New Parameterization of Neutron Absorption Cross Sections

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Abstract

Recent parameterization of absorption cross sections for any system of charged ion collisions, including proton-nucleus collisions, is extended for neutron-nucleus collisions valid from ~1 MeV to a few GeV, thus providing a comprehensive picture of absorption cross sections for any system of collision pairs (charged or uncharged). The parameters are associated with the physics of the problem. At lower energies, optical potential at the surface is important, and the Pauli operator plays an increasingly important role at intermediate energies. The agreement between the calculated and experimental data is better than earlier published results.

Introduction

Transportation of neutrons in matter is of direct interest in several technologically important and scientific areas (refs. 1 through 3), including space radiation, cosmic ray propagation studies in the galactic medium, nuclear power plants, and radiobiological effects that impact industrial and public health. For the proper assessment of radiation exposures, both reliable transport codes and accurate input data are needed.

An important ingredient of the input data is the total absorption (reaction) cross section (\(\sigma_R\)), defined as the total (\(\sigma_T\)) minus the elastic (\(\sigma_{el}\)) cross sections for two colliding ions:

\[
\sigma_R = \sigma_T - \sigma_{el}
\]

(1)

Recently, a simple, accurate formalism (refs. 4 and 5) for the total absorption cross sections for any system of charged colliding ions, including protons, was developed. The present work extends the formalism to neutron-nucleus collisions, thus presenting a simple accurate formalism for the absorption cross sections for any combination of colliding ions (charged or uncharged) that is valid for the entire energy range. The high-energy features of the earlier version (LaRC prior) of the model (ref. 6) developed here at Langley Research Center (LaRC) have been retained where it is accurate. Thus, the present model (LaRC) is an updated version of the earlier model (LaRC prior) that improves on its behavior at lower energies within the universal formalism recently developed (refs. 4 and 5). The other commonly used model (Letaw, Silberberg, and Tsao, ref. 7) gives good results for higher energies but shows increasingly larger deviations from the experiment for lower energies.

Model Description

Most of the empirical models approximate a total reaction cross section for charged ion collisions of the Bradt-Peters form (ref. 8) but have very different form for the neutron-nucleus collisions. We have maintained a uniform, consistent picture for all the systems: proton-nucleus, nucleus-nucleus, and now for the neutron-nucleus collisions and have written total reaction cross sections for neutron-nucleus collisions also of the Bradt-Peters form:

\[
\sigma_R = \pi r_0^2 \left( A_P^{1/3} + A_T^{1/3} - \delta \right)^2
\]

(2)

where \(r_0\) is the energy-independent parameter and \(\delta\) is either the energy-independent parameter or the energy-dependent parameter, and where \(A_P\) and \(A_T\) are the projectile and target mass numbers, respectively.

It is well-known that this type of parameterization is suited for higher energies and is modified through Coulomb interaction at lower energies (refs. 4 and 5) for the charged ion collisions. For the neutron-nucleus collisions there is no Coulomb interaction, but the total reaction cross section for these collisions is modified by the strength of the imaginary part of the optical potential at the surface. Because we are using this form of parameterization for the neutron-nucleus case for the first time (which helps to provide a unified, consistent, and accurate picture for the total reaction cross sections for any system of colliding nuclei for the entire energy range), we introduce a low-energy multiplier \(X_m\) that accounts for the strength of the optical model interaction. In addition, the effects of the transparency and of Pauli blocking are taken into account through the energy-dependent parameter \(\delta_E\), which is defined later in the text. We therefore propose the following similar form for the reaction cross sections for the neutron-nucleus:

\[
\sigma_R = \pi r_0^2 \left( A_P^{1/3} + A_T^{1/3} + \delta_E \right)^2 X_m
\]

(3)

where \(r_0 = 1.1\) fm. For \(A_T < 200\) the low energy multiplier \(X_m\) is given by

\[
X_m = 1 - X_1 e^{-[E/(X_1 S_L)]}
\]

(4)
\[ S_L = 0.6 \ \text{for} \ \left(A_T < 12\right) \]
\[ = 1.6 \ \text{for} \ \left(A_T = 12\right) \]
\[ = 1 \ \text{for} \ \left(A_T > 12\right) \]

with

\[ X_1 = 2.83 - 3.1 \times 10^{-2} A_T + 1.7 \times 10^{-4} A_T^2 \] (5)

Here \( E \) is the projectile energy (in MeV). For low energy, shell effects play an important role in light nuclei, and the optical potential at the surface varies differently with these effects, showing variation in \( S_L \).

For \( A_T \geq 200 \)

\[ X_m = \left( 1 - 0.3 e^{-\left(\frac{E}{115}\right)} \right) \left( 1 - e^{0.9 - E} \right) \] (6)

The effects of the energy dependence at intermediate and higher energies because of Pauli blocking and transparency are taken into account in \( \delta_E \), which is defined as

\[ \delta_E = 1.85 S + \frac{0.16 S}{E_{CM}^{1/3}} - C_E \]
\[ + \frac{0.91 (A_T - 2Z_T) Z_p}{A_T A_p} \] (7)

where \( E_{CM} \) is the center of mass energy, and \( Z_T \) and \( Z_p \) are the charge numbers of the target and the projectile ions. The mass asymmetry term \( S \) is given by

\[ S = \frac{A_p^{1/3} A_T^{1/3}}{A_p^{1/3} + A_T^{1/3}} \] (8)

and is related to the volume overlap of the collision system. For the results reported here \( A_p = 1 \). The last term on the right-hand side of equation (7) accounts for the isotope dependence of the reaction cross section. The term \( C_E \) is related to the transparency and to Pauli blocking and is given by

\[ C_E = D \left[ 1 - e^{-\left(\frac{E}{T_1}\right)} \right] -0.292 e^{-E/792} \]
\[ \times \cos \left( 0.229 E^{0.453} \right) \] (9)

where \( T_1 = 40 \) (except for \( 11 \leq A_T \leq 40 \), a value of \( T_1 = 30 \) is recommended for a better fit). Here \( D \) is related to the density dependence of the colliding system, and the global value is given by

\[ D_g = 2 \frac{\rho_n + \rho_A}{\rho_p + \rho_A} \] (10)

The numerator in equation (10) = 0.27 fm\(^{-3}\). The density of a nucleus is calculated in the hard sphere model and for a nucleus, \( A_i \) is given by

\[ \rho_{A_i} = \frac{A_i}{(4\pi/3)r_i^3} \] (11)

where \( r_i \) is the equivalent sphere radius and is related to the \( r_{rms,i} \) radius by

\[ r_i = 1.29 r_{rms,i} \] (12)

where \( r_{rms,i} \) values are taken from the experiment (ref. 9). The physics associated with the constant \( D \) is interesting. The constant \( D \) in effect simulates the modifications of the reaction cross sections because of Pauli blocking. This effect is new, was introduced in our charged-particles reaction cross-section work (refs. 4 and 5), and is being used for the first time in neutron-nucleus work. The introduction of constant \( D \) is an important physical ingredient of the model and helps present a unified picture of the reaction cross sections for any system of colliding particles.

It is well-known that the stability of the nucleus plays a significant role in the distribution of neutrons and protons in a nucleus and in turn, modifies the \( D_g \) value for different mass ranges.

For \( A_T \leq 40 \)

\[ D = D_g - 1.5 \frac{A_T - 2Z_T}{A_T} + \frac{0.25}{1 + e^{(E-170)/100}} \] (13)

For \( 40 < A_T < 60 \)

\[ D = D_g - 1.5 \frac{A_T - 2Z_T}{A_T} \] (14)

And for \( Z_T > 82 \)

\[ D = D_g - \frac{Z_T}{A_T - Z_T} \] (15)

The second term in equation (13) takes into account the isotopic differences in light nuclei. There is a significant dip in the cross sections for light nuclei \( \sim 170 \) A MeV, and the last term in eq. (13) accounts for this effect. This dip gradually fills in for heavier nuclei. On the other hand, for heavier targets neutron excess (or proton
deficiency) is an important effect, and the second term on the right-hand side of equation (15) has been introduced to take this neutron excess (or proton deficiency) into account.

Results

Figures 1 through 12 show the plot of the total absorption (reaction) cross sections for the neutron-nucleus collisions. The data for higher energies (379 to 1731 MeV) have been taken from reference 10, and the data for lower energies have been taken from the compilations of ENDF/B-VI and references 11 and 12. The agreement with the experiment is excellent for all nuclei for the entire energy range. We have also compared present results (figs. 2, 3, 6, 8, and 10) with the two popular models: the earlier Langley Research Center version (LaRC prior), developed by Wilson et al. (ref. 6) and that of Letaw, Silberberg, and Tsao (ref. 7) at the Naval Research Laboratory (NRL). We see from the plots that both these models agree very well with experiments at higher energies; however, there are departures at lower energies. In general, the LaRC prior model gives better results than the NRL model (Letaw, Silberberg, and Tsao, ref. 7) does at lower energies.

Concluding Remarks

Clearly, the present Langley model (LaRC) improves on earlier models and gives accurate results to much lower energies. This model is a welcome improvement for the transport model database. Neutrons are the dominant component of the radiation environment for the High-Speed Civil Transport (HSCT) mission in which the proposed database is immensely important.

We have successfully presented a simple, accurate, and unified model for the total absorption cross sections for any system of charged or uncharged collision systems that is valid for the entire energy range. The absorption cross sections are important ingredients for several databases, including transport. The model will also be useful where analytical forms of the cross sections are needed.

References


Figure 1. Total absorption cross sections for $n + {}^9_4\text{Be}$ as a function of neutron energy.

Figure 2. Total absorption cross sections for $n + {}^{12}_6\text{C}$ as a function of neutron energy.
Figure 3. Total absorption cross sections for $n + {^{27}_{13}}\text{Al}$ as a function of neutron energy.

Figure 4. Total absorption cross sections for $n + {^{56}_{26}}\text{Fe}$ as a function of neutron energy.
Figure 5. Total absorption cross sections for $n + ^{28}\text{Ni}$ as a function of neutron energy.

Figure 6. Total absorption cross sections for $n + ^{63}\text{Cu}$ as a function of neutron energy.
Figure 7. Total absorption cross sections for $n + ^{65}_{30}\text{Zn}$ as a function of neutron energy.

Figure 8. Total absorption cross sections for $n + ^{107}_{47}\text{Ag}$ as a function of neutron energy.
Figure 9. Total absorption cross sections for $n + ^{119}_{50}\text{Sn}$ as a function of neutron energy.

Figure 10. Total absorption cross sections for $n + ^{208}_{82}\text{Pb}$ as a function of neutron energy.
Figure 11. Total absorption cross sections for $n + ^{209}_{83}{\text{Bi}}$ as a function of neutron energy.

Figure 12. Total absorption cross sections for $n + ^{238}_{92}{\text{U}}$ as a function of neutron energy.
Recent parameterization of absorption cross sections for any system of charged ion collisions, including proton-nucleus collisions, is extended for neutron-nucleus collisions valid from ~1 MeV to a few GeV, thus providing a comprehensive picture of absorption cross sections for any system of collision pairs (charged or uncharged). The parameters are associated with the physics of the problem. At lower energies, optical potential at the surface is important, and the Pauli operator plays an increasingly important role at intermediate energies. The agreement between the calculated and experimental data is better than earlier published results.