A. Palmgren Revisited—A Basis for Bearing Life Prediction

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A. PALMGREN REVISITED—A BASIS FOR BEARING LIFE PREDICTION

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SUMMARY

Bearing technology, as well as the bearing industry, began to develop with the invention of the bicycle in the 1850's. At the same time, high-quality steel was made possible by the Bessemer process. In 1881, H. Hertz published his contact stress analysis. By 1902, R. Stribeck had published his work based on Hertz theory to calculate the maximum load of a radially loaded ball bearing. By 1920, all of the rolling bearing types used today were being manufactured. AISI 52100 bearing steel became the material of choice for these bearings. Beginning in 1918, engineers directed their attention to predicting the lives of these bearings. In 1924, A. Palmgren published a paper outlining his approach to bearing life prediction. This paper was the basis for the Lundberg-Palmgren life theory published in 1947. A critical review of the 1924 Palmgren paper is presented here together with a discussion of its effect on bearing life prediction.

INTRODUCTION

Rolling-element bearing technology has evolved over 4000 years to the present. H.T. Morton (1) in his 1965 book, "Anti-Friction Bearings," describes the evolution of rolling bearing technology, which I have summarized here. By the first century of the common era (C.E.), the forerunner of the ball thrust bearing had appeared. L. da Vinci, circa 1500 C.E., described a multiroller bearing. He also described a pivot bearing comprising a conical shaft pivoted upon three lower balls free to rotate in an angular-contact race. At this time, the bearing materials of choice were wood and bronze.

By 1556, "De Re Metallica" was published in Switzerland (2). The book describes rolling bearings for bucket pumps and gives friction data. Cast iron was added as another bearing material of choice. A. Ramelli wrote a book in 1588 entitled "Le Diverse et Artificiose Machine," which describes various rolling bearing types. According to Morton (1), the next important development was de Mondran's carriage in 1710 with wheels supported by rolling bearings.

The first English patent for a rolling bearing was issued to J. Rowe in 1734. Circa 1760 C.E., E. Coulomb constructed the first prototype of the modern ball bearing. The first English ball bearing patent was issued to P. Vaughn in 1791 for use with cart axles. In 1802, M. Cardinet was issued a French patent for the tapered-roller bearing. These first rolling-element bearings were hand crafted and probably custom made for specific applications.

Although rolling bearing innovation continued throughout the first half of the 19th century, it was not until the invention of the pedal bicycle that the rolling-element bearing industry became established. In 1868, A.C. Cowper made a bicycle with ball bearings, thereby starting the bearing industry. In the same year, E. Mischaux, a French bicycle builder, won a bicycle race from Paris to Rouen with a ball-bearing-equipped bicycle. According to Morton (1), W. Bown of Coventry, England, was the most successful bearing manufacturer. In 1880, he had a contract to produce 12 ball bearings a day for Singer and Company, a bicycle manufacturer.

In 1852, the forerunner of Kugelfischer George Schaefer & Co. (FAG), Fischer Bearing Manufacturing, Ltd., was established as a bicycle manufacturer in Schweinfurt, Germany. Its founder, P.M. Fischer, invented the first pedal bicycle. In 1883, his son, F. Fischer, invented the first ball mill. In that same year, they began manufacturing ball bearings. The Timken Company, established by H. Timken had its beginnings in 1898 in St. Louis, Missouri, as part of the Timken Carriage Company. In that year, Timken, began manufacturing tapered-roller bearings. Aktiebolaget Svenska Kullagerfabriken (now SKF), established in 1907 in Sweden by S. Wingquist, manufactured double-row, self-aligning ball bearings. The Torrington Company, which began in

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1866 as the Excelsior Needle Company, began manufacturing ball bearings in 1912. Many other bearing companies were established during this time. By 1920, most types of rolling-element bearing used today were in production.

Circa 1875, carbon and chromium steel became available for the manufacture of rolling-element bearings. A British patent was issued to J. Harrington and H. Brent in 1879 for a hardened steel bushing on an inner shaft fitted with a groove for balls. The chemistry of a French steel, No. 88, closely matched what later became known as American Iron and Steel Institute (AISI) specification 52100. AISI 52100 was first specified about 1920 and is the most used bearing steel today.

By the close of the 19th century, the bearing industry began to focus on sizing bearings for specific applications and determining bearing life and reliability. In 1896, R. Stribeck (3) began fatigue testing full-scale bearings. In 1912, J. Goodman published formulas based on fatigue data that would compute safe loads on ball and cylindrical roller bearings. The person most instrumental in developing life prediction methods for ball and roller bearings was Arvid Palmgren in Sweden (5). His work with Professor G. Lundberg (6,7), published in 1947 and 1952, resulted in the International Organization for Standardization (ISO) and the American National Standards Institute (ANSI)/Anti-Friction Bearing Manufacturers Association (AFBMA) standards for the load ratings and life of rolling-element bearings (8–10). The basis for the Lundberg-Palmgren theory and the ISO–ANSI/AFBMA standards was outlined in a German paper presented by Palmgren in 1924. My purpose here is to critically review the 1924 Palmgren paper with respect to what we know and accept today regarding bearing life and reliability.

LUNDBERG-PALMGREN THEORY

The genesis of the Lundberg-Palmgren theory for bearing life prediction traces back to the 1924 Palmgren paper (11). However, because the 1924 paper was missing two elements, it did not allow for a comprehensive bearing life theory. The first missing element was the ability to calculate the subsurface principal stresses and, hence, the shear stresses below the Hertzian contact of either a ball on a nonconforming race or a cylindrical roller on a race. The second missing element was a comprehensive life theory that would fit the observations of Palmgren. Palmgren, as will be discussed, distrusted Hertz theory (12) and depended on a load-life relation for ball and roller bearings (11). In 1930, V. Thomas and H. Hoersch (13) at the University of Illinois, Urbana, developed an analysis for determining subsurface principal stresses under Hertzian contact. In 1939, W. Weibull (14,15) published his theory of failure. Weibull was a contemporary of Palmgren and shared the results of his work with him. Palmgren, in concert with Lundberg, incorporated his previous work along with that of Weibull and what appears to be that of Thomas and Hoersch into what has become known as the Lundberg-Palmgren theory (6,7). (Lundberg and Palmgren do not reference the work of Thomas and Hoersch in their paper.)

Lundberg and Palmgren (6,7), using the theoretical work of Weibull (14,15), assumed that the logarithm of the reciprocal of the probability of survival $S$ could be expressed as a power function of orthogonal shear stress $\tau_o$, life $N$, depth to the maximum orthogonal shear stress $z_o$, and stressed volume $V$. That is,

$$\ln \frac{1}{S} = \frac{\tau_o^c N^e}{z_o^h} V$$  \hspace{1cm} (1)

where

$$V = alz_o$$  \hspace{1cm} (2)

$a$ is the semimajor axis of the Hertzian contact ellipse, and $l$ is the length of the running track of the race. Then,

$$\ln \frac{1}{S} = \frac{\tau_o^c N^e al}{z_o^{h-1}}$$  \hspace{1cm} (3)

Lundberg and Palmgren based their life theory on subsurface-originated fatigue. However, it is my opinion from experience that the theory is applicable to both surface and subsurface-originated fatigue.
Lundberg and Palmgren (6) obtained the following additional relation:

\[ L_{10} = \left( \frac{C_r}{P_{eq}} \right)^p \]  

where \( C_r \) is the basic dynamic load rating or capacity, is defined as the load that a bearing can carry for 1 million inner-race revolutions with a 90% probability of survival; \( P_{eq} \) is the equivalent bearing load; and \( p \) is the load-life exponent. In their analysis Lundberg-Palmgren (6,7) use values of \( p = 3 \) for ball bearings and \( p = 10/3 \) for cylindrical roller bearings. Most bearing manufacturers, ANSI/AFBMA (the latter now known as the American Bearing Manufacturers Association (ABMA)), and ISO have adopted these values. Substituting bearing geometry and Hertz stress for a given load and life into Eq. (3), with appropriate exponents, results in a value of \( C_r \) for a given bearing. All subsequent life calculations are based on bearing equivalent load \( P_{eq} \).

From Lundberg and Palmgren (6) and the ANSI/ABMA standard (9), the basic dynamic load rating of a radial ball bearing with a ball diameter less than 25 mm is

\[ C_r = f_{cm} (i \cos \alpha)^{0.7} Z^{2/3} d^{1.8} \]

where \( C_r \) is the basic dynamic load rating, \( f_{cm} \) is the bearing geometry and material coefficient per ANSI/ABMA standards, \( i \) is the number of rolling-element rows, \( \alpha \) is the bearing contact angle, \( Z \) is the number of elements per row, and \( d \) is the ball diameter.

In both the Lundberg-Palmgren life equations and the ISO-ANSI/ABMA standards (8,9), the bearing equivalent load is

\[ P_{eq} = X F_r + Y F_a \]

where \( F_r \) and \( F_a \) are, respectively, the radial and axial loads applied to the bearing and \( X \) and \( Y \) are factors calculated by Lundberg and Palmgren (6) to provide the proper equivalent load based on the ratio \( F_r/F_r \). For pure radial loads, \( P_{eq} \) equals \( F_r \).

**PALMGREN ON HERTZ STRESS**

Palmgren did not have confidence in the ability of the Hertzian equations to accurately predict rolling bearing stresses. Palmgren (11) states, “The calculation of deformation and stresses upon contact between the curved surfaces...is based on a number of simplifying stipulations which will not yield very accurate approximation values, for instance when calculating the deformations. Moreover, recent investigations (circa 1919–1923) made at A.-B. Svenska Kullager-Fabriken (SKF) have proved through calculation and experiment that the Hertzian formulae will not yield a generally applicable procedure for calculating the material stresses...As a result of the paramount importance of this problem to ball bearing technology, comprehensive in-house studies were performed at SKF in order to find the law that describes the change in service life that is caused by changing load, rpm, bearing dimensions, and the like. There was only one possible approach: tests performed on complete ball bearings. It is not acceptable to perform theoretical calculations only, since the actual stresses that are encountered in a ball bearing cannot be determined by mathematical means.”

Palmgren later recanted his doubts about the validity of Hertz theory. In their 1947 paper (6), Lundberg and Palmgren state, “Hertz theory is valid under the assumptions that the contact area is small compared to the dimensions of the bodies and that the frictional forces in the contact areas can be neglected. For ball bearings, with close conformity between rolling elements and raceways, these conditions are only approximately true. For line contact the limit of validity of the theory is exceeded whenever edge pressure occurs.”

Lundberg and Palmgren exhibited a great deal of insight as to the other variables modifying the resultant shear stresses calculated from Hertz theory. They state (6), “No one yet knows much about how the material reacts to the complicated and varying succession of (shear) stresses which then occur, nor is much known concerning the effect of residual hardening stresses or how the lubricant affects the stress distribution within the pressure area. Hertz theory also does not treat the influence of those static stresses which are set up by the expansion or compression of the rings when they are mounted with tight fits.” These effects are now understood and life factors are currently being used to account for them so as to more accurately predict bearing life and reliability (16).
EQUIVALENT LOAD

Palmgren (11) recognized that it was necessary to account for combined and variable loading around the circumference of a ball bearing. He proposed a procedure in 1924 “to establish functions for the service life of bearings under purely radial load and to establish rules for the conversion of axial and simultaneous effective axial and radial loads into purely radial loads.” Palmgren used Stribeck’s (3) equation to calculate what can best be described as a stress on the maximum radial loaded ball-race contact in a ball bearing. The equation attributed to Stribeck by Palmgren is as follows:

\[ k = \frac{5Q}{Zd^2} \] (7)

where \( Q \) is the total radial load on the bearing, \( Z \) is the number of balls in the bearing, \( d \) is the ball diameter, and \( k \) is Stribeck’s constant.

Palmgren modified Stribeck’s equation to include the effects of speed and load as well as modifying the ball diameter relation. For brevity, I have not presented this modification. It is not clear whether Palmgren recognized at that time that Stribeck’s equation was valid only for greater than zero diametral clearance with fewer than half of the balls being loaded. However, he stated that the corrected constant yielded good agreement with tests performed.

Palmgren (11) states, “It is probably impossible to find an accurate and, at the same time, simple expression for the ball pressure as a function of radial and axial pressure…” According to Palmgren, “Adequately precise results can be obtained by using the following equation:

\[ Q = R + yA \] (8)

where \( Q \) is the imagined, purely radial load that will yield the same service life as the simultaneously acting radial and axial forces, \( R \) is the actual radial load, and \( A \) is the actual axial load.”

For ball bearings, Palmgren presented values of \( y \) as a function of Stribeck’s constant \( k \). He said that these values were confirmed by test results, which I assume he generated. In the Lundberg-Palmgren (6) life equations and the ISO-ANSI/ABMA standards (8,9), Eq. (8) is replaced by Eq. (6).

In all of the above equations, I have omitted the units of the input variables and the resultant units used by Palmgren because they cannot be reasonably used or compared with engineering practice today. As a result, these equations should be considered only for their conceptual content and not for any quantitative calculations.

FATIGUE LIMIT

Palmgren (11) states that bearing “limited service life is primarily a fatigue phenomenon. However, under exceptional high loads there will be additional factors such as permanent deformations, direct fractures, and the like….” If we start out from the assumption that the material has a certain fatigue limit, meaning that it can withstand an unlimited number of cyclic loads on or below a certain, low level of load, the service life curve will be asymptotic. Since, moreover, the material has an elastic limit and/or fracture limit, the curve must yield a finite load even when there is only a single load value, meaning that the number of cycles equals zero. If we further assume that the curve has a profile of an exponential function, the general equation for the relationship existing between load and number of load cycles prior to fatigue would read:

\[ k = C(an + e)^{-x} + u \] (9)

where \( k \) is the specific load or Stribeck’s constant, \( C \) is the material constant, \( a \) is the number of load cycles during one revolution at the point with the maximum load exposure, \( n \) is the number of revolutions in millions, \( e \) is the material constant that is dependent on the value of the elasticity or fracture limit, \( u \) is the fatigue limit, and \( x \) is an exponent.”

According to Palmgren, “This exponent \( x \) is always located close to 1/3 or 0.3. Its value will approach 1/3 when the fatigue limit is so high that it cannot be disregarded, and 0.3 when it is very low.” Palmgren reported test results, which I assume he generated, that support a value of \( x = 1/3 \). Hence, Eq. (9) can be written as
Life (millions of stress cycles) = \left( \frac{C}{k-u} \right)^3 - e \quad (10)

This equation is similar in form to Eq. (4), which is used today. When Eq. (4) was published in 1947, Palmgren together with Lundberg (6) discarded the concept of a fatigue limit. The value \( e \) suggests a finite time below which no failure would be expected to occur.

By letting \( e = 0 \) and eliminating the concept of a fatigue limit for bearing steels, Eq. (10) can be rewritten as

\[ L \ (\text{millions of race revolutions}) = \left( \frac{CZd^2/5}{Q} \right)^3 \quad (11) \]

By letting \( f_{cm} = C/5, \ P_{eq} = Q, \ i = 1, \) and \( \alpha = 0, \) the 1924 version of the dynamic load rating for a radial ball bearing, when cast into the same form as Eq. (5), becomes

\[ C_r = f_{cm}Zd^2 \quad (12) \]

and Eq. (11) becomes

\[ L = \left( \frac{C_r}{P_{eq}} \right)^3 \quad (13) \]

Equations (12) and (13) are identical in format to Eqs. (5) and (4), respectively. This similarity suggests that the origins of the Lundberg-Palmgren equations are contained within the 1924 Palmgren paper.

**L_{10} LIFE**

The \( L_{10} \) life, or the time that 90\% of a group of bearings will exceed without failing by rolling-element fatigue, is the basis for calculating bearing life and reliability today. Accepting this criterion means that the bearing user is willing in principle to accept that 10\% of a bearing group will fail before this time. In Eq. (4) the life calculated is the \( L_{10} \) life.

The rationale for using the \( L_{10} \) life was first laid down by Palmgren in 1924. He states (11), “The (material) constant \( C \) (Eq. (9)) has been determined on the basis of a very great number of tests run under different types of loads. However, certain difficulties are involved in the determination of this constant as a result of service life demonstrated by the different configurations of the same bearing type under equal test conditions. Therefore, it is necessary to state whether an expression is desired for the minimum, (for the) maximum, or for an intermediate service life between these two extremes….In order to obtain a good, cost effective result, it is necessary to accept that a certain small number of bearings will have a shorter service life than the calculated lifetime, and therefore the constants must be calculated so that 90\% of all the bearings have a service life longer than that stated in the formula. The calculation procedure must be considered entirely satisfactory from both an engineering and a business point of view, if we are to keep in mind that the mean service life is much longer than the calculated service life and that those bearings that have a shorter life actually only require repairs by replacement of the part which is damaged first.”

Palmgren is perhaps the first person to advocate a probabilistic approach to engineering design and reliability. Certainly, at that time, engineering practice dictated a deterministic approach to component design. This approach by Palmgren is decades ahead of its time. What he advocated is designing for finite life and reliability at an acceptable risk. This concept is incorporated in the ISO-ANSI/ABMA standards (8-10) and is used for nonbearing aerospace designs today.

**LINEAR DAMAGE RULE**

Most bearings are operated under combinations of variable loading and speed. Palmgren recognized that the variation in both load and speed must be accounted for in order to predict bearing life. Palmgren reasoned:
“In order to obtain a value for a calculation, the assumption might be conceivable that (for) a bearing which has a life of \( n \) million revolutions under constant load at a certain rpm (speed), a portion \( m/n \) of its durability will have been consumed. If the bearing is exposed to a certain load for a run of \( m_1 \) million revolutions where it has a life of \( n_1 \) million revolutions, and to a different load for a run of \( m_2 \) million revolutions where it will reach a life of \( n_2 \) million revolutions, and so on, we will obtain

\[
\frac{m_1}{n_1} + \frac{m_2}{n_2} + \frac{m_3}{n_3} + \ldots = 1 \tag{14}
\]

“In the event of a cyclic variable load we obtain a convenient formula by introducing the number of intervals \( p \) and designate \( m \) as the revolutions in millions that are covered within a single interval. In that case we have

\[
p\left(\frac{m_1}{n_1} + \frac{m_2}{n_2} + \frac{m_3}{n_3} + \ldots\right) = 1 \tag{15}
\]

where \( n \) still designates the total life in millions of revolutions under the load and rpm in question.”

Equation (14) was independently proposed for conventional fatigue analysis by Miner (17) at Douglas Aircraft in 1945, 21 years after Palmgren. The equation has been subsequently referred to as the linear damage rule or the Palmgren-Miner rule. For convenience, the equation can be written as follows:

\[
\frac{1}{L} = \frac{X_1}{L_1} + \frac{X_2}{L_2} + \frac{X_3}{L_3} + \ldots + \frac{X_n}{L_n} \tag{16}
\]

and

\[
X_1 + X_2 + X_3 + \ldots X_n = 1 \tag{17}
\]

where \( L \) is the total life in stress cycles or race revolutions, \( L_1 \ldots L_n \) is the life at a particular load and speed in stress cycles or race revolutions, and \( X_1 \ldots X_n \) is the fraction of total running time at load and speed. This equation is the basis for most variable-load fatigue analysis and is used extensively in bearing life prediction.

CONCLUDING REMARKS

A first reading of the Palmgren paper (11) would not necessarily impress a reader unfamiliar with bearing life prediction. Palmgren had three objectives in writing his 1924 paper. These were “to establish functions for the service life of bearings under purely radial load; to establish rules for the conversion of axial and simultaneously effective axial and radial loads into purely radial loads; and to calculate the effect of different types of loads that are subject to change over time.” The relationships he established are empirical. They are based on intuition, experimental observation, analogy, and curve fitting. He mistrusted the Hertz formulas but, nevertheless, used them in the context of Stribeck’s equation. Palmgren realized that bearing size was a factor in determining bearing life. However, I do not believe that in 1924 he recognized the effect of stressed volume on bearing life (i.e., that for a given stress a larger bearing will have a lower life). The effect of stressed volume is unconsciously factored into his equations by the use of Stribeck’s constant (Eq. (7)), which incorporates \( d^2 \). By normalizing Palmgren’s equation to early bearing data, I suggest that reasonable life predictions were obtainable.

Palmgren based his analysis on pitting failure of the bearing races. He did, however, recognize that other factors can limit bearing life. He started with the assumption that the bearing material would have a certain fatigue limit. He then questioned whether this fatigue limit could be determined. He also questioned whether this fatigue limit is so significant that it will have a decisive effect on bearings run under low loads. He went on to ask, “If the analysis agrees with the experimental data, should a fatigue limit even be considered?” Palmgren answered these questions by saying that “(his) equations stipulate certain fatigue limits based on several years of experience acquired in practical operation which appears to confirm that the fatigue limit must in fact be taken into consideration.” However, 21 years later in his 1945 book (5) and together with G. Lundberg (6) in 1947, the concept of fatigue limit was not considered in their bearing life analysis.
I have scoured the literature without success for evidence or data that would justify the use of a fatigue limit for common bearing steels. In my opinion, some investigators (18-20) have mistaken the presence of compressive residual stresses in AISI 52100 as an indication of a fatigue limit. These compressive residual stresses, which result from material phase transformations, are induced during operation and can both extend bearing life and skew the load (stress)–life relation (21).

What is interesting is that Eq. (5) was presented and derived empirically by Palmgren and matched to test data in his 1945 book (5). The 1947 paper by Lundberg and Palmgren presented an analytical proof of Eq. (5), where the exponents in Eq. (3) were selected to match the relation shown in Eq. (5).

Palmgren suggested the concept of bearing repair instead of entire bearing replacement when he said that “the mean service life is much longer than the calculated service life and...those bearings which have a shorter life actually require only repairs by the replacement of the part which is damaged first.” The concept of bearing refurbishment is based on replacing the rolling elements and only those other components that are damaged. Undamaged components from separate bearings can be mixed and matched. Analysis and experience have shown that refurbished bearings can attain nearly the life and reliability of new bearings (22).

A. Palmgren also addressed life testing of rolling-element bearings. He recognized that different materials would produce different life results. He stated that conventional fatigue tests of materials cannot be used directly to compare different bearing materials because “the relationships involved in the fatigue of a ball bearing differ considerably from the fatigue phenomena observed on an ordinary test sample.” Palmgren recognized that the failure mode and the stress conditions differed between conventional and rolling-element fatigue. He cautioned that the constants given were determined for specific SKF bearings and materials and were not necessarily applicable to other bearing types and materials.

Arvid Palmgren was a person ahead of his time and perhaps the most important person in the field of rolling bearing technology in the first half of the 20th century. His pioneering work and contributions in bearing analysis led the way for those of us who came in the second half of the century. It was only on the foundation he laid that we were able to advance rolling bearing technology to where it is today.

REFERENCES


Bearing technology, as well as the bearing industry, began to develop with the invention of the bicycle in the 1850's. At the same time, high-quality steel was made possible by the Bessemer process. In 1881, H. Hertz published his contact stress analysis. By 1902, R. Stribeck had published his work based on Hertz theory to calculate the maximum load of a radially loaded ball bearing. By 1920, all of the rolling bearing types used today were being manufactured. AISI 52100 bearing steel became the material of choice for these bearings. Beginning in 1918, engineers directed their attention to predicting the lives of these bearings. In 1924, A. Palmgren published a paper outlining his approach to bearing life prediction. This paper was the basis for the Lundberg-Palmgren life theory published in 1947. A critical review of the 1924 Palmgren paper is presented here together with a discussion of its effect on bearing life prediction.