Penetration of the interplanetary magnetic field $B_y$ into Earth’s plasma sheet

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Abstract. There has been considerable recent interest in the relationship between the cross-tail magnetic field component $B_y$ and tail dynamics. The purpose of this paper is to give an overall description of the penetration of the interplanetary magnetic field (IMF) $B_y$ into the near-Earth plasma sheet. We show that plasma sheet $B_y$ may be generated by the differential shear motion of field lines and enhanced by the flux tube compression. The latter mechanism leads to a $B_y$ analogue of the pressure-balance inconsistency (Erickson and Wolf, 1980) as flux tubes move from the far tail toward the Earth. The growth of $B_y$, however, may be limited by the dawn-dusk asymmetry in the shear velocity as a result of plasma sheet tilting. $B_y$ penetration into the plasma sheet implies field-aligned currents flowing between hemispheres. These currents together with the IMF $B_y$ related mantle field-aligned currents effectively shield the lobe from the IMF $B_y$.

1. Introduction

There is abundant observational evidence supporting the open model of Earth’s magnetosphere. One indication is a high correlation between the interplanetary magnetic field (IMF) $B_y$ and the observed cross-tail, magnetic field component $B_y$ in the plasma sheet (e.g., Lui, 1984). This result, as suggested by Moses et al. [1985], may be interpreted in terms of high-latitude convection patterns observed during periods of large IMF $B_y$, which show opposite senses of rotational flow $v$, in conjugate hemispheres ($v_y$ has the same sign as the IMF $B_y$ in the northern hemisphere, as to be discussed in the next section). The asymmetric flow appearing on closed field lines causes an azimuthal tilt of nightside plasma sheet field lines, resulting in a $B_y$ with the same direction as the IMF $B_y$. While this conceptually simple model suggested by Moses et al. [1985] may be used to interpret the observational feature of a larger $B_y$ in the plasma sheet than the tail lobes [Fairfield, 1979; Lui, 1984; Kaynaza et al., 1994a, b], it does not directly address the questions of how $B_y$ enters the near-Earth plasma sheet (e.g., McComas et al., 1986; Nagai, 1987) from the far tail (e.g., Tsurutani et al., 1984; Sibeck et al., 1985) and of why the penetration is stronger in the near tail than in the far tail (e.g., Sergeev, 1987). As reported by McComas et al. [1986], there is an approximate hour correlation between the IMF $B_y$ and the $y$ component of the magnetic field in the plasma sheet. This delay is apparently due to the returning sunward convection in the tail, where the IMF-associated $B_y$ field appearing in the open tail lobes may enter the closed field lines following the reconnection process (e.g., Cowley, 1981). The purpose of this paper is to provide an overall description of how $B_y$ may be carried into the near-Earth plasma sheet via the convection process. We show that plasma sheet $B_y$ may be generated not only by the shear flow associated with the IMF $B_y$, as described by Moses et al. [1985] but also by flux tube compression during earthward convection. The latter mechanism leads to a $B_y$ analogue of the pressure-balance inconsistency (PBI) discussed by Erickson and Wolf [1980]. The PBI argument states that standard quiet time magnetic field models are inconsistent with the notion of steady state adiabatic lossless plasma sheet convection. The difficulty is primarily due to much shorter near-Earth field line lengths than those in the distant plasma sheet. This causes near-Earth flux tubes to have much smaller $pV^y$ values than those in the far tail due to much shorter flux tube volume $V = \int ds/B$ there. Both the original PBI argument and existing modeling results (e.g., Hau, 1991; Erickson, 1992) assume $B_y = 0$. Since the presence of $B_y$ leads to relatively longer field lines, one may speculate whether its presence can serve as a path to remove the PBI. Such a possibility will be examined in this paper. Our motivation is due to recent interest in the $B_y$ effect on tail dynamics. As reported by Sergeev et al. [1993], a significant increase in $B_y$ is occasionally observed in the near-Earth plasma sheet prior to the onset of a substorm. Hau and Voigt [1992] pointed out that within the context of two-dimensional MHD equilibrium theory, a sudden enhancement of $B_y$ in the plasma sheet can act as an additional pressure that is able to reduce the equatorial normal magnetic field component and thus lead to the current disruption in a highly stretched magnetic field configuration established by adiabatic sunward convection (Hau, 1991; Erickson, 1992).

2. Dynamic Evolution of $B_y$

In this section we give a general explanation for the enhancement of $B_y$ in the plasma sheet based on Faraday’s law and the frozen-in-flux condition

$$\frac{dB}{dt} = -\nabla \times E \quad (1)$$

$$E + v \times B = 0 \quad (2)$$

By use of $\nabla \cdot B = 0$ and the continuity equation.
of (5) thus vanishes. If initially \( B_y \) is absent, then the last term in (5) is responsible for the generation of \( B_y \) in the plasma sheet. In particular, for positive \( B_y \), as in Earth’s quiet magnetosphere, an antisymmetric shear flow in opposite hemispheres may give rise to a neutral sheet \( B_y \) in the same sense as \( v_y \) in the northern hemisphere. The antisymmetric feature of \( v_y \) has been observed in high-latitude convection patterns [e.g., Moses et al., 1985, and references therein] and in the magnetotail as well [McComas et al., 1986] during the periods of large IMF \( B_y \). The sense of \( v_y \) in correlation with the IMF \( B_y \) may be understood as follows. An IMF \( B_y \), associated with a north-south electric field component \( E_y \), in the solar wind which may enter the closed field lines of the distant magnetotail via reconnection process and implies a rotational flow \( v_y = E_y/B_y \) in the tail lobes. The north-south antisymmetric shear flow associated with the IMF \( B_y \) thus gives rise to the nightside plasma sheet \( B_y \) in the same direction as the IMF \( B_y \). This mechanism is indeed the qualitative model proposed by Moses et al. [1985] for the generation of \( B_y \) in the plasma sheet. Since \( B_y \) changes sign in the dayside, there the plasma sheet \( B_y \) produced by the last term of (5) is in the opposite direction as the IMF \( B_y \). Note also that the term is proportional to \( B_y \). As shown by Fairfield [1986a], the plasma sheet \( B_y \), resulting from the last term in (5) is thus larger at the flanks than in the center. Plasma sheet \( B_y \), resulting from (5) is responsible for the growth of \( B_y \), as discussed above. Experimental evidence for the tilting of the plasma sheet induced by the IMF \( B_y \) is provided, for example, by Tsyganenko [1990], who found that the current sheet is progressively twisted down the tail as predicted by Cowley [1981]. Kaymaz et al. [1994b] further showed that the current sheet at \( x = -33 R_E \) is tilted by \( 8.4^\circ \pm 1.7^\circ \) to \( 4.4^\circ \pm 1.3^\circ \), on average, for positive and negative IMF \( B_y \), respectively. The tilting of the plasma sheet, in the simplest way, can be explained in terms of the simple reconnection model of the open magnetosphere [e.g., Stern, 1973; Cowley, 1981]. As shown in Figure 1, the flow converges toward the tilted plasma sheet (reconnection line), as indicated by the thick arrows, resulting in an antisymmetric shear flow \( v_y \) at the equatorial plane. The sign of \( v_y \) is in agreement with the earlier argument that in general \( v_y \) has the same (opposite) sign as the IMF \( B_y \) in the northern (southern) hemisphere. As a result, the second term in (5), \( B_y v_y / \partial y \), is negative (positive) for positive (negative) IMF \( B_y \). The effect of dawn-dusk asymmetry in the shear flow is thus to limit the growth of \( B_y \) and the tilting of the plasma sheet. Note that the dawn-dusk asymmetric flow discussed here is entirely induced by the tilting of the plasma sheet associated with the IMF \( B_y \). There is in fact a regular feature of diverging flows toward the flanks in the central plasma sheet associated with the earthward convection [Huang and Frank, 1994]. The.

Figure 1. Magnetotail cross sections showing the tilting of the plasma sheet for positive (dawn to dusk) and negative (dusk to dawn) interplanetary magnetic field (IMF) \( B_y \). The plasma conves toward the neutral sheet, as indicated by the thick arrows, resulting in an asymmetric shear flow at the equatorial plane. The signs of the shear flow terms in (5) are also shown.

\[
(\partial \rho / \partial t) + \nabla \cdot (\rho v) = 0 \tag{3}
\]

the time evolution of the \( y \) component magnetic field vector may be written as

\[
\frac{dB_y}{dt} = \frac{B_z}{\rho} \frac{d\rho}{dt} + (B \cdot \nabla)v_y \tag{4}
\]

although the same equation may also be derived for \( B_x \) and \( B_z \). Note that the coordinate system to be used throughout the paper is GSM coordinates such that positive \( x \), \( y \), and \( z \) are directed toward the Sun, the dusk, and north side of the Earth. Equation (4) clearly indicates that the change of \( B_y \) may be caused by compressing or expanding the plasma and by the differential shear motion as well. We discuss these two mechanisms separately.

For incompressible plasmas, i.e., \( \rho = \text{const} \), the time evolution of \( B_y \) is entirely attributed to the shear flow term

\[
\frac{dB_y}{dt} = \left( B_y \frac{\partial}{\partial x} + B_z \frac{\partial}{\partial y} + B_z \frac{\partial}{\partial z} \right) v_y \tag{5}
\]

In the center of the plasma sheet where \( B_y = 0 \) (defining the so-called “neutral” sheet), the first term on the right-hand side
effect of this diverging flow is to enhance rather than to limit $B_x$. Nevertheless, this effect is probably important only in the very near-Earth plasma sheet, where the flow becomes azimuthal due to the three-dimensional geometry of the inner magnetosphere [e.g., Zhu, 1993], and relatively less pronounced during the periods of large IMF $B_x$.

On the other hand, $B_y$ resulting simply from the density compression may be calculated based on

$$
(d/dt)(B_y/p) = 0
$$

Equation (6) is analogous to the adiabatic energy equation, $d(\rho V^2)/dt = 0$, for the plasma pressure $p$. In particular, the two equations are interchangeable with respect to $p$ and $B_y^2/2\mu_0$, provided that particle motions have only two degrees of freedom, i.e., $\gamma = 2$. This is not a coincidence since (6) is derived under the assumption that $v_z$ is either zero or constant along each field line. For Earth's magnetotail, $\rho$ is in general decreasing down the tail. Equation (6) then implies that a far tail $B_z$ becomes enhanced as plasma move toward the Earth. To summarize thus far, $B_y$ may be generated by the differential shear motion of a field line and enhanced by the flux tube compression while the effect of dawn-dusk asymmetry in the shear flow, as a result of plasma sheet tilting, is to limit the growth of $B_x$. Since the last mechanism plays a role only in the presence of a finite $B_z$, the net result is that plasma sheet $B_z$ will have the same sign as the IMF $B_x$, consistent with the observations.

3. Plasma Sheet Convection With $B_y$

In this section we discuss the consequences of plasma sheet convection with a cross-tail magnetic $B_x$ component. We first examine how the presence of a $B_x$ may modify the pressure-balance inconsistency (PBI) argument. For this purpose, we check the consistency between an assumed steady state magnetic field configuration and the condition for adiabatic lossless convection

$$
(d/dt)(\rho V') = 0
$$

in the plasma sheet, where $V'$ is the volume of a magnetic tube of unit magnetic flux. In the absence of a $B_x$, Erickson and Wolf [1980] found that the pressure resulting from steady state, adiabatic compression during sunward convection is much higher than the equilibrium required to balance the tail lobe field. This is due to the fact that $V'$ near Earth is much smaller than $V'$ down tail in standard quiet-time magnetic field models. As shown in the appendix, the flux tube volume of an arbitrary magnetic field line is identical to that of its projection onto a plane, e.g., $y = \text{const}$. Thus, although the addition of $B_x$ increases the field line length, it also increases the total magnetic field strength $B$ such that the flux tube volume remains unchanged. (This has been verified numerically by adding an arbitrary $B_x$ to the noon-midnight plane of the Tsyganenko magnetic field models.) Figure 2 shows the variations of the equatorial normal magnetic field $B_{yz}$, flux tube volume $V'$, and field line length $S$ down the tail in the noon-midnight plane of Tsyganenko's [1987] model for $Kp = 0$ (hereafter referred to as the T87 model). As indicated, $V'$ decreases by a factor of 16 as a flux tube converges from $x = -40$ to $x = -12$ R$_E$. According to (7), as a flux tube moves from $x = -40$ to $x = -12$, its plasma pressure increases by a factor of 104 for $\gamma = 5/3$. Since this is much larger than the factor of 5 increase in the corresponding lobe magnetic pressure, it is impossible to maintain the force-balance equilibrium and steady state adiabatic convection at the same time in such a magnetic field configuration. We now add a $B_x$ to the model which, according to (A4) and (7), should not change $V'$ nor the increase of plasma pressure due to earthward convection. The same PBI thus persists provided that $B_x$ is small in the tail lobes [Fairfield, 1979]. Thus, although field lines may be lengthened, the addition of $B_x$ does not remove the PBI. Indeed, the PBI with a $B_x$ present seems more severe than in the original argument by Erickson and Wolf [1980]. Moreover, since this result is independent of the mechanism responsible for generating $B_x$ in the plasma sheet, one may conclude that the PBI exists in such magnetic field models, even though the convection process may be accompanied by field line tilting or twisting and the associated field-aligned currents. Note that the fact that a y component of the magnetic field does not invalidate the PBI argument does not imply that $B_x$ is irrelevant since the magnetospheric configuration will evolve differently during convection from the $B_x = 0$ case if a $B_x$ is present.

The implication of the PBI is that standard quiet time magnetic field configurations, such as T87, must change shape in response to plasma sheet convection [Erickson and Wolf, 1980]. We now examine how plasma sheet $B_x$ may evolve during earthward convection. Since quantitative distribution of $v_z$ in the plasma sheet is not available, we consider $B_x$ simply resulting from the density compression term in (4). As for the adiabatic energy equation (7), assuming lossless convection and $\rho \propto V^{-1}$, (6) becomes

$$
(d/dt)(B_x V') = 0
$$

Applying (8) to T87 model implies that the equatorial shear magnetic pressure $B_{yz}^2/2\mu_0$ will increase by a factor of 256 as a flux tube moves from $x = -40$ to $x = -12$ R$_E$, which is even larger than the increase in the plasma pressure (as a result of $\gamma$ less than 2). The flux tube compression thus leads to a $B_x$ analogue of the pressure-balance inconsistency in the plasma sheet. In this respect, the presence of $B_x$ not only fails to remove the PBI but exaggerates the problem. To estimate the additional lobe flux needed to balance the equatorial total pressure, $p^* = p + B_{yl}^2/2\mu_0$, resulting from adiabatic convection, we carry out the following experiment. Consider an equilibrium magnetic field configuration with $B_x = 0$ and a given...
flux tube in the far tail with \( p = p_0 \) and \( V = V_p \). We now add a uniform \( B_z = B_{z0} \) to this model which merely adds a constant background pressure and thus does not change the overall equilibrium nor the flux tube volume. Converting the same flux tube in each case to the same near-Earth location results in \( p^* = p_d(V_d/V)^2 + p_d(V_d/V)^2 + B_{z0}^2/2\mu_0(V_d/V)^2 \), respectively. After subtracting out the constant magnetic \( B_s \) pressure, the difference in lobe flux between the two cases is

\[
\Delta B_{\text{flux}} = B_{z0}(V_d/V)^2 - 1)^{1/2}
\]

(9)

Since the \( B_s \) correlation may be as high as 50% in the plasma sheet [Lui, 1984], for IMF \( B_z \) of the order of \(-5 \gamma \) and \( V_{d}(x = -40) \) as in the T87 model, \( \Delta B_{\text{flux}} \) may be as large as 40 \( \gamma \). This is much greater than the change of the magnetic field strength in the tail lobes typically observed during the substorm growth phase though an unusually large plasma sheet \( B_z \) on association with a very thin current sheet has also been observed prior to the onset of a substorm event [e.g., Sergeev et al., 1993]. As discussed earlier, the dawn-dusk asymmetry of the shear flow will act to reduce the growth of \( B_z \), while the differential shear motion of a field line will lead to further enhancement of \( B_z \). Note that the latter mechanism may play an important role in enhancing \( B_z \) of the far tail and perhaps the midtail plasma sheet where the plasma density is less variable. On the other hand, the flux tube compression resulting from earthward convection may be responsible for the strong \( B_z \) penetration in the near-Earth plasma sheet. In particular, the result that the penetration efficiency drops from 60% at \( x = -20 R_E \) to 30% at \( x = -6.6 R_E \) [Naqvi, 1987; Sergeev, 1987] may be due to the fact that the convection becomes azimuthal toward the inner magnetosphere.

After showing that the PBI cannot be removed by simply lengthening near-Earth field lines via the addition of \( B_z \), it becomes clear that the only way to achieve steady state convection with \( \rho V^\gamma = \text{const} \) is by inflating the flux tubes via the reduction of the normal magnetic field component there. We now show an example of self-consistent convection models with \( B_z \) which is constructed based on quasi-static two-dimensional \((\partial/\partial y = 0)\) MHD equations, described by the Grad-Shafranov equation [e.g., Hau and Voigt, 1992]

\[
\nabla^2 A + \mu_0 \left( dp^*(A)/dA \right) = -\mu_0 J_{\text{dipole}}
\]

(10)

with the generalized pressure

\[
p^*(A) = p(A) + \left[ B_z^2(A)/2\mu_0 \right]
\]

(11)

where the magnetic vector potential \( A \) is constant along a field line. The two-dimensional assumption may appear to be inconsistent with the dawn-dusk asymmetry in association with the \( B_z \) discussed above. Observations, however, have indicated that the neutral sheet tilting from the tail axis is usually within a few degrees [e.g., Kaymaz et al., 1994b], we may therefore neglect this effect in the equilibrium model. As before, the functional form \( p(A) \) needs to be determined by the adiabatic energy equation (7), while for closure reasons we consider \( B_z \) to be entirely due to flux tube compression such that \( B_z(A) \) is constrained by (8).

In general, numerical equilibrium solutions consistent also with the thermodynamic constraints (7) and (8) are difficult to construct (for numerical procedure and code description, see the papers by Hau [1991] and Erickson [1992]). This problem, however, is greatly simplified by the fact that for a given \( \gamma \) magnetic field projection, the flux tube volume \( V \) is unaffected by the addition of \( B_z \). In this case, previous convection models developed for the case of \( B_z = 0 \) may be used as a basis for this study. Figure 3 shows the variations of equatorial \( B_z, p, \) and \( \rho V^\gamma \) down the tail in two-dimensional steady state solutions for \( B_z = 0 \) (model A) and \( B_z \neq 0 \) (model B). Both models use the same \( p^*(A) \) and thus have the same \( x-z \) magnetic field projection and flux tube volumes. The difference between the two pressure curves represents the shear magnetic pressure due to \( B_z \) in model B, which is calculated based on (12). To achieve the same \( pV^\gamma = \text{const} \) as in model A, a higher \( p^* \) with \( p \) between the two pressure curves is required in model B. Such a solution will have a smaller \( B_z \), than the one shown here.

4. Discussion

The existence of \( B_z \) in the plasma sheet is attributed to the field-aligned currents flowing across the equatorial plane and along plasma sheet field lines. For positive (dawn to dusk) \( B_z \), the field-aligned currents flow away from the Earth in the southern hemisphere and toward Earth in the northern hemisphere. For negative (dusk to dawn) \( B_z \), the field-aligned currents flow in the opposite sense. Figure 4 is a sketch of magnetospheric field-aligned currents associated with positive IMF.
density. In the presence of an IMF

creases the magnetic field strength and dccrcascs the plasma

density. In the presence of an IMF $\mathbf{B}_y$, field-aligned currents must exist in association with the RD in order to shield the slow-mode expansion fan. The role of the RD is to insure the MHD coplanarity by rotating the

plasma mantle consists of a slow-mode expansion fan. The role

of the magnetopause is a rotational discontinuity (RD) and the

model developed by

Pudovkin and Zaitseva

[1993]. In that model the magnetopause is a rotational discontinuity (RD) and the plasma mantle consists of a slow-mode expansion fan. The role of the RD is to insure the MHD coplanarity by rotating the magnetic field vector while the slow-mode expansion fan increases the magnetic field strength and decreases the plasma density. In the presence of an IMF $\mathbf{B}_y$, field-aligned currents must exist in association with the RD in order to shield the slow-mode expansion fan from the $\mathbf{B}_y$. This explains the sense of field-aligned currents on open field lines in Figure 4. The expansion fan model of the plasma mantle has also been used to explain the tilting of the plasma sheet in association with the IMF $\mathbf{B}_y$. As shown by Siscoe and Sanchez [1987], the larger the $\mathbf{B}_y$ is, the thinner the slow-mode expansion fan is. For purely southward IMF, a $\mathbf{B}_y$ may be generated outside the magnetopause (an RD) away from the noon-midnight plane as a result of field line draping. The superposition of a positive IMF $\mathbf{B}_y$ on this draping magnetic field then yields a relatively smaller $\mathbf{B}_y$ and thicker plasma mantle on the dayside than on the dusk-side, which in turn causes the tilting of plasma sheet about the tail axis. This explains the sense of rotation of the neutral sheet shown in Figure 1.

On the other hand, the sense of field-aligned currents on the dayside open field line portion is a result suggested by Pudovkin and Zaitseva [1993] based on the reconnection model of the magnetopause in the case of large IMF $\mathbf{B}_y$. In particular, these authors suggested that because of the tilt of the magnetopause current from the reconnection line the current perpendicular to the reconnection line will be disrupted in a way such that for positive IMF $\mathbf{B}_y$ the current flows away from the Earth in the northern hemisphere and toward Earth in the southern hemisphere. They suggested that these currents may be closed through the dayside closed field lines or through the polar cap ionosphere to the nightside closed field lines and proposed a global field-aligned current system somewhat similar to that in Figure 4. Their model, however, does not include the field-aligned currents on the nightside open field lines, which as discussed earlier can be explained in terms of the open magnetotail model of Siscoe and Sanchez [1987]. These currents can indeed be produced by the same mechanism as that for the dayside field-aligned currents proposed by Pudovkin and Zaitseva [1993]. A complete model for the generation of field-aligned currents on association with the IMF $\mathbf{B}_y$ is thus emerged. Note that the sense of field-aligned currents on the closed field line portion may also be explained in terms of two-dimensional ($\partial / \partial y = 0$ but $\mathbf{B}_y \neq 0$) MHD equilibrium theory. In particular, the field-aligned currents in association with (10) and (11) have the form of [Voigt and Hilmer, 1987]

$$j = \left( \frac{1}{\mu_0} \frac{dB_\parallel(A)}{dA} + \frac{B_z(A)}{B^2} \frac{dp(A)}{dA} \right) \mathbf{B}$$

It is seen that $j_1$ vanishes for the case $\mathbf{B}_y = 0$. The sense of these currents can easily be illustrated by the linear solution of the form of $p \propto A^2$ and $B_z^2 \propto A^2$ [Voigt and Hilmer, 1987]. The result being in agreement with the plasma sheet portion of field-aligned currents in Figure 4 based on simple Ampere's law. It should be remarked that within the two-dimensional static MHD equilibrium theory, the choices of $p(A)$ and $B_z^2(A)$ are entirely arbitrary. The linear solutions not only are mathematically simple but also qualitatively resemble the average, quiet time magnetic field and plasma configurations of the Earth’s magnetotail.) The field-aligned currents described by (12) are in contrast to the field-aligned currents produced in general three-dimensional magnetic field models for the case $\mathbf{B}_y = 0$ in the plasma sheet, which do not flow across the equatorial plane. Nevertheless, such currents have the physical
meaning for the special case where \( B_y \) is present in the plasma sheet as might be expected during the periods of IMF \( B_y \) and have indeed been inferred from the observations [Sergeev, 1987]. One might also speculate that the superposition of these field-aligned currents onto a three-dimensional magnetic field model causes a north-south asymmetry in the overall magnetic field configuration due to the asymmetry in the net field-aligned currents.

The linear solutions of (10) and (11) give rise to a \( B_y \) decreasing down the tail and a larger \( B_y \) in the plasma sheet than in the tail lobes, a result implied by the relation \( B_y^2 + A^2 \) and the fact that \( |A| \) is decreasing down the tail and away from the equatorial plane. On the basis of this result, Voigt and Hiltner [1987] argued that the enhancement of \( B_y \) in the plasma sheet is a result of MHD equilibrium, rather than of IMF penetration. As mentioned earlier, Moses et al. [1985] have explained the relatively high correlation between the neutral sheet \( B_y \) and IMF \( B_y \) component in terms of the azimuthal tilting of closed field lines. Both arguments are in contrast to the direct IMF \( B_y \) penetration interpreted by Liu [1984], which should be stronger, not weaker, on open field lines. The discussion in section 2 and Figure 4 solve this dilemma, which shows how the IMF \( B_y \) is shielded less in the plasma sheet than in the tail lobes by the related field-aligned currents. The mantle currents in one hemisphere partially close through the ionosphere, either over the polar cap or along the polar cap boundary to field-aligned currents which flow through the plasma sheet to the other hemisphere and its mantle currents. The field-aligned current flowing between hemispheres is responsible for \( B_y \) in the plasma sheet, while field-aligned currents in the plasma sheet and mantle effectively shield the lobe from the IMF \( B_y \).

5. Conclusions

In this paper we have examined the generation and enhancement of plasma sheet magnetic \( B_y \) component based on a simple model of Faraday’s law and the frozen-in-flux condition and discussed several aspects of plasma sheet convection in the presence of \( B_y \). The results may be summarized as follows.

1. In a convective magnetotail, \( B_y \) is generated by differential shear motion of the field line resulting from north-south antisymmetry in the shear flow and enhanced by flux tube compression via earthward convection. Both mechanisms lead to the enhancement of \( B_y \) in the plasma sheet in the same direction as the IMF \( B_y \). The growth of \( B_y \), however, may be limited by the dawn-dusk asymmetry in the shear flow velocity as a result of plasma sheet tilting.

2. For a given magnetic field projection on the noon-midnight plane, the addition of \( B_y \) yields longer field lines, but it does not affect flux tube volumes. Therefore the conclusion that the pressure balance inconsistency (PBI) exists in standard quiet time, magnetic field models is applied to such a case as well.

3. In the presence of IMF \( B_y \), there exists a \( B_y \) analogue of the PBI as a result of earthward compression. The PBI resulting from the shear magnetic pressure is greater than the inconsistency due to the thermal pressure and may further be enhanced by the north-south asymmetry in the shear flow but reduced by the dawn-dusk asymmetry in the shear velocity.

4. For the same \( y^V \) in the far tail, the presence of \( B_y \) in the plasma sheet leads to relatively stronger tail lobe field and smaller equatorial normal magnetic field component \( B_y \).

5. The existence of plasma sheet \( B_y \) is associated with field-aligned currents flowing through the equatorial plane. These currents are connected to the IMF \( B_y \) related mantle field-aligned currents through the ionosphere and form a current closure which shields the tail lobe from the IMF \( B_y \). A superposition of these currents onto a general three-dimensional magnetic field model shall lead to a north-south asymmetry in the overall configuration.

Appendix

The flux tube volume \( V = \int dS \times B \) can be written as

\[
V = \int \frac{dt}{B_y} \sqrt{1 + (dV/dt)^2/(1 + (B_y/B_y)^2)}
\]

where \( ds^2 = d\xi_1^2 + d\xi_2^2 \) and \( B_y = B_y^x + B_y^y \). By use of the field line definition,

\[
d\xi_i = B_y^x \frac{d\xi_y}{B_y} = B_y^y \frac{d\xi_y}{B_y} = B_y^z
\]

and the resulting relation

\[
B_i = B_y^x \frac{d\xi_i}{B_y} + B_y^y \frac{d\xi_i}{B_y} + B_y^z
\]

the flux tube volume is thus simply

\[
V = \int \frac{dt}{B_y}
\]

which implies that for a given magnetic field projection, the flux tube volume is independent of the third component of the magnetic field vector. It is also obvious that the tangent vector to the projected magnetic field line, \( dt \), is simply the projection of the original tangent vector at each point.

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