TECHNICAL NOTE

D-1124

A COMPARISON OF THEORY AND OBSERVATION OF THE ECHO I SATELLITE

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON
December 1961
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SUMMARY

Observations of the Echo I balloon satellite have been compared with a theory including the following perturbing effects: (1) solar radiation pressure; (2) lunar and solar gravitation; (3) second, third, and fourth harmonics of the earth's gravitational potential; and (4) atmospheric drag. With a set of orbital elements at the 26th day of the lifetime of the satellite, it was possible to match the observational data to 180 days with root mean square residuals as follows:

\[ \Delta a = 17.9 \text{ km}, \quad \Delta e = 0.002, \quad \Delta i = 0.0177^\circ, \quad \Delta \omega = 1.1231^\circ, \quad \Delta \Omega = 0.4821^\circ, \quad \Delta \text{perigee height} = 7.50 \text{ km}. \] No differential correction has been applied as yet.

Values of atmospheric density between 1500 and 930 km, assuming neutral drag effects only, have been inferred from the orbital data. The connection between solar activity and drag is also examined.

As the Echo I perigee height continues to oscillate between 900 and 1500 km, more valuable orbital data will be obtained and atmospheric properties will be deduced. Further refinements in the mathematical model, especially in a time-dependent model atmosphere, should bring a substantial reduction in the residuals of the observations.
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INTRODUCTION

This report describes a continuation of the study of the orbit of the Echo I satellite (References 1, 2, and 3).

As reported earlier, the primary effect of solar radiation pressure on the Echo I satellite is to change the eccentricity and hence the altitudes of apogee and perigee. The period will also be changed when a portion of the orbit is in the shadow of the earth. The slow changes in period produced by the shadowing must be subtracted from the observed period changes in order to reveal the net effect produced by drag.

Assuming neutral drag effects only, the value of atmospheric density near perigee altitude is inferred from the net rate of change of the orbital period after the changes due to shadowing have been subtracted. Since the perigee altitude descended from 1500 km to 930 km and rose again to nearly 1500 km during the first year of the life of Echo I, it was possible to sample the atmospheric densities in this range of altitudes twice during that period. Thus, we can attempt to separate the time dependence from other effects in the atmosphere. Echo I is unique among the satellites launched thus far because of this characteristic of sampling the same altitude at different times.

COMPARISON OF THEORY AND OBSERVATION

Figure 1 shows a plot of the perigee altitude of Echo I as determined from Minitrack interferometer observations, Smithsonian Astrophysical Observatory Baker-Nunn optical observations, and optical observations obtained from modified calibration cameras at the Minitrack stations. Each orbit correction (i.e., determination of the mean elements from the observations) uses observational data from a four-day interval. The orbit correction
program is based on the modified Hansen satellite theory (Reference 4), and included the effects of the second and fourth harmonics of the earth's gravitational potential.

The solid curve in Figure 1 is the theoretical perigee height obtained from a theory of radiation pressure which includes the shadowing effect (Reference 5) combined with other perturbations produced by the second, third, and fourth harmonics of the earth's gravitational potential, lunar and solar terms, and atmospheric drag. The theory does not allow for radiation re-emitted by the earth. Also, it assumes a simple exponential model of atmospheric density. Several other authors have developed comparable theories. Wyatt (Reference 6) has considered the long-period effects of radiation pressure, and Zadunaisky, Shapiro, and Jones (Reference 7) have given a complete analysis of their data on Echo I in which they have included the above perturbations plus the effect of the fifth gravitational harmonic and the re-emission of radiation from the earth.

The starting point for the theoretical curve in Figure 1 is taken as 26 days after launch, when it is believed that outgassing had been completed and the mass had stabilized.
The rms residual in the perigee altitude is 7.5 km. The rms residuals for the individual orbital elements (data not included here) are: \( \Delta a = 17.9 \) km, \( \Delta e = 0.002 \), \( \Delta i = 0.0177^\circ \), \( \Delta \omega = 1.1231^\circ \), \( \Delta \Omega = 0.4821^\circ \).

Figure 2 shows the rate of change of the period of Echo I versus time, as determined by the orbit computing program at the Goddard Space Flight Center. Also included is a theoretical curve of the rate of change in period produced by the shadowing effect. Energy may be removed from or added to the satellite by this effect, depending on the orientation of the orbit, and it is necessary to separate such energy changes from those produced by drag before conclusions can be drawn about atmospheric density. Figure 2 shows both the total rate of change of period as observed and the net rate of change after the shadowing effect has been removed. The gap in the data during December is a period between the cessation of the Minitrack transmission and the receipt of routine observations from the Minitrack stations.

**SOLAR ACTIVITY AND ATMOSPHERIC DENSITY**

The data in Figure 2 show pronounced peaks in the rate of change of period centered on the dates of November 14 and December 5. Major solar flares were observed at approximately these same times.

If neutral drag only is assumed, atmospheric densities can be inferred from the rates of change of period. Figure 3 shows the densities obtained from the period changes of Echo I by application of the King-Hele formula (Reference 8). This formula relates atmospheric density at half a scale height above perigee height to the rate of change of period. By iterative application of this formula an apparent scale height of 260 km was obtained.

It must be pointed out in considering these density values that the atmosphere cannot be considered static at high altitudes. The diurnal effect can produce changes in density of as much as a factor of 30, and the effects associated with solar activity can produce changes of as much as a factor of 4 and possibly much greater. The seasonal effect may be as much as a factor of 1.5 (References 8, 9, and 10).

The diurnal effect could negate the validity of the formula for obtaining densities from the rates of change of period since the derivation of the formula is based on the assumption that most of the drag occurs in the vicinity of perigee. It is conceivable that a portion of the orbit other than the portion near perigee might be in the diurnal bulge and that the drag force could be greater along that portion than along the portion near perigee. For the period covered in this study, the local time at perigee changes from 18 hours to 11 hours; and it is considered, since the maximum diurnal effect is at about 14
Figure 2 – Rate of change of the period of Echo 1
Figure 3 - Atmospheric densities obtained from period change of Echo I

hours, that the perigee portion of the orbit is near enough to the diurnal bulge to sustain the validity of the densities obtained by the formula.

Figure 4 shows the flux of solar radiation at 20 cm, which constitutes a good indication of the level of solar activity. According to Figure 4, solar activity showed a downward trend from a flux of approximately 180 on the 14th of August to 120 on the 2nd of November. Jacchia (Reference 11) has obtained an empirical relation between atmospheric density and the 20-cm flux by analysis of several satellite orbits with perigee altitudes ranging between 200 and 700 km. According to Jacchia, the density is proportional to the first power of the 20-cm flux.

Priester (Reference 9) analyzed the Echo I data and found that the density is proportional to the square of the 20-cm flux at altitudes between 1000 and 1500 km. Through
analysis of the density variations in Figure 3, approximately the same relationship as that reported by Priester was found. The upper curve represents the density as perigee is descending, and the lower curve as perigee is rising. By using altitudes of 1065, 1040, 1020, 985, and 965 km (Figure 4), and the average flux values at the two times the satellite reached each of these altitudes, and adjusting for the diurnal effect by the Bonn density formula (Reference 12), we obtain values of 1.5, 8.5, 1.8 and 2.0 for the power law exponent in the density-flux relation. The value 8.5 is obtained at the epoch of December 2 and is undoubtedly influenced by the flare activity.

ACKNOWLEDGMENTS

The author is indebted to Dr. Robert Jastrow for valuable discussions on this problem. He also wishes to thank Robert Devaney and Arthur Smith, Jr., who carried out the orbit corrections and programmed the necessary theory for a large-scale computer.
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