1 to 2 times the cosmic mean density. In addition to the 12 targets, 29 companions were detected in HI. We find that the number of galaxies within 1 Mpc of the targets is the same to within a factor of two for void and control samples, and thus that the small scale clustering of galaxies is the same in regions that differ by a factor of ~ 6 in density on larger scales. A dynamical analysis of the galaxies in the void suggests that on scales of a few Mpc the galaxies are gravitationally bound, forming interacting galaxy pairs, loose pairs and loose groups. One group is compact enough to quality as a Hickson compact group. The galaxies found in the void are mostly late-type, gas rich systems. A careful scrutiny of their HI and optical properties shows them to be very similar to field galaxies of the same morphological type. This, combined with our finding that the small scale clustering of the galaxies in the void is the same as in the field, suggests that it is the near environment that mostly affects the evolution of the galaxies.

A NEW STATISTIC FOR REDSHIFT SURVEYS: THE REDSHIFT DISPERSION OF GALAXIES
Jeremy V. Kepner, F J Summers, and Michael A. Strauss
New Astronomy, in press

We present theoretical and N-body results on the correlation between velocity dispersion and local density measured in a redshift survey, as a means of constraining the cosmological density parameter, \( \Omega \). The Cosmic Virial Theorem relates the average pairwise velocity dispersion of a system of particles to \( \Omega \) and the two point correlation function. We show that the Cosmic Virial Theorem is valid for suitable particle subsets, including sets corresponding to surfaces of constant density. In particular, the velocity dispersion of particles on such a surface is proportional to \( \Omega \) and the density of the surface. We have calculated the velocity dispersion as a function of density for N-body simulations of several cosmological models. We find that the proportionality between velocity dispersion, density and \( \Omega \) holds over redshift scales in the range 50 to 500 km/s. This relationship is independent of the shape and the amplitude of the initial power spectrum for the range of models we address. We suggest an application of these results to redshift surveys by calculating the redshift dispersion of galaxies as a function of density on the sky. By not averaging over all densities, we can take advantage of the \( \Omega \) dependence of the velocity dispersion in a way that is insensitive to the effects of a small number of high-density regions. Currently available redshift surveys are too sparse to apply this method, but a volume-limited sub-sample of the Sloan Digital Sky Survey should be adequate to discriminate cleanly between low and high values of \( \Omega \).

TESTING THE HUBBLE LAW WITH THE IRAS LUMINOSITY FUNCTION
Daniel M. Koranyi and Michael A. Strauss
The Astrophysical Journal, 477, 36

We test and reject the claim of Segal et al (1993) that the correlation of redshifts and flux densities in a complete sample of IRAS galaxies favors a quadratic redshift-distance relation over the linear Hubble law. This is done, in effect, by treating the entire galaxy luminosity function (LF) as derived from the 60-\( \mu \)m 1.2 Jy IRAS redshift survey of Fisher et al (1995) as a distance indicator; equivalently, we compare the flux density
distribution of galaxies as a function of redshift with predictions under different redshift-distance cosmologies. This method does not assume a uniform distribution of galaxies. We find that this test has rather weak discriminatory power, and the differences between models are not as stark as one might expect a priori. Even so, we find that the Hubble law is indeed more strongly supported by the analysis than is the quadratic redshift-distance relation. Moreover, the galaxy density field derived assuming the Hubble law is close to homogeneous when averaged over solid angle; it is not in the case of the quadratic redshift-distance relation. We identify several methodological pitfalls which may lead to misleading results.

MEASURING HIGH-ORDER MOMENTS OF THE GALAXY DISTRIBUTION FROM COUNTS IN CELLS - THE EDGEWORTH EXPANSION
Rita S. Kim and Michael A. Strauss
The Astrophysical Journal, submitted

To probe the weakly nonlinear regime, past the point where simple linear theory is sufficient to describe the statistics of the density distribution, we measure the skewness ($S_3$) and kurtosis ($S_4$) of the Count Probability Distribution Function (CPDF) of the IRAS 1.2 Jy sample obtained from counts in cells. These quantities are free parameters in a maximum likelihood fit of an Edgeworth expansion convolved with a Poissonian to the observed CPDF. This method is appreciably less sensitive to the tail of the distribution than are measurements of $S_3$ and $S_4$ from moments of the CPDF. Moreover, unlike that method, determination of the errors of $S_3$ and $S_4$ is straightforward. We measure $S_3$ and $S_4$ to $l \sim 50 h^{-1}$Mpc; the data are consistent with scale invariance, yielding averages of $\langle S_3 \rangle = 2.83 \pm 0.06$, and $\langle S_4 \rangle = 6.89 \pm 0.48$. These values are higher than those found by Bouchet et al. (1993) using the moments method on the same data set, $\langle S_3 \rangle = 1.5 \pm 0.5$ and $\langle S_4 \rangle = 4.4 \pm 3.7$. Unlike the moments method, our results are quite robust to the fact that IRAS galaxies are under-represented in cluster cores. We use N-body simulations to show that our method yields unbiased results, while the moments method systematically underestimates $S_3$ and $S_4$.

CONSTRAINTS ON $\Omega$ FROM THE IRAS REDSHIFT SURVEYS
Shaun Cole, Karl B. Fisher, and David H. Weinberg
Monthly Notices of the Royal Astronomical Society, 275, 515

We measure the anisotropy of the redshift-space power spectrum in the 1.2-Jy and QDOT redshift surveys of IRAS-selected galaxies. On large scales, this anisotropy is caused by coherent peculiar motions, and gravitational instability theory predicts a distortion of the power spectrum that depends only on the ratio $\beta \equiv f(\Omega)/b \approx \Omega^{0.6}/b$, where $\Omega$ is the cosmological density parameter and $b$ is the bias parameter. On small scales, the distortion is dominated by the random velocity dispersion in non-linear structures. We fit the observed anisotropy with an analytic model that incorporates two parameters, $\beta$, and a small-scale velocity dispersion $\sigma_v$. Tests on N-body simulations show that this model recovers $\beta$ quite accurately on the scales accessible to the existing IRAS redshift surveys. Applying our procedure to the 1.2-Jy and QDOT surveys, we find $\beta = 0.52 \pm 0.13$ and
\[ \beta = 0.54 \pm 0.3, \text{ respectively. These results imply } \Omega \approx 0.35 \text{ if galaxies trace mass, or a bias factor of about 2 if } \Omega = 1. \]

LARGE-SCALE STRUCTURE IN COBE-NORMALIZED COLD DARK MATTER COSMOGONIES
Shaun Cole, David H. Weinberg, Carlos S. Frenk, and Bharat Ratra
Monthly Notices of the Royal Astronomical Society, in press

We study the clustering of the mass distribution in cold dark matter models using large cosmological N-body simulations. We investigate spatially-flat models with a cosmological constant and scale-invariant \((n = 1)\) primordial power spectra, as well as open-bubble inflation models. All the models we consider are normalized according to the fluctuation amplitude measured in the COBE-DMR microwave background anisotropy data. With an age of the universe \(t_0 \approx 14 \text{ Gyr (12 Gyr)}\) for the flat (open) models, a baryon mass density parameter \(\Omega_b = 0.0125h^{-2}\), and a reasonable assessment of the systematic uncertainties in the cumulative cluster mass function, the observed abundance of rich galaxy clusters leads to tight constraints on the mass density parameter \(\Omega_0\). The allowable ranges are \(0.4 \leq \Omega_0 \leq 0.5\) for open models and \(0.25 \leq \Omega_0 \leq 0.4\) for flat models. The upper limits on \(\Omega_0\) can be relaxed if one lowers the Hubble parameter and increases the age of the universe, but \(h \leq 0.25\) is required for \(\Omega_0 = 1\) to be allowed. The constraints also change if one allows tilted primordial power spectra. An \(\Omega_0 = 1\) cold dark matter model with \(h = 0.5\) can be constructed to satisfy both the cluster and DMR constraints, but it requires a tilted primordial power spectrum, with \(n \approx 0.8\) and a corresponding contribution to the DMR signal from gravitational waves that reduces the implied \(\sigma_8\) by a further 27%. We compare the evolved mass correlation functions and power spectra of the most promising of our N-body models with those of galaxies in the APM survey. The flat models have steep correlation functions at small scales and require the galaxy distribution to be antibiased on scales \(r \leq 8h^{-1}\text{Mpc}\). The open models require little or no antibias on small scales and a positive bias on large scales; these biases are small for \(\Omega \approx 0.4\), implying that, in this case, galaxies approximately trace the mass over a wide range of scales. The lack of a positive bias on small scales in almost all of these N-body models is difficult to reconcile with the mean mass-to-light ratio of cluster galaxies which, if \(\Omega_0 \geq 0.2\), implies that galaxies are overabundant in clusters relative to the field. The tilted \(\Omega_0 = 1\) model, on the other hand, does require that galaxies be positively biased on small scales, and that the bias to become stronger on larger scales. We also compute the topology of isodensity contours in these models, obtaining theoretical predictions that are less sensitive to the details of galaxy formation.