Image Tiling for Profiling Large Objects

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Abstract

Three dimensional surface measurements of large objects are required in a variety of industrial processes. The nature of these measurements is changing as optical instruments are beginning to replace conventional contact probes scanned over the objects. A common characteristic of the optical surface profilers is the trade off between measurement accuracy and field of view. In order to measure a large object with high accuracy, multiple views are required. An accurate transformation between the different views is needed to bring about their registration. In this paper, we demonstrate how the transformation parameters can be obtained precisely by choosing control points which lie in the overlapping regions of the images. A good starting point for the transformation parameters is obtained by having a knowledge of the scanner position. The selection of the control points are independent of the object geometry.

By successively recording multiple views and obtaining transformation with respect to a single coordinate system, a complete physical model of an object can be obtained. Since all data are in the same coordinate system, it can thus be used for building automatic models for free form surfaces.

When large objects have to be modeled, multiple views are needed to get complete surface information. Different views with overlapping regions must be integrated to form the solid object. This involves determining the transformation between the different views. This process is called the “solving for the correspondence problem” and can be accomplished by registering the multiple views. There has been work done on registration in the computer vision community[2, 3, 4]. In most of these cases, the object is moved relative to the sensor, without taking advantage of the object’s geometry for choosing vantage points. It is also important to focus on free form surfaces instead of classical surfaces to more accurately serve the needs of industry.

The objective of this work is to provide a starting point for the development of a system that would ultimately automatically identify overlapping regions and perform the registration without requiring precision fixtures or precise sensor motion control. The system should not be dependent on the object geometry and should be able to handle any free form surface.

We approach this problem by assuming that a good initial approximate transformation is available. This assumption prevents the need to search the entire transformation parametric space. This initial transformation can be obtained by tracking the camera position or by using machine vision techniques to identify surface features common to each view. Knowing corresponding points permits the use of the alignment method to obtain the initial transformation [5]. A good starting point brings the views to approximate registration and helps the registration algorithm find the global minimum.

Our approach is based on the method developed
Fig 1. The cross section of a plane containing ball bearings is shown in the overlapping region after surface A has been transformed with the registration transformation. The error in the overlapping region is plotted.

by Chen and Medioni[6]. The registration algorithm refines the initial six parameters, three in rotation and three in translation, using Newton's method of linearization and iteration to perform least-square minimization. Arbitrarily selected control points are chosen from the overlapping region to estimate the correction to the initial transformation. The distance of the control points from the reference surface is used as the error function.

To begin our study we used synthetic surface data. This data simulates range images from a laser scanner. The data consist of 640x480 element arrays containing the simulated object height \( Z = f(x,y) \) from the reference plane, where \( x \) and \( y \) are the pixel coordinates. We have initially addressed translation and rotation transformations, and intend to incorporate scale changes in the future.

II. Registration

The registration of multiple view range images have applications in 3D object recognition, terrain mapping, medical imaging and industrial inspection. Many methods that have been used utilize some kind of feature correspondence to obtain the transformation between views. Such methods require knowledge of point to point correspondence between surfaces and are very difficult for objects with free form surfaces.

Ideally, if the rigid transformation between the different views is known, there will be no error between the surfaces in the overlapping region. Figure 1. shows cross sections through the overlapping regions of two surfaces that have been registered. The error plot does not have discontinuity which is an indication of good registration. If we have two surfaces \( A \) and \( B \) that represent the same object, there exists a rigid transformation \( T \) between them such that

\[
\forall (a \in A, b \in B) \| Ta - b \| = 0
\]

i.e., when points \( a \) are transformed with a rigid transformation \( T \), all transformed as in the overlapping region match precisely with the corresponding point \( b \) of \( B \). \( T \) is of the form

\[
T =
\begin{bmatrix}
\cos \alpha \cos \gamma & \sin \alpha \sin \gamma & -\cos \alpha \sin \gamma \\
\cos \beta \sin \gamma & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & -\sin \beta \\
0 & \sin \beta \cos \beta & 1
\end{bmatrix}
\]

where \( t_x, t_y \) and \( t_z \) are the translation parameters and \( \alpha, \beta \) and \( \gamma \) are the rotation parameters. These parameters have to be evaluated such that equation (1) is satisfied. This problem can also be expressed as minimizing

\[
D(A, B) = \sum \| Ta - f(a) \|
\]
The distance $d_s$ of the transformed control point from the plane passing through the matching point on surface $B$ is shown. The normal at the control point is $n_{cp}$ and $n_s$ is the normal to the tangent plane.

**A. Selecting a criterion for registration**

When two surfaces are in registration, the distance between the surfaces in the overlapping region must ideally be zero. Since the ideal rigid transformation for registration is unknown, the registration parameters are evaluated such that the square of the distance between the two surfaces is a minimum. For $\mathbf{a} \in A$ and $\mathbf{b} \in B$

$$e = \sum d_s(\mathbf{T}_a - \mathbf{b})$$

(4)

where $d_s$ is the distance of the translated points of surface $A$ from $B$. If $N$ control points form surface $A$ are chosen and transformed, the corresponding points on $B$ must be identified to estimate the distance error. Points $b_j$ should be found such that $\sum d_s(\mathbf{T}_a - \mathbf{b})^2$ is a minimum for all $i=1$...$N$. Starting with a good initial transformation $\mathbf{T}^0$, an iterative algorithm can be formulated. For each iteration $k$, the previous value of $\mathbf{T}^{k-1}$ is used to find $b_j^k$:

$$e^k = \sum_{i=1}^N \| T^k a_i - b_j^k \|^2$$

(5)

with

$$b_j^k = b_{\text{min}} \| T^{k-1} a_i - \mathbf{b} \|$$

The corresponding points on $B$ must have approximately the same $z$ coordinate and the normals should have nearly the same slope. Chen and Medioni used the surface normal of the transformed control points and found the intersection on the reference surface by an iterative method. In our approach, the intersection point is found by searching the coordinate space of the reference surface $B$ in the neighborhood of the transformed control points. The normals can be compared by evaluating the scalar product of the normals of the control point and the matching point (see Figure 3) and seeing how close they are to one. This method avoids the problem of an iterative search.

For each iteration, the transformation matrix is corrected and a better matching point is found. The only restriction on this method is that the control points should be chosen in the overlapping region. The estimation of the correction to the registration
parameters is discussed below.

**B. Determining the correction to the parameters**

Let the control points chosen from surface A be \( a_{cp} \) and let \( a_{cp}' \) be the control points after they have been transformed by \( T \).

\[
a_{cp}' = T \cdot a_{cp}
\]  

Let the equation of the tangent plane at the approximate intersection point be

\[
Ax + By + Cz + D = 0
\]  

Each point from \( a_{cp}' \) should ideally lie on this plane. By substituting the values of \( x, y \) and \( z \) of \( a_{cp} \) from (7), the registration parameters have to be estimated. Instead of solving directly for the nonlinear parameters, we can use Newton's method to estimate the correction parameters as formulated by Lowe[7].

Let \( e \) be the error measurement between the reference image and the translated control points \( a_{cp}' \). Let \( h \) be the correction that has to be made to the parameters \( p \). Since we are in approximate registration, the assumption of local linearity is valid. Based on this assumption, the effect of each parameter correction \( h_i \) on the error measurement will be \( h_i \) multiplied by partial derivative of error with respect to that parameter.

\[
Jh = e
\]  

where \( J \) is the Jacobian matrix

\[
J_{ij} = \frac{\partial e_i}{\partial p_j}
\]  

Since the parameters are close to registration, small angle approximations can be made. The Jacobian matrix is given below

\[
\frac{\partial e_i}{\partial \alpha} = Bz - Cy, \quad \frac{\partial e_i}{\partial y} = Cx - Az, \quad \frac{\partial e_i}{\partial y} = Ay - Bx \\
\frac{\partial e_i}{\partial x} = A, \quad \frac{\partial e_i}{\partial y} = B, \quad \frac{\partial e_i}{\partial z} = C
\]  

Each measured error equals the sum of all changes in the error resulting from the parameter correction. By satisfying all these constraints simultaneously, the error can be reduced to zero. The number of control points must be greater than the number of parameters to be estimated. The more the number of points taken for estimation, the estimate will be more accurate. The correction \( h \) is estimated such that the residue

\[
\min ||Jh - e||
\]

is minimized.

This can be reduced to the form of normal equation

\[
h = (J^T J)^{-1} J^T e
\]  

For each iteration, \( h \) is evaluated and is subtracted from the parameters of the previous iteration.

\[
p^{(k+1)} = p^{(k)} - h
\]  

This assumes that the original nonlinear function is locally linear over the range of typical errors.

**C. The Registration Algorithm**

Take two surfaces \( A \) and \( B \) and consider \( B \) as the reference surface. N Control points are chosen from the overlapping region of \( A \) and are transformed by initial transformation \( T^0 \). The equation (6) now becomes

\[
e^k = \sum_{i=1}^{N} a_{cp}^2 (T \cdot T^{k-1} a_{cp}^i \cdot b_{kj})
\]  

where:

\( T \cdot T^{(k-1)} = T^k \) is the transformation matrix after \( k \)th iteration.

\( S_j^k \) is the tangent plane on surface \( B \) at the \( b_j^k \).

\( b_j^k \) is the matching point on \( B \) such that it has nearly the same z coordinate and \( n_{cp}' \cdot n_s \equiv 1 \).

\( n_{cp}' \) is the transformed normal of the control point and

\( n_s \) is the normal to the plane at \( S_j^k \).

\( n_{cp}' \cdot n_s \) is the scalar product of the control point normal and the surface normal at \( B \).

\( a_{cp} \) are the control points from \( A \).
\( d_i \) is the signed distance of \( \alpha_{cp}^i \) from the plane \( S_j^k \).

The registration algorithm is as follows:

1. Choose control points \( \alpha_{cp} \) from \( A \) such that they lie in the overlapping region. Compute the surface normal at each of these control points and store their position and orientation.
2. For each iteration repeat the following:
   i. For each control point
      - Apply \( T^{k-1} \) to \( \alpha_{cp}^i \) and all its normals.
      - Find the matching points \( b_j^i \) on surface \( B \) by comparing the z coordinate and the normals.
      - Find the error between \( \alpha_{cp}^i \) and the tangent plane at \( b_j^i \).
      - Find Jacobian
   ii. Find the correction parameters \( h \)
   iii. \( T^k = T \cdot T^{k-1} \)

The convergence measure is defined as
\[
\delta = \frac{|e^k - e^{k-1}|}{N} \leq \varepsilon \quad (e > 0) \quad (15)
\]

The threshold \( \varepsilon \) is a direct reflection of the noise level of the range image and \( N \) is the number of control points used.

D. Control Points

The registration algorithm uses control points in the overlapping region. Ideally, six control points are enough to estimate the six parameters, but the system of equations is usually overdetermined for a better estimate of parameters. One of the important features of this algorithm is that the choice of control points is not dependent on the object geometry. The only restriction on their selection is that they should lie in the overlapping region because matching points on the surfaces are located there. The control points have been chosen manually from the smooth regions of the surface to be transformed. It is most likely that the reference surface will be smooth in the corresponding region. The number of control points chosen were between 25 and 50 depending on the noise in the data and the extent of overlap of the surfaces. The registration is better with larger number of control points but the time taken for the estimation increases. A study of optimum number of control points has not been made as it will depend on the complexity of the object geometry.

III. Integration of multiple views

In order to integrate the different views in the overlapping region, we averaged the z values at particular values of \( x \) and \( y \) after performing the registration. Since the range measurements are of the form \( Z = f(x, y) \), the surface patches can be tiled together to get the complete object description. One surface is considered to be the reference surface. By registering the other images with respect to the reference image, all the images are brought to one reference coordinate system. This problem can be solved by bringing about registration between different views at the same time. In this manner, information of all previous views are used instead of just the neighboring views.

IV. Results and discussion

Synthetic 8-bit, 640x480 range images were generated to test the registration algorithm. Gaussian noise with a standard deviation of 0.945 was added to these images to simulate conditions close to real range data. The algorithm was first tested on two frames of data simulating a plane surface studded with hemispheres. The two views have three columns of hemispheres in the overlapping region. Figure 4 shows the two images to be registered. We need to have prior knowledge of the approximate region of overlap as more than one solution for this problem exists. The actual transformation between the two views was 200 pixels in the \( x \) direction, 0 in the \( y \) direction, and 20 units in \( z \). The initial guess was given with an offset of up to 21 pixels in the neighborhood of the actual transformation. The actual transformation between different views were 200 pixels in the \( x \) direction, 0 in the \( y \) direction and 20 units in \( z \). The initial guess were given as

\[
\begin{bmatrix}
1 & 0 & 0 & 210 \\
0 & 1 & 0 & 8 \\
0 & 0 & 1 & 30 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

and the final registration transformation was found to be

\[
\begin{bmatrix}
0.999989 & -0.006532 & 0.004343 & 199.7546 \\
0.006473 & 0.999890 & -0.001103 & 0.4576 \\
0.004205 & 0.00473 & 0.998999 & 19.7856 \\
0.0 & 0.0 & 0.0 & 1.0
\end{bmatrix}
\]
Fig 4. The two images of the plane with the ball bearings is shown. They have three columns of ball bearings in the overlapping region.

Fig 5(a). The error image in the region of overlap is shown. For a perfect registration, the image will be completely dark. (b) The histogram of the residual error in the overlapping region.

The algorithm converged in 4 iterations. It can be seen that the algorithm works well for pure translation. Figure 5 shows the error image in the overlapping region and a plot of the histogram in the overlapping region.

Another test case was taken which involved rotation. Figure 6(a) shows an object which is asymmetrical and (b) shows the same object which has been rotated about its axis. The initial guess was given as an identity matrix and the registration matrix was found to be

$$T_{reg} = \begin{bmatrix} 0.984697 & 0.174148 & -0.006636 & 2.801439 \\ -0.174012 & 0.984589 & 0.017416 & -2.108876 \\ 0.009567 & -0.015994 & 0.999826 & 0.526710 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

The error image after registration is shown in figure 6(c). The histogram of the error is shown in figure 6(d). The angle by which the object is rotated comes to about 9 degrees.
Fig 6. The object shown in (a) is taken to test the capability of the algorithm for rotation. The object rotated about its own z axis is shown in (b). (c) The error image after registration. (d) The histogram of the z error in the overlapping region.

The results with the synthetic data show that the algorithm is capable of handling translation and rotation transformations. The algorithm converges to a good registration. This is important because noise is present in the data from all actual sensors. The speed of convergence depends on the amount of noise and the number of control points chosen.

One of the main requirements for this algorithm is the necessity of a good initial transformation. The importance of this can be seen from the first example where the possibility of more than one solution exists. For complicated surfaces, the algorithm can converge to a local solution but miss the global solution. This requirement of a good initial guess is one of the main drawbacks of the algorithm. Either the relative motion between the object and the camera must be monitored, or machine vision techniques must be used to identify features common to the multiple views. Also, the views must have an overlapping region for a solution to be found.
V. Concluding Remarks

This registration algorithm has been shown to be a good starting point for the development of a system to construct a free form surface from multiple sensor views with minimal constraints. Control points are chosen from the overlapping regions of a given surface and the registration transformation is found by minimizing their distance from a reference surface. One of the main advantages of this algorithm is that the choice of control points is not dependent on surface features.

The extension of this algorithm to cover scaling transformations will be studied next to handle errors caused by focus changes between views.

References


