Cyclic fatigue of brittle materials with an indentation-induced flaw system

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Abstract

The ratio of static to cyclic fatigue life, or 'h ratio', was obtained numerically for an indentation flaw system subjected to sinusoidal loading conditions. Emphasis was placed on developing a simple, quick lifetime prediction tool. The solution for the \( h \) ratio was compared with experimental static and cyclic fatigue data obtained from as-indented 96 wt.% alumina specimens tested in room-temperature distilled water.

Keywords: Cyclic fatigue; Static fatigue; Indentation-induced flaw; Ceramics

1. Introduction

For glass and ceramic materials which have slow crack growth (stress corrosion) as the unique, time-dependent failure mechanism, it is possible to predict the fatigue life under one loading condition from another. Prediction of cyclic fatigue lifetime from static lifetime for Griffith flaw system can be done analytically by using the ratio of static fatigue to cyclic fatigue life, introduced by Evans and Fuller [1]. In the case of the indentation flaw system, the solution requires a numerical approach since an additional driving force appears in the net stress intensity factor. This term is attributed to residual contact stress produced by the elastic/plastic deformation of indentation [2]. Analyses of dynamic and static fatigue of the indentation flaw system were carried out previously for both postthreshold [3,4] and subthreshold [5] indentation flaws. However, no general solution for the cyclic fatigue of such a flaw system has been found, although some cyclic fatigue data on ceramic specimens containing indentation cracks exist [6–9].

The purpose of this study is to analyze cyclic fatigue of the indentation flaw system under sinusoidal loading conditions so that lifetime prediction from one fatigue condition to another is readily feasible. For this purpose, the complete solution of the ratio of static to cyclic fatigue life was obtained numerically in conjunction with fatigue parameter (n) and stress ratio (R ratio). The solution was compared with experimental data that were obtained from static and cyclic fatigue testing of indented alumina specimens in room-temperature distilled water. The analysis was carried out for material exhibiting a flat R curve and a power-law crack propagation as a delayed failure mechanism.

2. Analysis

In many cases slow crack growth of glass and ceramic materials under Mode I loading conditions is described by the following empirical, power-law relation [4]

\[
\frac{da}{dt} = A \left( \frac{K_1}{K_{IC}} \right)^n
\]

where \( r, a, \) and \( t \) are crack velocity, crack size, and time, respectively. \( A \) and \( n \) are the fatigue parameters which depend on material and environment. \( K_1 \) is the...
Mode I stress intensity factor, and $K_{IC}$ is the Mode I critical stress intensity factor or fracture toughness of the material with a flat $R$ curve.

2.1. Natural flaws

In static fatigue testing a constant applied stress ($\sigma$) is employed. Noting that $n \geq 10$ for most glass and ceramics, one can derive the following static fatigue equation [10]

$$t_{fs} = B S_i^{n-2} \sigma_{max}^{-n}$$

(2)

where $t_{fs}$ is the time to failure, $S_i$ is the inert strength, and $B$ is the parameter associated with fracture toughness, crack geometry and fatigue parameters.

In cyclic fatigue testing a time-varying, periodic stress ($\sigma(t)$) is applied. The time to failure ($t_{fc}$) in cyclic fatigue can be expressed as [1]

$$t_{fc} = B S_i^{n-2} \sigma_{max}^{-n} \frac{1}{\tau} \int_{0}^{\tau} [f(t)]^n \, dt$$

(3)

where $\sigma_{max}$ is the maximum applied stress, $f(t)$ is a periodic function defined as $\sigma(t) = \sigma_{max} f(t)$ with a range of $0 \leq f(t) \leq 1$, and $\tau$ is the period. The ratio of static fatigue to cyclic fatigue life, $h$, with a condition of $\sigma$ in static loading equal to $\sigma_{max}$ in cyclic loading ($\sigma = \sigma_{max}$) can be obtained from Eqs. (2) and (3)

$$h = \frac{t_{fs}}{t_{fc}} = \frac{1}{\tau} \int_{0}^{\tau} [f(t)]^n \, dt$$

(4)

Sinusoidal loading is the most common and popular wave form used in cyclic fatigue testing. The periodic function $f(t)$ for a sinusoidal wave is expressed as $f(t) = [(1 + R)/2 + [(1 - R)/2] \sin \omega t]$, where $R$ is the stress (or load) ratio, defined as $R = \sigma_{max}/\sigma_{min}$ with $\sigma_{min}$ being the minimum applied stress. $\omega$ is the angular velocity. Eq. (4) can be solved either analytically or numerically for the sinusoidal wave. The $h$ ratio for any other periodic loading wave form such as trapezoidal, triangular or square can be solved analytically with a straightforward procedure [1,11]. Eq. (4) was solved analytically for a sinusoidal wave by using the series expansion technique of Evans and Fuller [1]. Their analytical solution is expressed as

$$h = \sum_{k=0}^{n} \left[ \frac{n!}{(n-2k)! (k!)^2} \right] \left[ 1 - \frac{R}{2(1 + R)} \right]^{2k} \left( \frac{1 + R}{2} \right)^n$$

(5)

Eq. (5) is still complicated to solve and valid only for integer values of $n$. Therefore, it is desirable to solve Eq. (4) numerically to cover any integer or real value of $n$ for a full range of $R = 0$ to $1.0$, even though some limited data on the $h$ ratio exist [1,12].

Fig. 1 shows the results of the numerical solution of Eq. (4) as a function of $n$ for $R$ ratios from $R = 0$ to $R = 1.0$. If $\log h$ is treated as a linear function of $\log n$, coefficients of correlation of $r_{corr}$ $= 0.9966$ result. Thus it is possible to obtain an approximate (but accurate) relation between $\log h$ and $\log n$ for a given $R$ ratio by using the regression analysis

$$\log h = ax \log n + \beta$$

(6)

where $a$ and $\beta$ are the regression coefficients.

Table 1 shows the regression coefficients $a$ and $\beta$. For $n \geq 10$, the maximum error in $h$ ratio, as compared with the exact solution, was 1.0% for $R = 0$ to 0.7, and 4.5% for $R = 0.8$ to 0.9. Therefore, the table gives a convenient and accurate means of determining the $h$ ratio for a natural flaw system for any given value of $n$, either integer or real. The complication of a lack of a simple analytical solution to Eq. (5) for real numbers can be eliminated with Eq. (6) and Table 1.

2.2. Indentation flaws

Because of the residual contact stress produced by elastic/plastic indentation deformation [2], an addi-
A typical example of the normalized time to failure as a function of normalized maximum applied stress for the indentation flaw system.

Note that the normalized net stress intensity factor consists of two terms: the first is a function of the remote applied stress and the second is related to the residual contact stress [2,3]. The differential form of Eq. (6) was solved numerically using a fourth-order Runge–Kutta method. The initial condition was $C^* = 0.3967$ at $J = 0$ and the instability conditions were $K^* = 1$ and $dK^*/dC^* > 0$. The solution was initiated to determine the normalized time to failure ($J_f$) as a function of normalized maximum applied stress for the selected values of $n = 5–160$. This procedure was continued for the range of stress ratios from $R = 0.1$ to $1.0$. Since the solution is independent of frequency ($f$) as long as $\omega a_m/A \geq 2\pi f$, any particular values of $\omega a_m/A$ satisfying $\omega a_m/A \geq 2\pi$ can be chosen. A value of $\omega a_m/A = 100$ was used in this analysis.

Fig. 2 shows a typical example of the normalized time to failure as a function of normalized maximum applied stress for stress ratios of $R = 1.0$, 0.5 and 0.1. Fatigue susceptibility increases with increasing $R$ ratio, yielding a maximum at $R = 1.0$ (static fatigue). Note that regardless of $R$ ratio the curves converge to $\sigma^* = 1.0$, in which the inert strength with no slow crack growth is defined. Similar to the previous static fatigue analysis [4], the slope in Fig. 2 is not representative of a ‘true’ fatigue parameter of $n$, due to the effect of residual contact stress. The slope in Fig. 2, which is called ‘apparent’ fatigue parameter ($n'$), was found to have the following approximate relation

$$n' = \frac{4}{3}n - 2/3$$

which reduces to the relationship in static fatigue of indentation cracks [4]. This indicates that the relationship between the true ($n$) and apparent ($n'$) fatigue parameters is independent of $R$ ratio.

Based on the results as shown in Fig. 2, the $h$ ratio was determined for a given $R$ using the relation $h = J_{f1}/J_{fc} (= t_{f1}/t_{fc})$, where $J_{f1}$ and $J_{fc}$ are the normalized time to failure corresponding to static and cyclic fatigue, respectively. Fig. 3 shows a summary of the $h$ ratio as a function of $n$ for $R$ ratios from $R = 0.1$ to $R = 1.0$, where $\log h$ was plotted against $\log n$ as for the natural flaw system (Fig. 1). Again the excellent coefficients of correlation of $r_{s0} \geq 0.9911$ allow one to obtain a relationship based on Eq. (6). The results of such regression analysis for $x$ and $\beta$ are shown in Table 1. The maxi-
mum error associated in the $h$ ratio was observed to be about 4%, occurring at $R = 0.9$ for $n \geq 10$. Hence, lifetime prediction of an indentation flaw system with one fatigue loading condition can be done quickly and accurately from another by using Eq. (6) in conjunction with Table 1.

2.3. Evaluation of fatigue parameters for indentation flaws

The "true" fatigue parameter $n$ for both static and cyclic fatigue can be determined using Eq. (9) once the "apparent" fatigue parameter $n'$ is determined from the slope of fatigue data. The fatigue parameter $A$ in static fatigue was determined previously [4]

$$A = \left( \frac{2\pi}{n} \right)^{1.2} \sigma_m^{0.3} \frac{1}{\lambda_c'}$$

(10)

where $\lambda_c'$ is the intercept of static fatigue data, expressed as $t_o \sigma_c' = \lambda_c'$. Since the following relation for the indentation flaw system holds

$$J_{tc} = \frac{t_o}{t_c} \frac{\lambda_c \sigma_c}{\lambda_c' \sigma_{c_{\text{max}}}} = \frac{h}{\lambda_c}$$

(11)

where $\lambda_c$ is the intercept of cyclic fatigue data as $t_o \sigma_{c_{\text{max}}} = \lambda_c$. Hence, from Eqs. (10) and (11), the parameter $A$ in cyclic fatigue is obtained

$$A = \left( \frac{2\pi}{n} \right)^{1.2} \sigma_m^{0.3} \frac{1}{\lambda_c h}$$

(12)

3. Experimental procedure

Static and cyclic fatigue tests of indented alumina (96 wt.% ALSIMAG 614, General Electrical Ceramics) flexure specimens were carried out in room-temperature distilled water using a four-point bend fixture of 6.05 mm inner and 19.05 mm outer spans. The nominal dimensions of the test specimens were 4 mm by 5 mm by 25 mm, respectively, in height, width, and length. The center (5 mm side) of each specimen was indented in air for about 20 s using a Vickers microhardness indenter (Zwick, model 3212, Germany) with an indentation load of 49 N. Static fatigue testing was conducted by loading indented specimens in a lever-arm creep machine (ATS) with constant stress levels of 95 to 133 MPa. Cyclic fatigue testing of indented specimens was conducted by sinusoidal loading with a stress ratio of $R = 0.5$ and a frequency of $f = 5$ Hz with a servohydraulic testing machine (Instron, Model 8562). The maximum applied stress in cyclic fatigue ranged from 100 to 130 MPa. The sinusoidal wave shape was frequently checked and verified by a digital storage oscilloscope.

4. Results and discussion

The results of the static and cyclic fatigue testing of the indented 96 wt.% alumina specimens in room-temperature distilled water are depicted in Fig. 4. Consistent with the results shown in Fig. 3, the alumina is slightly more susceptible to fatigue in static loading than in cyclic loading within the experimental range used. The "apparent" fatigue parameter was obtained from the data as $n' = 32.92 \pm 6.01$ and $35.37 \pm 4.79$, respectively, for static and cyclic fatigue. The corresponding "true" fatigue parameter was obtained from Eq. (9) as $n = 43.23 \pm 8.01$ and $46.49 \pm 6.39$, respectively, for static and cyclic fatigue. The prediction of cyclic fatigue from static fatigue data can be done using Eq. (6) together with Table 1. The prediction thus made is presented in Fig. 5, where the cyclic fatigue data obtained from the experiments are included for comparison. The prediction somewhat overestimates (less

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Fig. 4. Results of static and cyclic fatigue testing obtained from indented 96 wt.% alumina flexure specimens tested in room-temperature distilled water. The solid lines represent the best-fit lines in a plot of log $t$ vs. log $\sigma_{c_{\text{max}}}$.

Fig. 5. Prediction of cyclic fatigue lifetime from static fatigue data for indented 96 wt.% alumina specimens tested in room-temperature distilled water.
than one order of magnitude) the actual cyclic time to failure.

It has been reported that, for certain ceramic materials, damage accumulation and/or fatigue synergisms can be active in cyclic fatigue, resulting in more fatigue susceptibility in cyclic than in static loading [8,13–16]. If such synergies exist or if creep at elevated temperatures is simultaneous with cyclic fatigue, the analysis given in this paper may not be valid. However, the analysis may still provide, by comparing with experimental data and by using fractographic analysis, some clues with which the prevailing mechanism associated with failure can be pinpointed. It should be noted that most of glass and ceramic materials under static fatigue loading conditions in a room-temperature moisture environment are subjected to one failure mechanism, stress corrosion. Therefore, in view of the reasonable agreement between the static and cyclic fatigue data shown here, it can be stated that stress corrosion is a governing failure mechanism, either in static or in cyclic loading conditions. This is supported by the fact that the material contains a large amount of glassy phase, which is highly susceptible to stress corrosion in an aqueous environment. The fatigue parameters for this material system, therefore, can be obtained by using either static or cyclic fatigue testing. In terms of testing economy, however, static fatigue is preferred because of the much higher testing costs associated with cyclic fatigue testing.

As mentioned before, the analytical solution of the h ratio for other loading functions such as trapezoidal, triangular and square wave forms can be easily obtained for the natural flaw system because of their mathematical simplicity [1,11]. However, the conventional numerical solution for these loading functions may not be simple for the indentation flaw system, since the functions, unlike the sinusoidal wave, are not continuous but discontinuous in nature. In this case, each discontinuous periodic function should be converted into a continuous periodic function so that numerical solution is feasible. This may be done by using the Fourier analysis from which a discontinuous function can be approximated into a continuous function.

4. Conclusions

The ratio of static to cyclic fatigue life, or h ratio, was obtained numerically with an emphasis on the indentation flaw system subjected to sinusoidal loading conditions. The h ratio decreases with increasing n and decreasing R ratio. The solution provides a simple and quick methodology to predict the lifetime of one fatigue condition from another for indentation flaw systems (as well as the natural flaw system) when the governing failure mechanism is stress corrosion. The solution was compared with static and cyclic fatigue data obtained from indented 96 wt.% alumina specimens tested in room-temperature distilled water.

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