Biaxial Testing of 2219-T87 Aluminum Alloy Using Cruciform Specimens

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ABSTRACT

A cruciform biaxial test specimen was designed and seven biaxial tensile tests were conducted on 2219-T87 aluminum alloy. An elastic-plastic finite element analysis was used to simulate each test and predict the yield stresses. The elastic-plastic finite analysis accurately simulated the measured load-strain behavior for each test. The yield stresses predicted by the finite element analyses indicated that the yield behavior of the 2219-T87 aluminum alloy agrees with the von Mises yield criterion.

KEYWORDS
Biaxial, Finite Element Analysis, 2219-T87, von Mises, Yield Surface, Yield Locus, Aluminum Alloy.

INTRODUCTION

Many structural applications are subjected to multi-axial states of stress. For example, a pressurized cylinder will have both hoop and axial stress components and proper structural design sizing would require knowledge of the behavior of the material under biaxial loading. A design sizing procedure will first determine the local state of stress, then translate the stresses to the principal plane, and finally evaluate whether the principal stresses fall within a predetermined yield surface, as illustrated in Figure 1.

Several different theories have been suggested to describe the shape of the yield surface. The von Mises yield criterion has been successfully used to describe the yield surface for many ductile, isotropic materials. This yield criterion requires the determination of only one parameter (obtained from uniaxial tensile tests) and forms an ellipse, as given by equation 1. Other higher order yield criterion may require the determination of additional parameters.

\[ \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \sigma_y^2 \]  

where:

\( \sigma_1, \sigma_2 = \) principal stresses
\[ \sigma_{ys} = \text{uniaxial tensile yield stress} \]

Biaxial testing is required to determine the yield surface for a particular material. Two of the most common types of biaxial tests are axial loading of pressurized thin-walled cylinders and tension-tension testing of cruciform specimens. In testing of pressurized thin-walled cylinders, the combination of the axial loading with the hoop and axial stresses due to the pressurization can result in nearly any ratio \((\sigma_1/\sigma_2)\) of tensile biaxial stresses. For isotropic materials, the stresses in the cylinder can be easily determined from measurements of axial load and internal pressure. The disadvantage of this method is that it requires the material to be in the form of a thin-walled cylinder.

Biaxial testing of cruciform specimens characterizes flat sheet and plate material, but presents additional complications. In a uniaxial tensile test the stress can be calculated simply knowing the applied load and the cross sectional area. The cruciform specimen is more complex and requires numerical analysis to determine stress from the applied loads. Other considerations in using the cruciform specimen include: specimen alignment, size and uniformity of the biaxial stress, stress concentrations outside the test region, and stress redistribution after localized yielding.

Tensile tests, both biaxial and uniaxial, require a definition of the yielding. The two most common definitions of yielding are proportional limit and the 0.2\% offset. For both, yielding is defined as the stress that results in a deviation of a set amount from the linear-elastic behavior of the material, as illustrated in Figure 2. For the proportional limit, the offset is very small, around 0.01\% plastic strain. The 0.2\% offset defines yield as the stress at 0.2\% plastic strain.

The objective of this paper is to describe the characterization of the biaxial yield surface of 2219-T87 aluminum alloy using a cruciform biaxial test. The specimen design is described and the procedures for conducting the tests are outlined. The numerical procedures required to determine the load-stress relationship, 0.2\% and proportional limit, and the resulting tensile biaxial yield surface for 2219-T87 aluminum alloy are presented.

**SPECIMEN DESIGN**

The design of a cruciform specimen needs to consider the following constraints: (a) maximize the region of uniform biaxial stress, (b) minimize the shear strains in the test section, (c) minimize the stress concentration outside of the test section, (d) minimize the change in the load-stress relationship after yielding begins, and (e) size the specimen to obtain yielding in the gage section before failure elsewhere in the specimen. The specimen thickness is constrained by both the thickness of the raw material and thickness of the end product. The cruciform specimen used in this study was designed using an elastic finite element analysis [1] to meet the above criteria. The 2219-T87 aluminum alloy material was available in 0.25-inch thick plate and the thickness of the gage section was 0.08-inch.
The biaxial test frame used in this study had a 100 kip by 150 kip (orthogonal directions) capacity and a maximum specimen size of roughly a 4-foot by 4-foot square, as shown in Figure 3. The cruciform specimen was loaded by 4 actuators loading simultaneously in two orthogonal directions. The specimen has 4 orthogonal griped regions, as shown in Figure 4, each connected to one of the actuators. To reduce the cross stiffness (and minimize load transfer into the griped region), 12 slots were machined into each of the griped regions. These slots increased the cross-section compliance, but created stress concentration sites at the ends of the slots. Furthermore, large shear stresses (about 20% of the normal stresses in the loading directions) developed in each corner of the specimen. A series of thickness reductions was used to decrease the likelihood that these stresses would result in specimen failure before the desired plastic strain was obtained in the center of the specimen. The material in the wings was the thickest at 0.25-inch. In a 9-inch by 9-inch square center section, the thickness was reduced to 0.125-inch, within this square region was a 0.08-inch thick, 8-inch diameter circular gage section. This approach resulted in thickest material around the stress concentration sites at the end of the slots and the corners of the central square region were thicker than the gage region, reducing the effect of the shear stresses.

A two-dimensional, elastic finite element analysis was conducted to evaluate the stresses in the circular gage section and the stress concentrations at the corners. The finite element analysis assumed isotropic material behavior and a von Mises yield criterion. Symmetry conditions required that only one quarter of the cruciform specimen be modeled. The grips that were attached to the actuators were modeled and connected to the specimen model with stiff spring elements. The thickness changes in the specimen were explicitly modeled. The total model consisted of 1466 nodes and 2482 8-noded quadrilateral elements, as shown in Figure 5.

The model was loaded using displacement control under equal biaxial tension loading of $P_x=P_y=0.85$ kips, which resulted in an equal biaxial stress of $\sigma_{xx}=\sigma_{yy}=1$ ksi in the center of the specimen. The stress contours at the notch, the $\sigma_{xx}$ contours in the gage section, the contours of the ratio of $\sigma_{xx}/\sigma_{yy}$, and the $\tau_{xy}$ contours are given in Figures 6a, 6b, 6c, and 6d, respectively. The analysis of the notch root indicated that the stresses were about 1.7 times the stress at the center of the gage. Although these were the highest stresses in the specimen, they were extremely localized and nearly equal for all notches. Thus, the material at the notch roots should yield uniformly, minimizing load redistribution around the gage section. The $\sigma_{xx}$ contours given in Figure 6b and the $\sigma_{xx}/\sigma_{yy}$ contours given in Figure 6c indicated that the biaxial stresses varied by less than 2% for most of the circular gage section. A steep gradient occurs at the step from the circular gage section to the square reduced section, and the stresses $\sigma_{xx}$ in the thicker corner region are about 25% less than those of the gage section. The $\tau_{xy}$ contours given in Figure 6d indicated that the shear stresses were negligible in the gage section. The shear stresses in the corners of the specimens were significant, but with the lower $\sigma_{xx}$ and $\sigma_{yy}$ stresses, the principal stresses were less than in the gage section.

The resulting specimen design had a large region of uniform stress with negligible shear stresses. The greatest stresses were concentrated at the notch roots, but the regions of elevated stress were highly confined and nearly equal for all notch roots. Specimen
failure will most likely begin at the notch root, but for ductile materials the strains in the
gage section should exceed 0.2% plastic strain well before failure. The stresses in the
circular gage section (assuming linear elastic behavior) would reach more than 117 ksi at
maximum load capacity (100 kips), more than enough load to fail the specimen.

TESTING PROCEDURE

The biaxial test stand is shown in Figure 3. The four actuators are gimbaled to the
frame and therefore capable of applying only tensile loads. One opposing pair of actuators
has a maximum load capacity of 100 kips and the other pair has a maximum of 150 kips.
The cruciform specimen was bolted to grips that were attached to the ends of the actuators.
An example of this attachment is shown on an early version of a cruciform specimen in
Figure 7.

Each actuator was independently directed by a separate controller, but each was
provided the same external set point to ensure the specimen was centered. All tests were
conducted in stroke (displacement) control to ensure the specimen stayed centered during
testing.

The cruciform specimens were instrumented with 52 strain gages, as shown in
Figure 8. All of the gages were back-to-back with the odd numbered gages on the front of
the specimen and the even on the back. Stacked rosette gages were used in the biaxial
region to obtain the principal strains. In the corners, rosettes were used for specimen
alignment. Single gages were placed at the roots of two slots to evaluate the elevated
strains due to the stress concentration. An 80 channel data acquisition system, with a
collection rate of 30 data points per second, was used to collect the strain gage data.

Proper alignment of the specimen was critical to obtaining a uniform biaxial stress
field in the test region. Specimen misalignment could result in out-of-plane bending, in-
plane bending, and shear stresses in the gage section. An alignment check was performed
prior to testing. The specimen was loaded, using 1:1 stroke ratio, to approximately 10 kips
several times to remove backlash in the load train. Load and strain gage data were recorded
for the last 10 kip loading cycle. The back-to-back strain gauge readings were compared to
confirm that minimal out-of-plane bending was present. The shear strains were calculated
from the corner rosette strain gages, in-plane bending would appear as unequal shear in the
corner gages, as shown in Figure 9a. Figure 9b shows the shear strains from the same
gages after the specimen was properly aligned.

After the alignment was complete, the stroke ratios and rates were programmed into
the controlling computer and the data acquisition system was programmed to collect strain
gage, load, and stroke data every 5 seconds. The specimen was loaded at a stroke rate of
0.003 inch/second until failure. In several tests, the specimen was loaded until the strain in
the center gage was 12,000 \(\mu\)strain. Then, the specimen was unload to a low (but positive)
load, to determine the plastic strain, and re-loaded back to 12,000 \(\mu\)strain. The ramp rate
was dropped to 0.0015 inch/second until the specimen failed to collect more strain gage data near the failure load.

The $\varepsilon_{xx}$, $\varepsilon_{yy}$, and $\gamma_{xy}$ strains from the center strain gage of a 1:1 stroke ratio test are shown in Figures 10, 11 and 12 respectively. The measured $\varepsilon_{xx}$ and $\varepsilon_{yy}$ strains were nearly equal and were linear with applied load up to about 40 kips. At loads above 40 kips, the $\varepsilon_{xx}$ and $\varepsilon_{yy}$ strains increased nonlinearly with applied load until the strain gages failed at about 1% strain. This behavior indicates either material yielding with strain hardening, or a change in the relationship between applied load and local strain due to load redistribution resulting from yielding outside of the gage section, or a combination of the two. The results from tensile tests on the same material (0.2% plastic strain at about 5,000 $\mu$strain) indicated that the gage section exceeded 0.2% plastic strain. Figure 11 shows that the shear strains were negligible.

Numerical Analysis

The cruciform biaxial test produces measurements of strain as a function of applied load, but the determination of a biaxial yield surface requires strain as a function of stress. For a cruciform specimen, a numerical analysis was required to determine the relationship between applied load and gage section stresses. An elastic finite element analysis will not account for any load redistribution due to material yielding, and is therefore inappropriate if significant load redistribution occurs. To evaluate the effect of load redistribution, an elastic-plastic, two-dimensional finite element analysis was conducted to determine the local stresses for the biaxial specimen. The elastic-plastic finite element analysis used the same mesh as used in the elastic finite element analysis used in the specimen design.

The elastic-plastic finite element analysis [2] was based on incremental flow theory and small strain assumptions. A multi-linear representation of the uniaxial stress-strain curve for 2219-T87 was used in the analysis and is shown in Figure 13. This material had a 0.2% offset yield stress of 55 ksi. A von Mises yield criterion was assumed in the elastic-plastic finite element analysis.

Elastic and elastic-plastic finite element analyses were conducted to simulate a biaxial test with an applied displacement ratio of 1.18:1 (i.e., 1.18 inches of displacement in the y-direction for every 1 inch of displacement in the x-direction). The local stresses in the center of the specimen were calculated as a function of major axis applied load ($P_y$), as shown in Figure 14a. The elastic-plastic analysis indicated that the plastic component of the major axis strain ($\varepsilon_{yy}$) was 0.2% at a load of $P_y$=54.2 kips, as shown in Figure 14b. At this load level the stresses calculated by elastic-plastic finite element analysis were about 10% less than the elastic results. This result indicates that significant load redistribution occurs before the 0.2% plastic strain is achieved, decreasing the stresses in the gage section from those predicted by the elastic analysis. Thus, an elastic-plastic finite element analysis simulation of each biaxial test was required to obtain the gage section stresses.
A comparison between the measured load-strain behavior and that from the elastic-plastic finite element analysis was performed to verify that the von Mises yield criterion described the behavior of the 2219-T87 material. If the finite element analyses were able to simulate the load-strain behavior for tests conducted at several biaxial loading ratios, then the von Mises yield criterion assumption would be valid for this material. On the other hand, if the finite element analyses failed to simulate the measured load-strain behavior, then a higher order yield criterion would be required. The parameters of the higher order yield criterion could be determined through an optimization study that minimized the difference between the experimental measurements and finite element simulations for several different biaxial loading ratios.

The approach taken to determine the biaxial yield surface required 3 steps, as illustrated in Figure 15. (1) An elastic-plastic finite element analysis of the biaxial test was conducted and the load-strain behavior simulated. The major axis load that resulted in a 0.2% major axis plastic strain was determined. (2) The stresses at this load were determined from the elastic-plastic finite element analysis. (3) The major and minor axis stresses were plotted to determine the yield surface.

RESULTS

Biaxial tensile tests were conducted at seven different applied displacement ratios, as summarized in Table 1. Elastic-plastic finite element analyses were conducted for each test and the load-strain behavior was predicted using the von Mises yield criterion. The load and strain measurements and the elastic-plastic finite element simulations for the biaxial test with the applied displacement ratio of 2.5x1 are shown in Figure 16. This test had an applied displacement of 2.5 inches in the y-direction for every 1.0 inch of displacement in the x-direction. The load plotted in Figure 16 is the applied load in the major axis (y-direction) and the plotted strains are measured at the center of the gage section. The elastic-plastic finite element simulation showed excellent agreement with the experimental measurements well beyond 0.2% plastic strain. The yield load was defined as the major axis load that results in a 0.2% plastic strain in a principle axis (the lack of shear strains make $\varepsilon_{yy}$ and $\varepsilon_{xx}$ principle strains). For the 2.5x1 test, the yield load was 55.7 kips. Since the elastic-plastic finite element analysis was able to accurately simulate the measured load-strain behavior for this test (and the tests at other biaxial ratios, see the Appendix A), it was assumed that the analysis properly characterized the yield behavior with the von Mises criterion. The stresses in the gage section were obtained by determining the $\sigma_{yy}$ and $\sigma_{xx}$ stresses at the yield load from the elastic-plastic finite element analysis, as shown in Figure 17. Again, the lack of shear stresses in the center of the gage section indicate that $\sigma_{yy}$ and $\sigma_{xx}$ are the principle stresses $\sigma_1$ and $\sigma_2$, respectively. The yield stresses for the 2.5x1 test were 63.4 ksi and 36.6 ksi for stresses $\sigma_1$ and $\sigma_2$, respectively.

The yield stresses from each test were plotted along with the von Mises ($\sigma_{ys} = 55$ ksi) in Figure 18. As expected, the experiments all fall along the von Mises yield surface. If the von Mises yield criterion did not accurately describe the yield behavior of this material, then the finite element analysis would not be able to simulate the load-strain
behavior for all of the tests. Appendix A contains plots of measured and simulated load-strain behavior, at the center of the gage section, for the 1x0, 20x1, 5x1, 2.5x1, 1.18x1, and 1x1 tests. Also included are the stresses at the center of the gage section obtained from the finite element analyses.

SUMMARY AND CONCLUSIONS

A cruciform biaxial test specimen was designed using elastic finite element analysis. Seven biaxial tensile tests were conducted on 2219-T87 aluminum alloy. The tests were simulated using an elastic-plastic finite element analysis that assumed a von Mises yield criterion. The results of this study indicate:

(1) The cruciform specimen that was designed had a gage section of uniform biaxial stress that would achieve 0.2% plastic strain before failure.
(2) A test procedure was developed that would allow for quick and accurate alignment and testing of cruciform specimens at various stroke ratios.
(3) An elastic-plastic finite element analysis was required to determine the stresses in the cruciform specimen because significant load redistribution, due to material yielding, occurred before achieving 0.2% plastic strain.
(4) The elastic-plastic finite element analyses that assumed a von Mises yield criterion and isotropic material behavior (obtained from uniaxial tensile tests) accurately predicted the load-strain behavior for all seven tests.
(5) The von Mises yield criterion was shown to accurately describe the behavior of the 2219-T87 aluminum alloy.

References
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<th>Minor Axis Load (kips)</th>
<th>Major Axis Stress at 0.2% Plastic Strain (ksi)</th>
<th>Minor Axis Stress at 0.2% Plastic Strain (ksi)</th>
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Table 1: 2219 Biaxial Test Summary
Figure 1  Schematic of the determination of biaxial yielding using a von Mises yield criterion.
Figure 2  Definition of proportional limit and 0.2% offset yield stress
Figure 3
Photograph of biaxial loading frame.
Figure 6a \( \sigma_{xx} \) stress contours for a slot in the biaxial specimen loaded in equal biaxial tension (1x1).
\( p_0 = 0.85 \) kips

(linear elastic response)

\( \sigma_{xx} = 1.0 \) ksi

Figure 6b

\( \sigma_{xx} \) stress contours for the biaxial specimen loaded in equal biaxial tension (1x1).
Figure 6c Contours of the ratio of $\sigma_{xx}/\sigma_{yy}$ for the biaxial specimen loaded in equal biaxial tension (1x1).
Figure 6d  $\tau_{xy}$ stress contours for the biaxial specimen loaded in equal biaxial tension ($1\times1$).
Figure 8  Locations of strain gages on the biaxial specimen.
Figure 9a  Shear strains from the corner strain gage rosettes in a poorly aligned specimen under equal biaxial tension loading (stroke ratio: 1:1).
Figure 9b  Shear strains from the corner strain gage rosettes in a well aligned specimen under equal biaxial tension loading (stroke ratio: 1:1).
Figure 10 Load against $\varepsilon_{xx}$ strain from the center 4 rosettes for a biaxial specimen loaded under equal biaxial tension (stroke ratio: 1:1).
Figure 11  Load against $\varepsilon_{yy}$ strain from the center 4 rosettes for a biaxial specimen loaded under equal biaxial tension (stroke ratio: 1:1).
Figure 12  Load against $\gamma_{xy}$ strain from the center 4 rosettes for a biaxial specimen loaded under equal biaxial tension (stroke ratio: 1:1).
Figure 13  Uniaxial stress-strain behavior of 2219-T87 aluminum alloy.
Figure 14b  The local strain for the applied major axis load in the center of the specimen.
Figure 14a  Load against stress behavior from elastic and elastic-plastic finite element simulations of a biaxial specimen subjected to biaxial loading (stroke ratio: 1.18:1).
Figure 15  Schematic of yield surface calculation.
Figure 16  Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio: 2.5:1).
Figure 17  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio: 2.5:1).
Figure 18  Yield surface for biaxial specimens conducted various tensile stroke ratios.
Biaxial tests were conducted at load ratios of 1:1.18, 1:0, 1:1, 2.5:1, 2.5:1, 26.4:1, and 5:1. The load-strain measurements and the finite element simulation for each test are given in Figures A-1a to A-7a, respectively. The load given in these figures is the applied load in the major axis (y-direction) and the plotted strains are measured at the center of the gage section. The yield load (at 0.2% plastic strain) is shown in each figure.

The gage section $\sigma_{xx}$ and $\sigma_{yy}$ stresses determined from the elastic-plastic finite element analyses are given in Figures A-1b to A-7b for each of the biaxial tests. The stresses at the yield load are shown in each figure. These stresses correspond to the data points shown in Figure 18.

Figure A-1a: Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 1.18:1).
Figure A-1b  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 1.18:1).
Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 1:0).
Figure A-2b  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 1:0).
Figure A-3a  Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 1:1).
Figure A-3b  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 1:1).
Figure A-4a Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 2.5:1).
Figure A-4b  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 2.5:1).
Figure A-5a Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 2.5:1).
Figure A-5b  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 2.5:1).
Figure A-6a Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 26.4:1).
Figure A-6b  Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 26.4:1).
Figure A-7a  Predicted and experimental load against strain behavior for the center gage of a biaxial specimen loaded in tension (stroke ratio 5:1).
Figure A-7b Load against stress behavior, obtained from elastic-plastic finite element analysis, for the center of a biaxial specimen loaded in tension (stroke ratio 5:1).
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## Abstract
An elastic-plastic finite element analysis was used to simulate each test and predict the yield stresses. The elastic-plastic finite analysis accurately simulated the measured load-strain behavior for each test. The yield stresses predicted by the finite element analyses indicated that the yield behavior of the 2219-T87 aluminum alloy agrees with the von Mises yield criterion.

## Subject Terms
- Biaxial
- Finite Element Analysis
- 2219-T87
- von Mises
- Yield Surface
- Yield Locus
- Aluminum Alloy