Spherical Harmonics Analysis of the ECMWF Global Wind Fields at the 10-Meter Height Level During 1985: A Collection of Figures Illustrating Results

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ABSTRACT

Half-daily global wind speeds in the east-west (u) and north-south (v) directions at the 10-meter height level were obtained from the European Centre for Medium Range Weather Forecasts (ECMWF) data set of global analyses. The data set covered the period 1985 January to 1995 January.

A spherical harmonic expansion to degree and order 50 was used to perform harmonic analysis of the east-west (u) and north-south (v) velocity field components.

The resulting wind field is displayed, as well as the residual of the fit, at a particular time. The contribution of particular coefficients is shown. The time variability of the coefficients up to degree and order 3 is presented. Corresponding power spectrum plots are given. Time series analyses were applied also to the power associated with degrees 0-10; the results are included.
ACKNOWLEDGMENTS

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INTRODUCTION

The general circulation of the ocean is driven by various types of forces, including surface wind stress. The results presented in this publication were obtained as part of an investigation of the wind-driven variability of the ocean circulation. It is possible that these results might be of interest to other investigators, not only in the limited context of ocean circulation, but also from the larger perspective of geodynamics, Earth rotation, and space geodesy. With such possibility in mind, it was decided to document the results. However, the intention is only to present the results obtained in graphical form; no interpretation is attempted here.

Half-daily global wind speeds in the east-west \((u)\) and north-south \((v)\) directions at the 10-meter height level were obtained from the European Centre for Medium Range Weather Forecasts (ECMWF) data set of global analyses. The data set covered the period 1985 January to 1995 January. The original global grid is 2.5° latitude by 2.5° longitude, and the data set used in this investigation has been reprocessed and interpolated onto a 2° latitude by 2.5° longitude grid, as described by Schubert et al. (1990).

A spherical harmonic expansion to degree and order 50 was used to perform harmonic analysis of the east-west \((u)\) and north-south \((v)\) velocity field components. The formal validity of this procedure was established by Simmonds (1974). Formal developments are given in Appendix I.

The software used in the analysis implements a technique developed by O. L. Colombo (1981), based on the Fast Fourier Transform.
GLOBAL WIND FIELDS

Figures 1-10 show the vectors for the wind fields corresponding to various cases. With the exception of figures 7 and 8, the time epoch is January 1, 0 hour GMT, 1985. This is the time of the first data entry for 1985.

Figure 1 shows the wind field as obtained from the ECMWF data files, the maximum magnitude is 22 meters/second. This wind field was fitted by means of an expansion in spherical harmonics including coefficients up to degree and order 10. The resulting field is shown in figure 2; maximum wind speed is 12 meters/second. The difference between the data and the fit is displayed in figure 3; the largest discrepancy has a magnitude of 16 meters/second.

Figure 4 exhibits the wind field obtained by fitting the data with spherical harmonics up to degree and order 50, note the improvement as compared to the degree-10 fit; the maximum wind speed is now 20 meters/second. The residuals are shown in figure 5; the maximum error has a magnitude of 4.5 meters/second. Note that the largest discrepancies are concentrated around the poles.

Figures 6 and 7 exhibit the wind fields from the ECMWF data files at January 1, 0 hour GMT, 1985 and at July 1, 0 hour GMT, 1985; the wind vectors are plotted with equal length in order to show clearly the wind direction, the magnitudes are indicated by the shading, the lower (higher) wind speeds corresponding to the darker (brighter) shades. Figure 8 depicts the average wind field for 1985.

Figures 9 and 10 are included to display the effect of some of the coefficients in the spherical harmonic expansion. Figure 9 brings to view the wind field corresponding to the $C_{10}$ coefficient by itself. Figure 10 makes apparent the wind field produced by the sum of the $C_{11}$ and $S_{11}$ coefficients.
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WIND VELOCITY YEAR 1985: JAN 1 00 DATA

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WIND VELOCITY YEAR 1985: JAN 1 00 FIT (10X10)

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The time series analysis was applied also to the power associated with degrees 0-10, as given by the last equation in Appendix I. The results are displayed in figures 35-45. Each figure presents the results for the east-west (u) and north-south (v) velocity components. As shown in part (a) of the figures, most of the power is contained in the zero-frequency term. Part (b) makes evident the power in the other frequencies by setting the constant part equal to zero.
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References


Let \( f(\theta, \lambda) \) represent a function defined globally at a set of discrete points, that is,

\[
f(\theta, \lambda) = f(\theta_i, \lambda_j)
\]

\( i = 0, 1, \ldots, n-1 \)

\( j = 0, 1, \ldots, m-1 \)

\( \lambda_j \) : longitude \hspace{1cm} \lambda_{j+1} - \lambda_j = 2\pi/m

\( \theta_i \) : colatitude \hspace{1cm} \theta_{i+1} - \theta_i = \pi/n

Assume that \( f(\theta, \lambda) \) can be expanded in a series of spherical harmonics,

\[
f(\theta, \lambda) = \sum_n \sum_m \left\{ (C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)) \right\} P^{n, m}_{nm}(\theta)
\]

where,

\( P^{n, m}_{nm}(\theta) \) : fully normalized Legendre functions of degree \( n \), order \( m \), i.e.

\[
\int \int P^{n, m}_{nm}(\theta) \left\{ \cos(m\lambda), \sin(m\lambda) \right\} P^{k, l}_{kl}(\theta) \left\{ \cos(l\lambda), \sin(l\lambda) \right\} \sin(\theta) d\theta d\lambda
= (4\pi) \delta_{nk} \delta_{ml}
\]

Then,

\[
C_{nm} = (1/4\pi) \int \int f(\theta, \lambda) P^{n, m}_{nm}(\theta) \cos(m\lambda) \sin\theta d\theta d\lambda
\]

\[
S_{nm} = (1/4\pi) \int \int f(\theta, \lambda) P^{n, m}_{nm}(\theta) \sin(m\lambda) \sin\theta d\theta d\lambda
\]

The power associated with the terms of degree \( n \) is given by,

\[
W_n = \sum_m (C_{nm})^2 + (S_{nm})^2
\]
POWER SPECTRUM OF FUNCTIONS DEFINED ON THE GLOBE

Let F denote a function such as $C_{nm}$, $S_{nm}$, or $W_n$, as defined in Appendix I. If certain basic conditions are satisfied, then F can be expanded in a Fourier series:

$$F = a_0 + a_1 \cos(t) + b_1 \sin(t) + a_2 \cos(2t) + b_2 \sin(2t) + \ldots + a_{365} \cos(365t) + b_{365} \sin(365t).$$

for $0 \leq t \leq 2\pi$

If the data covers a time span of one year, then the zero frequency term corresponds to the average over the year, frequency 1 corresponds to a periodicity of one year, frequency 2 corresponds to a periodicity of 6 months, and so on. The power corresponding to each frequency is given by:

$$P_i = (a_i)^2 + (b_i)^2$$
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