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## TRAJECTORY DESIGN STRATEGIES THAT INCORPORATE INVARIANT MANIFOLDS AND SWINGBY

J. J. Guzmán,<sup>\*</sup> D. S. Cooley,<sup>†</sup> K. C. Howell,<sup>‡</sup> and D. C. Folta<sup>§</sup>

Libration point orbits serve as excellent platforms for scientific investigations involving the Sun as well as planetary environments. Trajectory design in support of such missions is increasingly challenging as more complex missions are envisioned in the next few decades. Software tools for trajectory design in this regime must be further developed to incorporate better understanding of the solution space and, thus, improve the efficiency and expand the capabilities of current approaches. Only recently applied to trajectory design, dynamical systems theory now offers new insights into the natural dynamics associated with the multi-body problem. The goal of this effort is the blending of analysis from dynamical systems theory with the well established NASA Goddard software program SWINGBY to enhance and expand the capabilities for mission design. Basic knowledge concerning the solution space is improved as well.

### INTRODUCTION

The trajectory design software program SWINGBY, developed by the Guidance, Navigation and Control Center at NASA's Goddard Space Flight Center, is successfully used to design and support spacecraft missions. Of particular interest here are missions to the Sun-Earth collinear libration points. Orbits in the vicinity of libration points serve as excellent platforms for scientific investigations including solar effects on planetary environments. However, as mission concepts become more ambitious, increasing innovation is necessary in the design of the trajectory. Although SWINGBY has been extremely useful, creative and successful design for libration point missions still relies heavily on the experience of the user. In this work, invariant manifold theory and SWINGBY are combined in an effort to improve the efficiency of the trajectory design process. A wider range of trajectory options is also likely to be available in the future as a result.

Design capabilities for libration point missions have significantly improved in recent years. The success of SWINGBY for construction of trajectories in this regime is evidence of the improvement in computational capabilities. However, conventional tools, including SWINGBY, do not currently incorporate any theoretical understanding of the multi-body problem and do not exploit the dynamical relationships. Nonlinear dynamical systems theory (DST) offers new insights in multi-body regimes, where qualitative information is necessary concerning sets of solutions and their evolution. The goal of this effort is a blending of dynamical systems theory, that employs the dynamical relationships

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<sup>\*</sup> Graduate Student, School of Aeronautics and Astronautics, Purdue University.

<sup>†</sup> Aerospace Engineer, Flight Dynamics Analysis Branch, Guidance, Navigation and Control Center, Goddard Space Flight Center.

<sup>‡</sup> Professor, School of Aeronautics and Astronautics, Purdue University.

<sup>§</sup> Aerospace Engineer, Systems Engineering Branch, Guidance, Navigation and Control Center, Goddard Space Flight Center.

to construct the solution arcs, and SWINGBY, with its strength in numerical analysis. Dynamical systems theory is, of course, a broad subject area. For application to spacecraft trajectory design, it is helpful to first consider special solutions and invariant manifolds, since this aspect of DST offers immediate insights. An understanding of the solution space then forms a basis for computation of a preliminary solution; the end-to-end approximation can then be transferred to SWINGBY for final adjustments. Accomplishing this objective requires an exchange of information between two software packages. At Purdue, various dynamical systems methodologies are included in an internal software tool called GENERATOR. GENERATOR includes several programs that *generate* different types of solution arcs, some based on dynamical systems theory; the user then collects all the arcs together and differentially corrects the trajectory segments to produce a complete path in a complex dynamical model. A two level iteration scheme is utilized whenever differential corrections are required; this approach produces position continuity (first level), then velocity continuity (second level).<sup>1-4</sup> SWINGBY, on the other hand, is an interactive *visual* tool that allows the user to model launches and parking orbits, as well as design transfer trajectories utilizing various targeting schemes.<sup>5</sup> SWINGBY is also an excellent tool for prelaunch analysis including trajectory design, error analysis, launch window calculations and ephemeris generation.<sup>6</sup> SWINGBY has proven to be an improvement over previous non-GUI (Graphical User Interface) programs. The goal here is a procedure to use the tools in combination for mutual benefit.

## INVARIANT MANIFOLDS

The geometrical theory of dynamical systems is based in phase space and begins with special solutions that include equilibrium points, periodic orbits, and quasi-periodic motions. Then, curved spaces (differential manifolds) are introduced as the geometrical model for the phase space of dependent variables. An invariant manifold is defined as an  $m$ -dimensional surface such that an orbit starting on the surface remains on the surface throughout its dynamical evolution. So, an invariant manifold is a set of orbits that form a surface. Invariant manifolds, in particular stable, unstable, and center manifolds, are key components in the analysis of the phase space. Bounded motions (including periodic orbits) exist in the center manifold, as well as transitions from one type of bounded motion to another. Sets of orbits that approach or depart an invariant manifold asymptotically are also invariant manifolds (under certain conditions) and these are the stable and unstable manifolds, respectively.

In the context of the three body problem, the libration points, halo orbits, and the tori on which Lissajous trajectories are confined are themselves invariant manifolds. First, consider a collinear libration point, that is, an equilibrium solution in terms of the rotating coordinates in the three-body problem. The libration point itself has a one-dimensional stable manifold, a one dimensional unstable manifold, and a four dimensional center manifold. As has been described in more detail in Ref. 7, there exist periodic and quasi-periodic motions in this center manifold. Two types of periodic motion are of interest here, i.e., the planar Lyapunov orbits as well as the nearly vertical (out of plane) orbits. The familiar periodic halo orbits result from a bifurcation along the planar family of Lyapunov orbits as the amplitude increases. Also in the center subspace are quasi-periodic solutions related to both the planar and the vertical periodic orbits. These three-dimensional, quasi-periodic solutions are those that have typically been denoted as Lissajous trajectories. Although not of interest here, a second type of quasi-periodic solution is the motion on tori that envelop the periodic halo orbits.

The periodic halo orbits, as defined in the circular restricted problem, are used as a reference solution for investigating the phase space in this analysis. It is possible to exploit the hyperbolic nature of these orbits by using the associated stable and unstable manifolds to generate transfer trajectories as well as general trajectory arcs in this region of space. (The results can also be extended to more complex dynamical models.<sup>4,8</sup>) Developing expressions for these nonlinear surfaces

is a formidable task, one that is unnecessary in the context of their role in this particular design process. Rather, the computation of the stable and unstable manifolds associated with a particular halo orbit is accomplished numerically in a straightforward manner. The procedure is based on the availability of the monodromy matrix (the variational or state transition matrix after one period of the motion) associated with the halo orbit. This matrix essentially serves to define a discrete linear map of a fixed point in some arbitrary Poincaré section. As with any discrete mapping of a fixed point, the characteristics of the local geometry of the phase space can be determined from the eigenvalues and eigenvectors of the monodromy matrix. These are characteristics not only of the fixed point, but of the halo orbit itself.

The local approximation of the stable (unstable) manifolds involves calculating the eigenvectors of the monodromy matrix that are associated with the stable (unstable) eigenvalues. This approximation can be propagated to any point along the halo orbit using the state transition matrix. Recall that the eigenvalues of a periodic halo orbit are known to be of the following form:<sup>9</sup>

$$\lambda_1 > 1, \quad \lambda_2 = (1/\lambda_1) < 1, \quad \lambda_3 = \lambda_4 = 1, \\ \lambda_5 = \lambda_6^*, \quad \text{and } |\lambda_5| = |\lambda_6| = 1,$$

where  $\lambda_5$  and  $\lambda_6$  are complex conjugates. Stable (and unstable) eigenspaces,  $E^S$  ( $E^U$ ) are spanned by the eigenvectors whose eigenvalues have modulus less than one (modulus greater than one). There exist local stable and unstable manifolds,  $W_{loc}^S$  and  $W_{loc}^U$ , tangent to the eigenspaces at the fixed point and of the same dimension.<sup>10,11</sup> Thus, for a fixed point  $\bar{X}^H$  defined along the halo orbit, the one-dimensional stable (unstable) manifold is approximated by the eigenvector associated with the eigenvalue  $\lambda_2$  ( $\lambda_1$ ). First, consider the stable manifold. Recall that a periodic orbit appears as one fixed point in a Poincaré map; thus, the halo orbit is identified as  $\bar{X}^H$  in the two dimensional representation in Figure 1. Let  $\bar{Y}^{W^s}$  denote a six-dimensional vector that is coincident with the stable eigenvector and is scaled such that the elements corresponding to position in the phase space have been normalized. This vector serves as the local approximation to the stable manifold ( $W^S$ ). Remove the fixed point  $\bar{X}^H$  from the stable manifold and there remain two half-manifolds,  $W^{S+}$  and  $W^{S-}$ . Each half-manifold is itself a manifold consisting of a single trajectory. Now, consider some point  $\bar{X}_o$  that lies exactly on  $W^{S+}$ . Integrating forward and backward in time from  $\bar{X}_o$  produces  $W^{S+}$ . Of course, the stable manifold approaches the fixed point asymptotically, so  $W^{S+}$  reaches  $\bar{X}^H$  only in infinite time. Nevertheless, conceptually, calculating a half manifold is composed of the following two steps: locating or approximating a point on  $W^{S+}$ , and numerically integrating from this point.

To numerically generate the stable manifold, an algorithm originally developed for second order systems has been employed.<sup>12</sup> The algorithm, however, does not possess any inherent limit to the

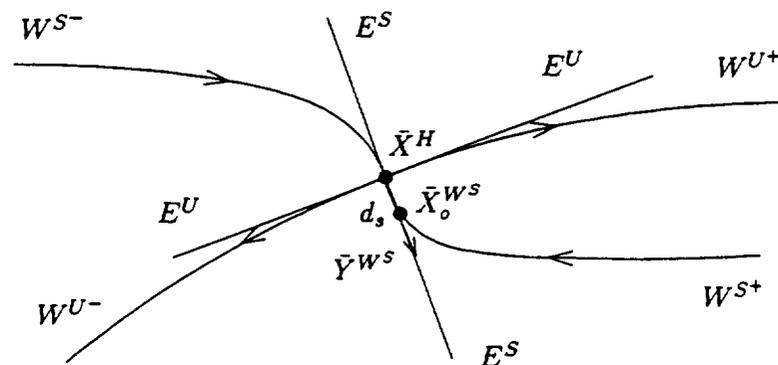


Figure 1 Stable and Unstable Manifolds Associated with a Fixed Point  $\bar{X}^H$

order of the system and has been used successfully here. Near the fixed point  $\bar{X}^H$ , the half-manifold  $W^{S^+}$  is determined, to first order, by the stable eigenvector  $\bar{Y}^{W^*}$ . The next step is then to globalize the stable manifold. This can be accomplished by numerically integrating backwards in time. It also requires an initial state that is on  $W^{S^+}$  but not on the halo orbit. To determine such an initial state, the position of the spacecraft is displaced from the halo in the direction of  $\bar{Y}^{W^*}$  by some distance  $d_s$  such that the new initial state, denoted as  $\bar{X}_o^{W^*}$ , is calculated as

$$\bar{X}_o^{W^*} = \bar{X}^H + d_s \bar{Y}^{W^*} \quad (1)$$

Higher order expressions for  $\bar{X}_o^{W^*}$  are available but not necessary. The magnitude of the scalar  $d_s$  should be small enough to avoid violating the linear estimate, yet not so small that the time of flight becomes too large due to the asymptotic nature of the stable manifold. This investigation is conducted with a nominal value of 200 km for  $d_s$  since this application is in the Sun-Earth system. A suitable value of  $d_s$  should be determined for each application. Note that a similar procedure can be used to approximate and generate the unstable manifold. One additional observation is notable. The stable and unstable manifolds for any fixed point along a halo orbit are one-dimensional and this fact implies that the stable/unstable manifolds for the entire halo orbit are two-dimensional. This is an important concept when considering design options.

## APPLICATION TO MISSION DESIGN

Trajectory design has traditionally been initiated with a baseline mission concept rooted in the two-body problem and conics. For libration point missions, however, a baseline concept derived from solutions to the three-body problem is required. Since no such general solution is available, the goal is to use dynamical systems theory to numerically explore the types of trajectory arcs that exist in the solution space. Then, various arcs can be "patched" together for preliminary design; the end-to-end solution is ultimately computed using a model that incorporates ephemeris data as well as other appropriate forces (e.g., solar radiation pressure).

### Force Models

The dynamical model that is adopted to represent the forces on the spacecraft includes the gravitational influences of the Sun, Moon and Earth. (Additional gravitational bodies can certainly be added. This subset, however, includes the dominant gravitational influences and is a convenient set for this discussion and demonstration.) All planetary, solar, and lunar states are obtained from the GSFC Solar Lunar and Planetary (SLP) files. The SLP files describe positions and velocities for nine solar system bodies (excluding Mercury) in the form of Chebyshev polynomial coefficients at 12 day intervals. These files are based on the Jet Propulsion Laboratory's Definitive Ephemeris (DE) 118 and 200 files.<sup>13</sup>

Solar radiation pressure is also included in the differential equations. It is modeled as follows:<sup>5,14</sup>

$$\bar{F} = \frac{F_s M}{r_{\text{sun-spacecraft}}^3} \bar{r}_{\text{sun-spacecraft}} = \left( \frac{kA}{M} \right) \cos^2(\beta) \left( \frac{S_o D_o^2}{c} \right) \frac{M}{r_{\text{sun-spacecraft}}^3} \bar{r}_{\text{sun-spacecraft}}, \quad (2)$$

where  $M$  is the spacecraft mass,  $\bar{r}_{\text{sun-spacecraft}}$  is the vector from the Sun to the spacecraft, and the scalar variable  $F_s$  is used to represent all other predetermined constants in the model. The scalar quantity  $F_s$  includes information regarding the characteristics of the spacecraft and certain physical constants. For instance, the parameter  $k$  represents the absorptivity of the spacecraft surface over the range  $0 \leq k \leq 2$ ;  $A$  is the effective cross sectional area;  $c$  is the speed of light;  $S_o$  is the solar light flux at 1 A.U. from the Sun;  $D_o$  is the nominal distance associated with  $S_o$ ; and  $\beta$  is the angle of incidence which can be calculated (for Sun radiating radially outward) as follows

$$\beta = \cos^{-1} \left( -\frac{\hat{\mathbf{n}} \cdot \bar{\mathbf{r}}_{\text{sun-spacecraft}}}{r_{\text{sun-spacecraft}}} \right), \quad (3)$$

where  $\hat{\mathbf{n}}$  is the unit vector normal to the incident area. In this study, the solar radiation pressure model will be simplified by assuming that the force is always normal to the surface, i.e.,  $\beta = 0$ . In terms of the spacecraft engine, only impulsive maneuvers are considered. Of course, the analysis must be consistent across all analysis tools.

### Nominal Baseline Trajectory

Assume a mission concept that involves departure from a circular Earth parking orbit and transfer along a direct path to arrive in a halo or Lissajous trajectory associated with an  $L_1$  libration point, defined in terms of a Sun-Earth/Moon barycenter system. Thus, the baseline trajectory is composed of two segments: (a) the Earth-to-halo transfer, and (b) the Lissajous trajectory. The design strategy is based on computing the halo/Lissajous trajectory *first*, since this type of orbit enables the flow (the stable/unstable manifolds) in the region between the Earth and  $L_1$  to be represented relatively straightforwardly in configuration space using the invariant manifolds. An appropriate Lissajous orbit, i.e., one that meets the science and communications requirements, is computed using GENERATOR.<sup>1</sup> A Lissajous trajectory is quasi-periodic; however, two revolutions along the path can be assumed as a nearly periodic orbit for construction of a monodromy matrix. The transfer design process then consists of identifying the subspace (or surface) that flows from the vicinity of the Earth to the Lissajous trajectory by computation of the associated stable manifold. Using the stable manifold to construct the transfer trajectory from Earth implies an asymptotic approach to the “periodic” orbit and, even in actual practice, may result in no insertion maneuver. So, rather than a targeting problem to reach a specified insertion point on the halo orbit, the transfer design problem becomes one of insertion onto the stable manifold, directly from an Earth parking orbit, if possible. The flight time along such a path is actually very reasonable.

Unfortunately, not every halo/Lissajous orbit possesses stable manifolds that pass at the precise altitude of a specified Earth parking orbit. However, the stable/unstable manifolds control the behavior of all nearby solutions in this region of the phase space. Thus, the behavior of the manifolds provides insight into optimal transfers and serves as an excellent first approximation in a differential corrections scheme.<sup>3</sup> Of course, altitude is not the only launch constraint. Once an appropriate initial transfer path is available, a series of patch points (“control points”) are automatically inserted. A two-level iteration scheme then shifts positions and times to satisfy constraints on launch altitude, launch date, and launch inclination as well as placement of the transfer trajectory insertion point as close to perigee as possible.<sup>15</sup> Note that this process for computation of the transfer leaves the Lissajous trajectory intact. This is extremely difficult to accomplish solely in SWINGBY (as it is currently structured).

After the transfer is produced, it is successfully transferred to SWINGBY. Note that, in this process, the transfer path emerges without a random search. Thus, this critical initial approximation is extremely important for the design of more complex missions (that might include phasing loops and/or gravity assists), since transferring to the nominal Lissajous orbit is, in general, a challenge for the trajectory analysts.<sup>16</sup> Once a suitable trajectory associated with a particular Lissajous trajectory is identified, SWINGBY can be further utilized for final adjustments, maneuver error analysis, and exploration of changes in the mission specifications. Understanding both the traditional design methodology and invariant manifold theory demonstrates that a tool that integrates manifold theory into the mission design process is very beneficial. Furthermore, like SWINGBY, this tool must possess an excellent graphical user interface.

## Implementation Issues

For comparison and data exchange between GENERATOR and SWINGBY, it is imperative that a consistent match exists in the following aspects: Coordinate Systems, Time Standards, and Integrators. To accomplish this task, SWINGBY is assumed as the reference and GENERATOR is modified to meet the conditions in the reference as closely as possible.

*Coordinate Systems.* To perform the integrations, the geocentric inertial (GCI) frame is used. This frame is defined with an origin at the Earth's center and an equatorial reference plane. For visualization, the Sun-Earth Rotating (SER) and the Rotating Libration Point (RLP) frames also prove to be invaluable. The SER frame uses an origin at the Earth and an ecliptic reference plane. The RLP frame also defines an origin at the moving libration point ( $L_1$  or  $L_2$ ) and, like the SER frame, uses an ecliptic reference plane.

*Time Standards.* Julian days, in atomic time standard, are assumed to advance the integration. The Julian Date system numbers days continuously, without division of years and months.<sup>17</sup> The atomic time standard is defined in terms of the oscillations of the cesium atom at mean sea level.<sup>5</sup>

*Integrators.* For the numerical integration scheme, a Runge-Kutta-Verner 8(9) integrator is incorporated. This Runge-Kutta integrator is, of course, based on the Verner methodology.<sup>18</sup> The Verner formulas provide an estimate of the local truncation errors that allow the development of an adaptive step size control scheme.<sup>5</sup> It is important to note that, when performing differential corrections, GENERATOR also integrates the 36 first order scalar differential equations from the state transition matrix that is associated with the equations of motion governing the position and velocity states. As a result, a total of 42 equations are simultaneously integrated. Therefore, for adequate error control, the scaling of the variables is very important.

## EXAMPLES

Given sets of mission specifications, two sample trajectories are computed below. The blended procedure is employed to demonstrate its implementation. The results can be compared to known solutions, if available. For the following examples, it is assumed that communication requirements impose minimum and maximum angles of 3 and 32 degrees, respectively, between the Sun/Earth line and the Earth-Vehicle vector (SEV angle) during the transfer from the Earth parking orbit to the vicinity of the libration point. The parking orbit is specified as circular with a 28.5 degree inclination (Earth equatorial) and an altitude of 185 km. (Deep Space Network coverage and shadowing/eclipse constraints will not be considered at this time.)

### SOHO Mission

On December 2, 1995, the Solar Heliospheric Observatory (SOHO) spacecraft was launched. Built by the European Space Agency to study the Sun, SOHO is part of the International Solar-Terrestrial Physics (ISTP) program.<sup>19</sup> To meet the science requirements, SOHO requires an uninterrupted view of the Sun and the minimization of the background noise due to particle flux. A halo/Lissajous orbit similar to the libration point ( $L_1$ ) orbit utilized for the ISEE-3 mission<sup>20</sup> is assumed. The science and communications requirements generate the following Lissajous amplitude constraints:  $A_x = 206,448$  km,  $A_y = 666,672$  km, and  $A_z = 120,000$  km. A Class I (northern) Lissajous, obtained<sup>1</sup> numerically with the appropriate amplitude characteristics, appears in Figure 2.

Given this Lissajous orbit, a transfer trajectory is sought. Initially, a limited set of points is selected along some specified part of the Lissajous trajectory; this specific region along the orbit in Figure 2 is identified as all the points in the shorter arc defined by the symbols "x". It is already known that the manifolds associated with these points will pass close to the Earth. This particular region along the nominal path is designated as the "Earth Access region".<sup>3,9</sup> Each point in the

Earth Access region can be defined as a fixed point  $\bar{X}^H$  and the corresponding one-dimensional stable manifold globalized. Together, these one-dimensional manifolds form a two-dimensional surface associated with this region of the nominal orbit. The projection of this surface onto configuration space appears in Figure 3. (Note that the manifolds in Figure 3 pass closest to the Earth as compared to those associated with any other region along the nominal orbit; altitude is the only characteristic used in determining this region.) From this invariant subspace, the one trajectory that passes closest to the Earth is selected as the initial guess for the transfer path. Some of the notable characteristics of this approximation are listed in Table 1 and a plot appears in Figure 4. Note in Figure 4 that the constraint on the SEV angle is met.

Given the initial guess and utilizing continuation, the transfer is differentially corrected to meet the requirements on the other constraints. This correction process can occur in GENERATOR or SWINGBY, although the methodology differs between the two algorithms; numerical data corresponding to the final solution that appears in Table 1 is from GENERATOR. (A plot of the final solution is indistinguishable from Figure 4.) Although there is no guarantee that this result represents an optimal solution, all constraints have been met and the solution process is automated. This transfer compares most favorably with the transfer solution actually used by SOHO. From this point, the solution is input directly into SWINGBY and appears in Figure 5. SWINGBY can now be used for further analysis including visualization, launch and maneuver error investigations, as well as midcourse corrections. Data can still be exchanged and new transfers computed as needed.

**Table 1**  
**SOHO EXAMPLE: TRANSFER TRAJECTORY DESIGN**

Initial Approximation	
Transfer Trajectory Insertion Date	12/03/95
Closest Approach (Altitude)	5,311 km
Inclination	15.58 degrees
Final Transfer Trajectory	
Transfer Trajectory Insertion Date	12/02/95
Closest Approach (Altitude)	185 km
Inclination	28.5 degrees
Ascending Node	292.63 degrees
Argument of Perigee	145.77 degrees
Transfer Insertion Cost	3193.9 m/s
Lissajous Insertion Cost	33.8 m/s
Time of Flight <sup>a</sup>	204.7 days

<sup>a</sup>The time of flight is calculated as follows: from transfer trajectory insertion until the point along the path such that the vehicle is within 200 km of the nominal Lissajous. This point is indicated in Figure 4 with a symbol '\*'.

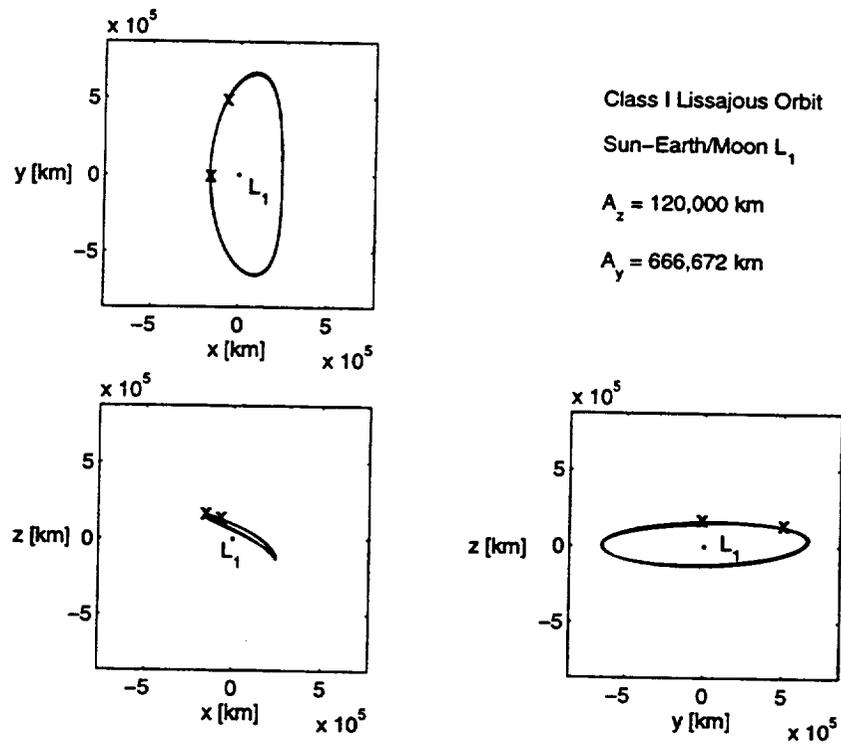


Figure 2 SOHO Example: Nominal Lissajous

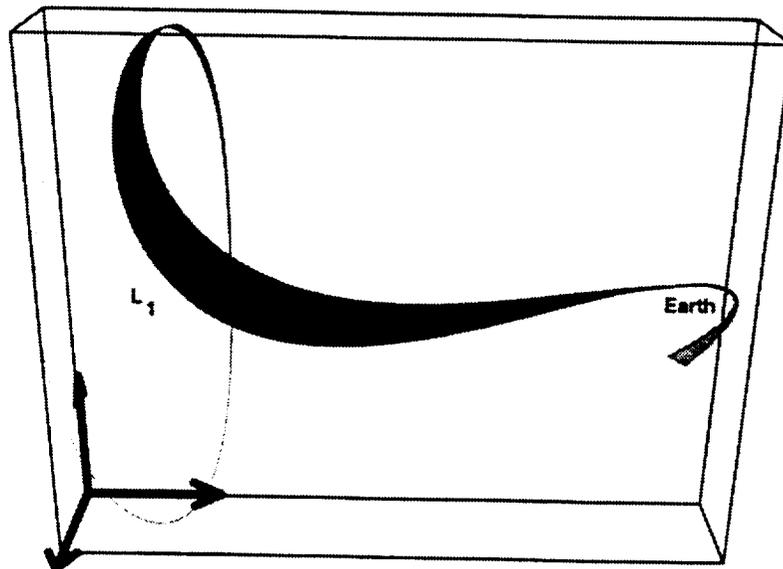


Figure 3 SOHO Example: Manifold Surface Section

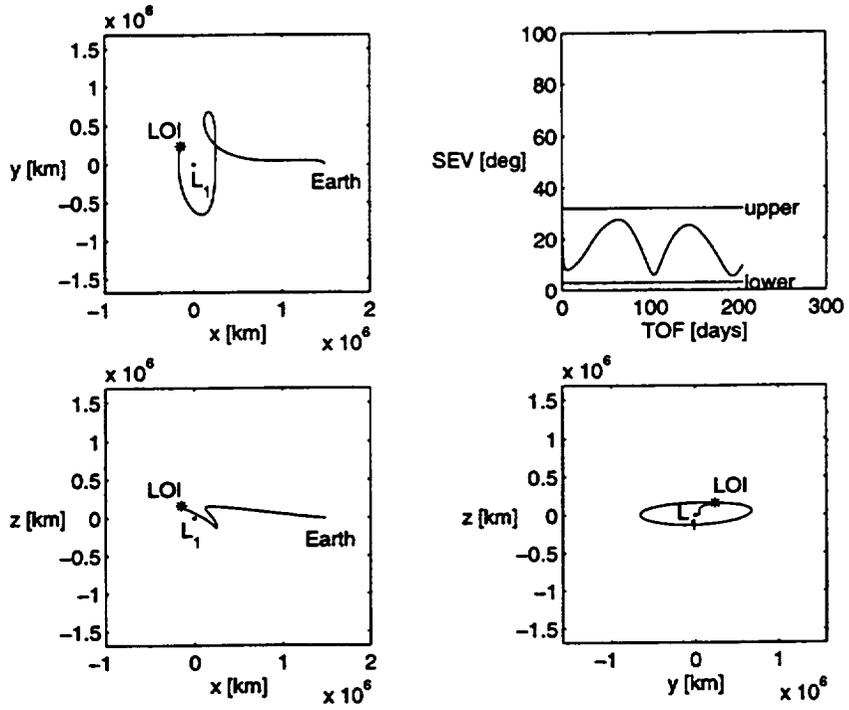


Figure 4 SOHO Example: Initial Approximation

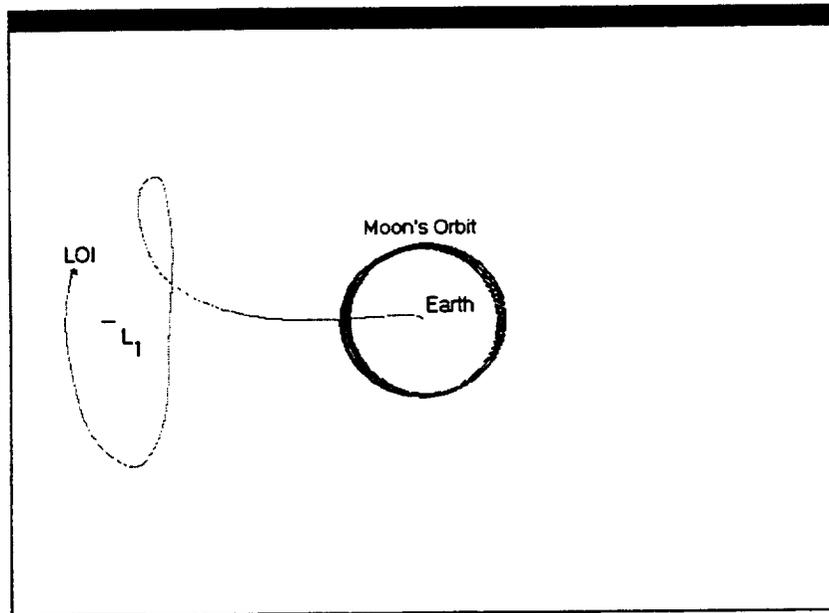


Figure 5 SOHO Example: SWINGBY

## NGST Mission

The Next Generation Space Telescope (NGST),<sup>21</sup> part of the NASA Origins Program, is designed to be the successor to the Hubble Space Telescope. Since for NGST the majority of the observations by the instruments aboard the spacecraft will be in the infrared part of the spectrum, it is important that the telescope be kept at low temperatures. To accomplish this, an orbit far from Earth and its reflected sunlight is desirable. There are several orbits that are satisfactory from a thermal point of view, and, in this study, an orbit in the vicinity of the  $L_2$  point is considered. Based on this information, the following Lissajous amplitudes<sup>21</sup> are incorporated:  $A_x = 294,224$  km,  $A_y = 800,000$  km, and  $A_z = 131,000$  km. A Class I (northern) Lissajous, obtained numerically, with the appropriate amplitude characteristics appears in Figure 6; note that the trajectory is 2.36 years in duration.

Again, given this Lissajous orbit, a transfer trajectory is sought. Using invariant manifold theory, several transfer paths can be computed; a surface is projected onto configuration space and the three-dimensional plot appears in Figure 7. Again, this particular section of the surface is associated with the "Earth Access region" along the  $L_2$  libration point orbit.<sup>4,9</sup> An interesting observation is apparent as motion proceeds along the center of the surface. The smoothness of the surface is interrupted because a few of the trajectories pass close to the Moon upon Earth departure. Lunar gravity was not incorporated into the approximation for the manifolds; but no special consideration was involved to avoid the Moon either. This information concerning the lunar influence can probably be exploited with further development of the methodology. From information available in Figure 7, the one trajectory that passes closest to the Earth is identified and used as the initial guess for the transfer path. Some of the notable characteristics of this approximation are listed in Table 2 and a plot appears in Figure 8.

**Table 2**  
**NGST EXAMPLE: TRANSFER TRAJECTORY DESIGN**

Initial Approximation	
Transfer Trajectory Insertion Date	09/30/2007
Closest Approach (altitude)	-2,520.6 km
Inclination	30.1 degrees
Final Transfer Trajectory	
Transfer Trajectory Insertion Date	10/01/2007
Closest Approach (altitude)	185 km
Inclination	28.5 degrees
Ascending Node	342.65 degrees
Argument of Perigee	210.74 degrees
Transfer Insertion Cost	3195.1 m/s
Lissajous Insertion Cost	15.4 m/s
Time of Flight <sup>b</sup>	210.8 days

<sup>b</sup>The time of flight is calculated as follows: from transfer trajectory insertion until the point along the path such that the vehicle is within 200 km of the nominal Lissajous. This point is indicated in Figure 8 with a symbol '\*'.

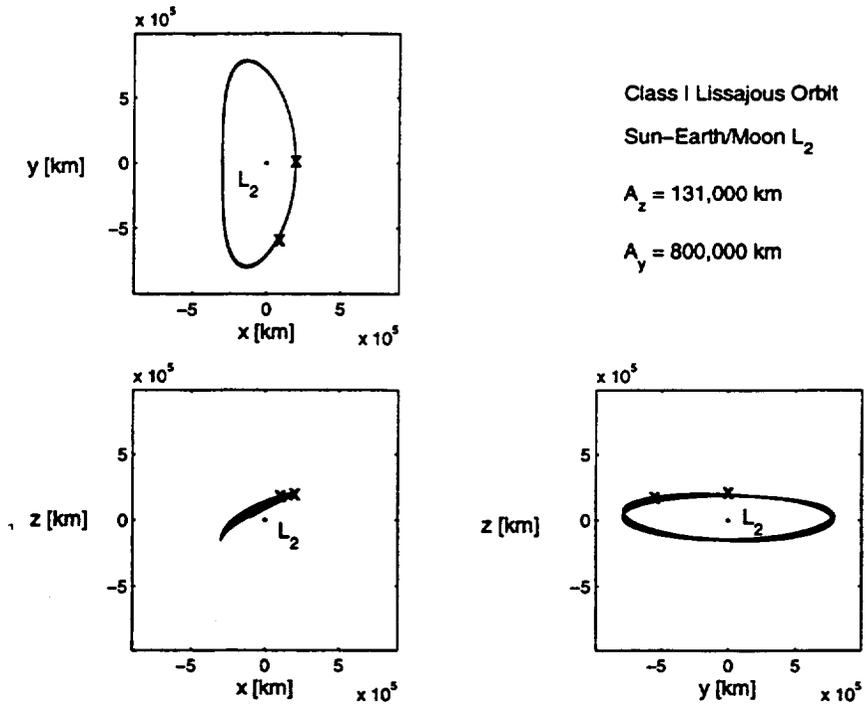


Figure 6 NGST Example: Nominal Lissajous

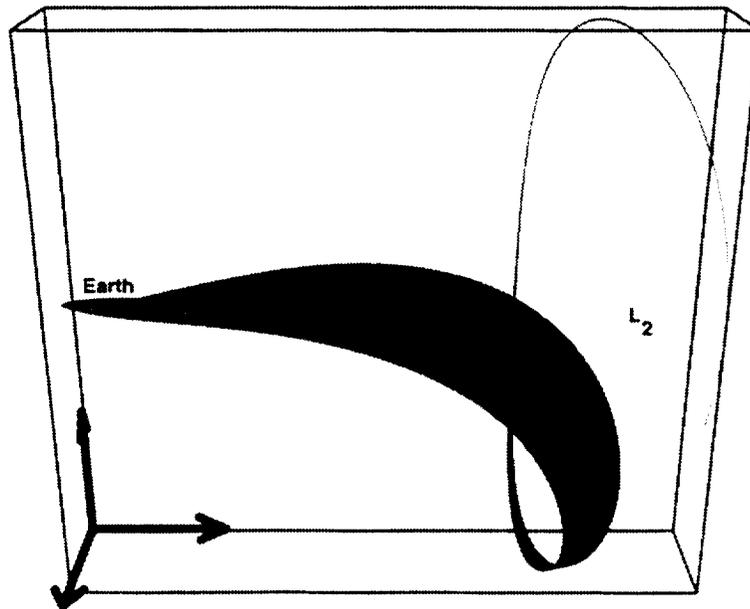


Figure 7 NGST Example: Manifold Surface Section

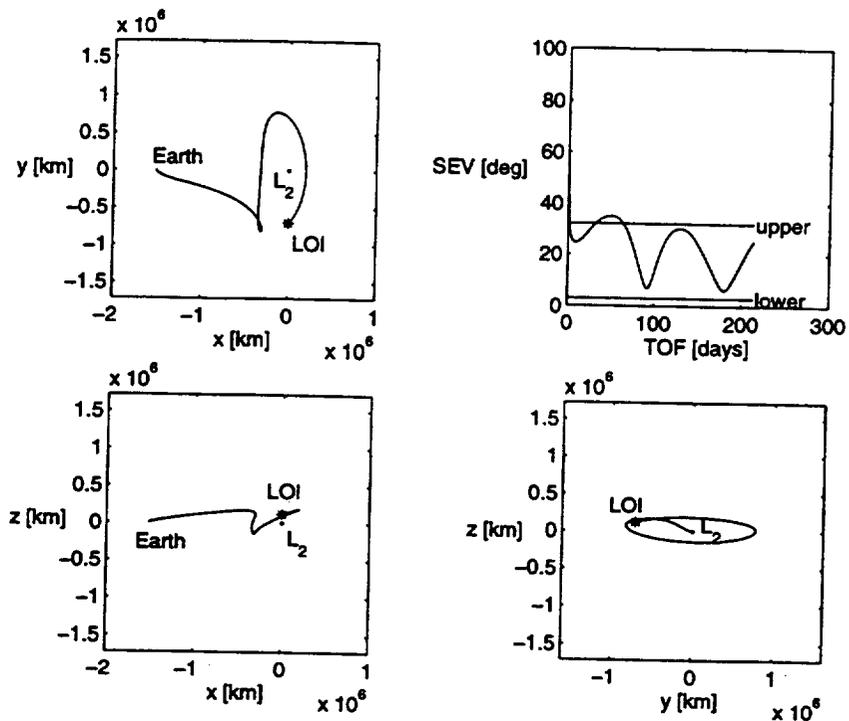


Figure 8 NGST Example: Initial Approximation

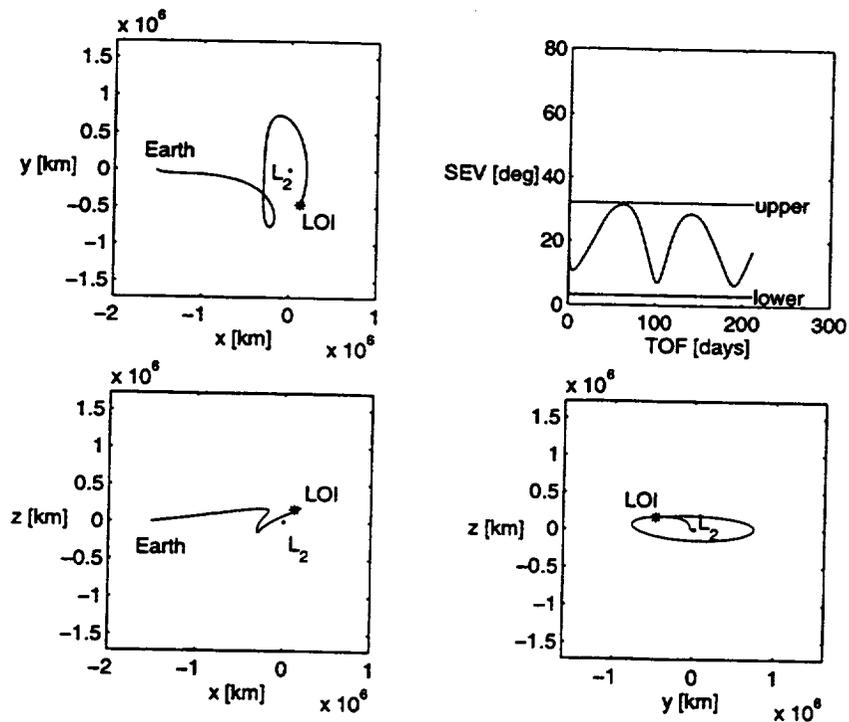
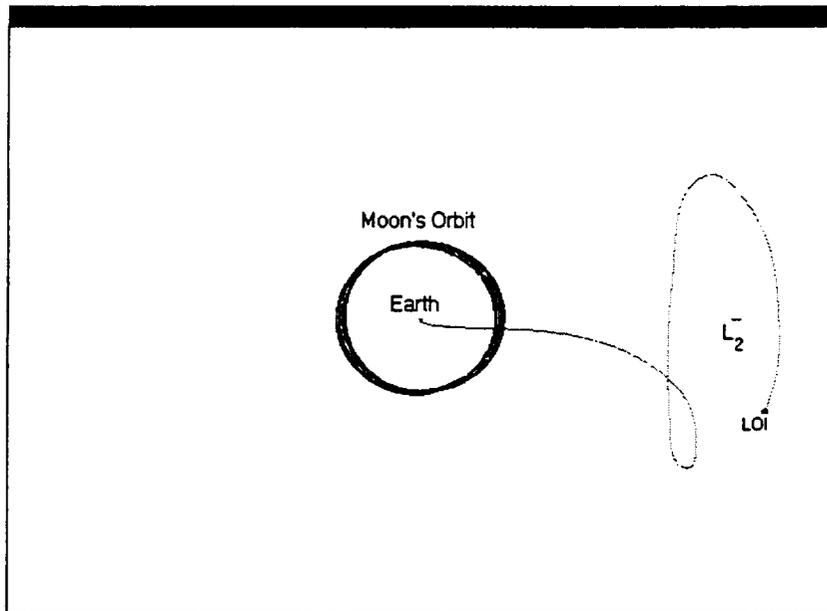


Figure 9 NGST Example: Final Trajectory



**Figure 10 NGST Example: SWINGBY**

Note from Table 2 that this particular approximation passes below the Earth's surface. The larger size of this Lissajous orbit, as compared to the SOHO example, reduces the Earth passage distance. Furthermore, note in Figure 8 that the constraint on the SEV angle is not met. Given the initial guess, the transfer is differentially corrected to meet the requirements on all the constraints except the SEV angle. In this case, after this process, the SEV constraint is met. The SEV constraint could certainly be added to the differential correction process, although it has not yet been incorporated. The final solution as seen in Figure 9 is from GENERATOR. From this point, the solution is input directly into SWINGBY and appears in Figure 10. Similar to the previous example, SWINGBY can now be used for further visualization, analysis of launch and maneuver errors, midcourse corrections, and other investigations.

### CONCLUDING REMARKS

The primary goal of this effort is the blending of analysis from dynamical systems theory with the well established NASA Goddard software program SWINGBY to enhance and expand the capabilities for mission design. Dynamical systems theory provides a qualitative and quantitative understanding of the phase space that facilitates the mission design. SWINGBY can then utilize this information to visualize and complete the end-to-end mission analysis. Combination of these two tools proves to be an important step towards the next generation of mission design software.

### ACKNOWLEDGMENTS

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### REFERENCES

1. K. C. Howell and H. J. Pernicka, "Numerical Determination of Lissajous Trajectories in the Restricted Three Body Problem," *Celestial Mechanics*, Vol. 41, 1988, pp. 107-124.

2. G. Gómez, A. Jorba, J. Masdemont, and C. Simó, "Study of the Transfer from the Earth to a Halo Orbit Around the Equilibrium Point  $L_1$ ," *Celestial Mechanics and Dynamical Astronomy*, Vol. 56, No. 4, 1993, pp. 541-562.
3. K. C. Howell, D. L. Mains, and B. T. Barden, "Transfer Trajectories from Earth Parking Orbits to Sun-Earth Halo Orbits," *AAS/AIAA Spaceflight Mechanics Meeting*, Cocoa Beach, Florida, February 1994. AAS Paper 94-160.
4. K. C. Howell, B. T. Barden, and M. W. Lo, "Application of Dynamical Systems Theory to Trajectory Design for a Libration Point Mission," *Journal of the Astronautical Sciences*, Vol. 45, No. 2, 1997, pp. 161-178.
5. D. McGiffin and C. Roberts, *Mission Analysis and Design Tool (SWINGBY) Mathematical Principles*. Goddard Space Flight Center, Greenbelt, Maryland, September 1995. Revision 1.
6. J. Carrico, D. Conway, D. Ginn, D. Folta, and K. Richon, "Operational Use of SWINGBY-an Interactive Trajectory Design and Maneuver Planning Tool-for Missions to the Moon and Beyond," *AAS/AIAA Astrodynamics Specialists Conference*, Halifax, Nova Scotia, Canada, August 1995. AAS 95-323.
7. B. T. Barden and K. C. Howell, "Formation Flying in the Vicinity of Libration Point Orbits," *AAS/AIAA Spaceflight Mechanics Meeting*, Monterey, California, February 1998. AAS Paper 98-169.
8. K. C. Howell, B. T. Barden, R. S. Wilson, and M. W. Lo, "Trajectory Design Using a Dynamical Systems Approach with Application to GENESIS," *AAS/AIAA Astrodynamics Specialists Conference*, Sun Valley, Idaho, August 4-7 1997. AAS Paper 97-709.
9. B. T. Barden, "Using Stable Manifolds to Generate Transfers in the Circular Restricted Problem of Three Bodies," M.S. Thesis, Purdue University, West Lafayette, Indiana, December 1994.
10. J. Guckenheimer and P. Holmes, *NonLinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. New York: Springer-Verlag, 1983.
11. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*. New York: Springer-Verlag, 1990.
12. T. S. Parker and L. O. Chua, *Practical Numerical Algorithms for Chaotic Systems*. New York: Springer-Verlag, 1989.
13. "Solar, Lunar and Planetary (SLP) Files," <http://fdd.gsfc.nasa.gov/SLP.html>. Goddard Space Flight Center.
14. K. C. Howell and J. L. Bell, "Solar Radiation Pressure and Sun-Earth  $L_1$  Libration Point Orbits," School of Aeronautics and Astronautics, Purdue University, Technical Report, West Lafayette, Indiana, 1992.
15. K. C. Howell and R. S. Wilson, "Trajectory Design in the Sun-Earth-Moon System Using Multiple Lunar Gravity Assists," *AAS/AIAA Astrodynamics Specialists Conference*, San Diego, California, pp. 630-640, July 1996. AIAA Paper 96-3642.
16. K. Richon and M. Mathews, "An Overview of the MAP Trajectory Design," *AAS/AIAA Astrodynamics Specialists Conference*, Sun Valley, Idaho, August 1997. AAS Paper 97-728.
17. *Cambridge Astronomy Dictionary*. New York, New York: Cambridge University Press, 1996.
18. J. H. Verner, "Explicit Runge-Kutta Methods with Estimates of the Local Truncation Error," *Society for Industrial and Applied Mathematics (SIAM) Journal of Numerical Analysis*, Vol. 15, August 1978, pp. 772-790.

19. S. Stalos, D. Folta, B. Short, J. Jen, and A. Seacord, "Optimum Transfer to a Large Amplitude Halo Orbit for the Solar and Heliospheric Observatory (SOHO) Spacecraft," *AAS/GSFC International Symposium on Space Flight Dynamics*, Greenbelt, Maryland, April 1993. AAS Paper 93-553.
20. R. W. Farquhar, D. P. Muhonen, C. R. Newman, and H. S. Heuberger, "Trajectories and Orbital Maneuvers for the First Libration-Point Satellite," *Journal of Guidance and Control*, Vol. 3, No. 6, 1980, pp. 549-554.
21. "NGST Homepage," <http://ngst.gsfc.nasa.gov/>. Goddard Space Flight Center.

