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MEMORANDUM

CHARTS AND TABLES FOR ESTIMATING THE STABILITY OF THE COMPRESSIBLE LAMINAR BOUNDARY LAYER WITH HEAT TRANSFER AND ARBITRARY PRESSURE GRADIENT

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SUMMARY

The minimum critical Reynolds numbers for the similar solutions of the compressible laminar boundary layer computed by Cohen and Reshotko and also for the Falkner and Skan solutions as recomputed by Smith have been calculated by Lin's rapid approximate method for two-dimensional disturbances. These results enable the stability of the compressible laminar boundary layer with heat transfer and pressure gradient to be easily estimated after the behavior of the boundary layer has been computed by the approximate method of Cohen and Reshotko.

The previously reported unusual result (NACA Technical Note 4037) that a highly cooled stagnation point flow is more unstable than a highly cooled flat-plate flow is again encountered. Moreover, this result is found to be part of the more general result that a favorable pressure gradient is destabilizing for very cool walls when the Mach number is less than that for complete stability. The minimum critical Reynolds numbers for these wall temperature ratios are, however, all larger than any value of the laminar-boundary-layer Reynolds number likely to be encountered. For Mach numbers greater than those for which complete stability occurs a favorable pressure gradient is stabilizing, even for very cool walls.

INTRODUCTION

In reference 1 a useful method for calculating the compressible laminar boundary layer with heat transfer and arbitrary pressure gradient is presented. This method is based on the similar solutions of the laminar boundary-layer equations obtained in reference 2.

Because of the importance of the problem of transition from laminar to turbulent flow, it is often desirable to have an estimate of the stability of the laminar boundary layer. In order to obtain such an estimate easily, the minimum critical Reynolds numbers for the similar solutions presented in references 2 and 3 have been calculated for the Mach number range between 0 and 2.8 by the rapid approximate method of reference 4. The results are presented in tables and charts so that, after a calculation of the laminar boundary has been made by the method of reference 1, the distribution of the minimum critical Reynolds number over the surface can be easily estimated. The present investigation is limited to two-dimensional disturbances. (See ref. 4 for a discussion of three-dimensional disturbances.)

The distribution of the minimum critical Reynolds number and the distribution of the boundary-layer Reynolds number enables the stability of the laminar boundary layer with respect to the small-disturbance Tollmien-Schlichting type of waves (ref. 4) to be estimated. The boundary layer is stable when the boundary-layer Reynolds numbers are less than the minimum critical Reynolds numbers and unstable when they are greater. If the boundary layer is unstable, the Tollmien-Schlichting waves will amplify and eventually cause transition somewhere downstream of the location where the boundary layer first becomes unstable.

It is known that, even though the boundary layer is stable, transition can still occur if surface imperfections or other sources of disturbances generate disturbances sufficiently large to be outside the scope of the linear theory (ref. 4) or if the type of disturbances that lead to transition are different from those postulated (for example, see ref. 5). Moreover, experiments seem to indicate that extreme cooling may cause early transition (ref. 6) although the theory based on the Tollmien-Schlichting type of waves predicts that the laminar boundary layer on a very cool surface is stable; this phenomenon is not understood at present.

SYMBOLS

$\bar{A}$ constant

$\bar{a}$ velocity of sound

$\bar{c}$ wave velocity of disturbance

$c = \dfrac{\bar{c}}{\bar{u}_e}$

$\bar{c}_p$ specific heat at constant pressure

$f = \bar{\psi}\sqrt{\dfrac{m + 1}{2\bar{\nu}_0\bar{U}_e X}}$

$\bar{h}_s = \bar{c}_p\bar{t} + \dfrac{\bar{u}^2}{2}$

$\bar{h}_0 = \bar{c}_p\bar{t}_0$

m exponent from $\bar{U}_e = \bar{A}X^m$ (ref. 2)

M_e local Mach number at outer edge of boundary layer, $\dfrac{\bar{u}_e}{\bar{a}_e}$

$n \equiv - \dfrac{\dfrac{d\bar{U}_e}{dX}\,\bar{\theta}_{tr}^2}{\bar{\nu}_0}$ correlation number (ref. 1)

$R_{\delta^*,c}$ minimum critical boundary-layer Reynolds number based on displacement thickness $\bar{\delta}^*$

$R_{\theta,c}$ minimum critical boundary-layer Reynolds number based on momentum thickness $\bar{\theta}$

S enthalpy function, $\dfrac{\bar{h}_s}{\bar{h}_0} - 1$

$\bar{t}$ temperature

$t = \dfrac{\bar{t}}{\bar{t}_e}$

$\bar{u}$ velocity component parallel to surface

$u = \dfrac{\bar{u}}{\bar{u}_e}$

$\bar{U} = \dfrac{\bar{u}\bar{a}_0}{\bar{a}_e}$, transformed velocity component parallel to surface (ref. 2)

$U = \frac{\bar{U}}{\bar{U}_e}$

X transformed distance along wall (ref. 2)

$\bar{y}$ distance from wall

$y = \frac{\bar{y}}{\bar{\theta}}$

Y transformed distance from wall (ref. 2)

$\beta = \frac{2m}{m + 1}$, pressure gradient parameter

γ ratio of specific heats (taken equal to 1.4)

$\bar{\delta}$ boundary-layer thickness

$\bar{\delta}^*$ boundary-layer displacement thickness, $\bar{\delta}^* = \int_0^\infty \left(1 - \frac{\bar{\rho}\bar{u}}{\bar{\rho}_e\bar{u}_e}\right)d\bar{y}$

$\eta = \frac{Y}{X}\sqrt{\frac{m + 1}{2} \frac{\bar{U}_e X}{\bar{\nu}_0}}$, similarity variable

$\bar{\theta} = \int_0^\infty \frac{\bar{\rho}\bar{u}}{\bar{\rho}_e\bar{u}_e}\left(1 - \frac{\bar{u}}{\bar{u}_e}\right)d\bar{y}$, boundary-layer momentum thickness

$\bar{\theta}_{tr} = \int_0^\infty \frac{\bar{U}}{\bar{U}_e}\left(1 - \frac{\bar{U}}{\bar{U}_e}\right)dY$, transformed momentum thickness (ref. 1)

$\Lambda = \int_0^\infty f'(1 - f')d\eta$

$\bar{\mu}$ viscosity

$\bar{\nu}$ kinematic viscosity

$\bar{\rho}$ density

N_{Pr} Prandtl number

$$\phi = -\frac{\pi f_w''}{t_w^2}\left[\frac{f't}{f''^3}(tf''' - 2t'f'')\right]$$

$\bar{\Psi}$ stream function (ref. 2)

Subscripts:

e at outer edge of boundary layer

0 stagnation value outside boundary layer

c at critical layer inside boundary layer, where $\bar{u} = \bar{c}$

∞ value at which $R_{\theta,c} = \infty$ when $f_c' = 1 - \frac{1}{M_e}$

w value at surface

Primes denote differentiation with respect to η. Barred quantities are dimensional and X, Y are dimensional.

ANALYSIS

Derivation of Equations

In order to calculate the minimum critical Reynolds numbers for the similar solutions of references 2 and 3, equations (5.4.3) and (5.4.4) of reference 4 are used; these equations can be written as

$$R_{\delta^*,c} = \frac{25\left(\dfrac{\partial\dfrac{\bar{u}}{\bar{u}_e}}{\partial\dfrac{\bar{y}}{\bar{\delta}}}\right)_w \dfrac{\bar{\delta}^*}{\bar{\delta}}\left(\dfrac{\bar{t}}{\bar{t}_e}\right)_c^{1.76}}{\left(\dfrac{\bar{c}}{\bar{u}_e}\right)^4\sqrt{1 - M_e^2\left(1 - \dfrac{\bar{c}}{\bar{u}_e}\right)^2}} \tag{1}$$

and

$$\frac{-\pi\left(\dfrac{\partial\dfrac{\bar{u}}{\bar{u}_e}}{\partial\dfrac{\bar{y}}{\bar{\delta}}}\right)_w\left(\dfrac{\bar{c}}{\bar{u}_e}\right)}{\left(\dfrac{\bar{t}}{\bar{t}_e}\right)_w}\left[\frac{\left(\dfrac{\bar{t}}{\bar{t}_e}\right)^2}{\left(\dfrac{\partial\dfrac{\bar{u}}{\bar{u}_e}}{\partial\dfrac{\bar{y}}{\bar{\delta}}}\right)^3}\frac{\partial}{\partial\dfrac{\bar{y}}{\bar{\delta}}}\left(\dfrac{\dfrac{\partial\dfrac{\bar{u}}{\bar{u}_e}}{\partial\dfrac{\bar{y}}{\bar{\delta}}}}{\dfrac{\bar{t}}{\bar{t}_e}}\right)\right]_{\bar{u}=\bar{c}} = 0.58 \tag{2}$$

when $M_e \geqq 1$, the supplementary condition $\dfrac{\bar{c}}{\bar{u}_e} \geqq \left(1 - \dfrac{1}{M_e}\right)$ (see eq. (5.3.24) of ref. 4) must also be satisfied. It is remarked that the quantity 0.76 in the exponent 1.76 in equation (1) follows from the use of a power law for the viscosity, with exponent equal to 0.76, in the derivation of equation (1).

When the reference length is changed from $\bar{\delta}$ to $\bar{\theta}$, equations (1) and (2) can be written as

$$R_{\theta,c} = \frac{25\left(\dfrac{\partial u}{\partial y}\right)_w t_c^{1.76}}{c^4\sqrt{1 - M_e^2(1 - c)^2}} \qquad \left(\begin{matrix} c \geqq 1 - \dfrac{1}{M_e} \\ \text{when } M_e \geqq 1\end{matrix}\right) \tag{3}$$

and

$$\frac{-\pi\left(\frac{\partial u}{\partial y}\right)_w c}{t_w}\left[\frac{t^2}{\left(\frac{\partial u}{\partial y}\right)^3}\frac{\partial}{\partial y}\left(\frac{\frac{\partial u}{\partial y}}{t}\right)\right]_{u=c} = 0.58$$

or

$$\frac{-\pi\left(\frac{\partial u}{\partial y}\right)_w c}{t_w}\left[\frac{t\,\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\,\frac{\partial t}{\partial y}}{\left(\frac{\partial u}{\partial y}\right)^3}\right]_{u=c} = 0.58 \qquad (4)$$

In order to write equations (3) and (4) in the notation of references 1 and 2, note that from references 1 and 2

$$\bar{u} = \frac{\bar{a}_e}{\bar{a}_0}\,\bar{U}$$

and

$$\bar{u}_e = \frac{\bar{a}_e}{\bar{a}_0}\,\bar{U}_e$$

thus

$$\frac{\bar{u}}{\bar{u}_e} = \frac{\bar{U}}{\bar{U}_e}$$

where

$$\bar{U} = \frac{\partial\bar{\psi}}{\partial Y}$$

However,

$$\bar{\psi} = f(\eta)\sqrt{\frac{2\bar{\nu}_0\bar{U}_eX}{m + 1}}$$

(ref. 2) thus,

$$\frac{\partial\bar{\psi}}{\partial Y} = f' \frac{\partial\eta}{\partial Y}\sqrt{\frac{2\bar{\nu}_0\bar{U}_eX}{m + 1}}$$

Therefore,

$$\bar{U} = f' \frac{\partial\eta}{\partial Y}\sqrt{\frac{2\bar{\nu}_0\bar{U}_eX}{m + 1}} \tag{5}$$

where

$$\eta = Y\sqrt{\frac{m + 1}{2}\frac{\bar{U}_e}{\bar{\nu}_0X}}$$

so that

$$\bar{U} = \bar{U}_ef'$$

Then

$$u = \frac{\bar{u}}{\bar{u}_e} = \frac{\bar{U}}{\bar{U}_e} = f' \tag{6}$$

In order to obtain the expression for $\partial u/\partial y$ note that

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \frac{\bar{y}}{\bar{\theta}}} = f'' \frac{\partial \eta}{\partial Y} \frac{\partial Y}{\partial \bar{y}} \bar{\theta} = f'' \sqrt{\frac{m+1}{2} \frac{\bar{U}_e}{\bar{\nu}_0 X}} \frac{\partial Y}{\partial \bar{y}} \bar{\theta} \tag{7}$$

The definition of $\bar{\theta}$ in equation (7) is

$$\bar{\theta} = \int_0^\infty \frac{\bar{\rho}\bar{u}}{\bar{\rho}_e \bar{u}_e}\left(1 - \frac{\bar{u}}{\bar{u}_e}\right) d\bar{y}$$

or

$$\bar{\theta} = \frac{\bar{\rho}_0}{\bar{\rho}_e} \int_0^\infty f'(1 - f') \frac{\bar{\rho}}{\bar{\rho}_0} \, d\bar{y} \tag{8}$$

but

$$\frac{\bar{\rho}}{\bar{\rho}_0} \, d\bar{y} = \frac{\bar{a}_0}{\bar{a}_e} \, dY \tag{9}$$

(eq. 6(b) of ref. 2) so that equation (8) becomes

$$\bar{\theta} = \frac{\bar{\rho}_0}{\bar{\rho}_e} \frac{\bar{a}_0}{\bar{a}_e} \int_0^\infty f'(1 - f') dY$$

but

$$d\eta = dY \sqrt{\frac{m+1}{2} \frac{\bar{U}_e}{\bar{\nu}_0 X}} \tag{10}$$

Then

$$\bar{\theta} = \frac{\dfrac{\bar{\rho}_0}{\bar{\rho}_e}\,\dfrac{\bar{a}_0}{\bar{a}_e}}{\sqrt{\dfrac{m + 1}{2}\,\dfrac{\bar{U}_e}{\bar{\nu}_0 X}}} \int_0^\infty f'(1 - f')d\eta \tag{11}$$

When equations (9), (10), and (11) are used, equation (7) becomes

$$\frac{\partial u}{\partial y} = \frac{\bar{\rho}}{\bar{\rho}_e} f'' \int_0^\infty f'(1 - f')d\eta \tag{12}$$

But

$$\frac{\bar{\rho}}{\bar{\rho}_e} = \frac{\bar{t}_e}{\bar{t}}$$

where

$$\frac{\bar{t}}{\bar{t}_e} = t = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S) - \frac{\gamma - 1}{2} M_e^2 f'^2 \tag{13}$$

(which is eq. 31 of ref. 2). Then equation (12) becomes

$$\frac{\partial u}{\partial y} = \frac{f''}{t} \Lambda \tag{14}$$

where

$$\Lambda = \int_0^\infty f'(1 - f')d\eta$$

From equation (14) there is obtained

$$\frac{\partial^2 u}{\partial y^2} = \frac{\Lambda}{t^2}(tf''' - t'f'')\frac{\partial \eta}{\partial y}$$

where

$$\frac{\partial \eta}{\partial y} = \frac{\partial \eta}{\partial Y}\frac{\partial Y}{\partial \bar{y}}\bar{\theta} = \frac{\bar{\rho}}{\bar{\rho}_e}\Lambda$$

With

$$\frac{\bar{\rho}}{\bar{\rho}_e} = \frac{\bar{t}_e}{\bar{t}} = \frac{1}{t}$$

it follows that

$$\frac{\partial \eta}{\partial y} = \frac{\Lambda}{t} \tag{15}$$

Then

$$\frac{\partial^2 u}{\partial y^2} = \frac{\Lambda^2}{t^3}(tf''' - t'f'') \tag{16}$$

where, from equation (13),

$$t' = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)S' - 2\,\frac{\gamma - 1}{2} M_e^2 f'f'' \tag{17}$$

By the use of equation (15) there is obtained

$$\frac{\partial t}{\partial y} = t'\,\frac{\Lambda}{t} \tag{18}$$

When equations (6) and (14) are used, equation (3) becomes

$$R_{\theta,c} = \frac{25\Lambda f_w'' t_c^{1.76}}{t_w (f_c')^4 \sqrt{1 - M_e^2 (1 - f_c')^2}} \qquad \left(\begin{array}{l} f_c' \geqq 1 - \frac{1}{M_e} \\ \text{when } M_e \geqq 1 \end{array} \right) \tag{19}$$

where from equation (13) it follows that

$$t_w = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S_w) \tag{20}$$

and

$$t_c = \left[\left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S) - \frac{\gamma - 1}{2} M_e^2 (f')^2\right]_c \tag{21}$$

When equations (6), (14), (16), and (13) are used, equation (4) becomes

$$\frac{-\pi f_w''}{t_w^2} \left[\frac{f't}{f''^3}(tf''' - 2t'f'')\right]_c = 0.58 \tag{22}$$

The expressions for t, t', t_w, and t_c in equations (19) and (22) are given by equations (13), (17), (20), and (21), respectively.

Calculation Procedure

The values of the minimum critical Reynolds number $R_{\theta,c}$ were calculated by means of equations (19) and (22) for the Mach number range between 0 and 2.8 for all the solutions with $f_w'' > 0$ presented in table I of reference 2 except those for $S_w = -1$, and for all the solutions presented in table VI of reference 3. All the solutions of reference 3 are for $S_w = 0$. The special case $S_w = -1$ is discussed later. The values of $R_{\theta,c}$ were also calculated for solutions that are not included in table I of reference 2 but which are listed in table II of reference 2, namely, the solutions for $\beta = 0$ and $S_w = 1, 0, -0.4,$ and -0.8. These solutions were obtained by using the solution for $\beta = 0$ in reference 3 together with the Crocco relation for $\beta = 0$, that is, $S = S_w(1 - f')$. (See page 3 of ref. 2.)

The calculations were made with the aid of the IBM type 704 electronic data processing machine. Because the value of $R_{\theta,c}$ depends on f_c' raised to the fourth power (see eq. (19)) and is thus sensitive to the value of f_c' and because a high-speed computing machine was available, an iterative method was used to find f_c'. The method was to compute ϕ, the left-hand side of equation (22), for a range of values of η beginning with $\eta = 0$. Upon reaching a value of η for which ϕ was greater than 0.58, this value of ϕ and the two preceding values were used in a second-order divided-difference interpolation procedure to find the value of η at which $\phi = 0.58$.

In a few cases the value 0.58 lay between $\eta = 0$ and the first value of η in table I of reference 2; in these cases two values of ϕ beyond 0.58 were used. Interpolations were made in the tables of given data to find f, f', f'', S, and S' at this value of η (called η_c). The value of f''' was also needed (see eq. (22)); this value was obtained by the use of equation (18a) of reference 2 which can be written as

$$f''' = \beta\left[f'^2 - (1 + S)\right] - ff'' \tag{23}$$

The value of $R_{\theta,c}$ was then computed.

Because near η_c, the functions f, f', f'', S, and S' are usually either monotonically increasing or decreasing whereas the function ϕ often has a maximum and a minimum, the accuracy of the interpolation was improved by using the values of f_c, f_c', and so forth to calculate the value of ϕ for η_c; this value of ϕ usually differed slightly from 0.58. A new interpolation to find η_c was then made. In this interpolation the value of ϕ that differed slightly from 0.58 was included in the interpolation and the value of ϕ that differed most from 0.58 was dropped. When the new value of η_c was found, interpolations were again made in the tables of given data to find f, f', f'', S, and S'. A new value of $R_{\theta,c}$ and a new value of ϕ were then computed. This procedure was continued until

$$\left|\frac{R_{\theta,c_2} - R_{\theta,c_1}}{R_{\theta,c_1}}\right| \leqq 0.0001$$

but never more than six times. Because the data in table I of reference 2 are given to four significant figures, the final results of the present computations were rounded off to four significant figures and are so presented in table I. In order to provide "working charts" and to show more readily the dependence of $R_{\theta,c}$ on β, M_e, and S_w, these results are also presented in figure 1.

The case $S_w = -1$ is a special case because the left-hand side of equation (22) cannot be used to compute ϕ numerically because the quantity t_w in the denominator is zero for $S_w = -1$. (See eq. (20).) L
Equation (22) indicates that, in order that $\phi = 0.58$ when $t_w = 0$, it 2
is necessary that either $f_c' = 0$ or $(tf''' - 2t'f'')_c = 0$. First con- 2
sider the condition $f_c' = 0$; the condition $(tf''' - 2t'f'')_c = 0$ is dis- 6
cussed later. For $M_e < 1$, the requirement $f_c' \geqq 1 - \frac{1}{M_e}$ does not apply; thus, any value of f_c' between zero and unity is allowable. If the quantities that occur in equation (22) are expanded in powers of η and only the first power of η is retained, these quantities become

$$f' = f_w''\eta$$

$$f'' = f_w'' + f_w'''\eta$$

$$f''' = -\beta(1 + S_w + S_w'\eta)$$

$$t = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S_w + S_w'\eta)$$

$$t' = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)S_w' - 2\,\frac{\gamma - 1}{2} M_e^2 f_w''^2\eta$$

where the result that $S_w'' = 0$ has been used. (See eq. (18b) of ref. 2.)

When these expressions are substituted into equation (22) and powers of η greater than the first are neglected, the result for ϕ, the left-hand side of equation (22), is

$$\phi = \frac{\pi\eta}{(1 + S_w)f_w''}\left[\beta(1 + S_w)^2 + 2S_w'f_w''\right] \tag{24}$$

When S_w is sufficiently near -1, the quantities S_w' and f_w'' are both positive and the term $2S_w'f_w''$ is much greater than $\beta(1 + S_w)^2$. The quantity ϕ therefore increases linearly with η from zero for all values of β. As S_w approaches -1, the slope of the curves for ϕ against η approaches infinity so that the value $\phi = 0.58$ occurs at $\eta = 0$. Therefore, $\eta_c = 0$ and $f_c' = 0$ are allowable values.

The form of equation (19) that is valid when η_c is near zero is

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$$R_{\theta,c} = \frac{25\Lambda\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{0.76} (1 + S_w + S_w'\eta_c)^{1.76}}{(1 + S_w)f_w''^3\eta_c^4\sqrt{1 - M_e^2(1 - f_w''\eta_c)^2}} \tag{25}$$

If the term $\beta(1 + S_w)^2$ is neglected with respect to $2S_w'f_w''$ in equation (24), a value of 0.58 is substituted for ϕ and equation (24) is solved for η_c, the result is

$$\eta_c = \frac{0.58(1 + S_w)}{2\pi S_w'} \tag{26}$$

If this value of η_c is substituted into equation (25), the result is an equation for $R_{\theta,c}$ that is valid for S_w near -1 and $M_e \leqq 1$; namely,

$$R_{\theta,c} = 40 \times 10^4 \frac{\Lambda\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{0.76} (S_w')^4}{(1 + S_w)^{3.24}(f_w'')^3\sqrt{1 - M_e^2\left(1 - 0.0923f_w'' \frac{1 + S_w}{S_w'}\right)^2}} \tag{27}$$

If S_w is placed equal to -1 in equation (27), the result is that $R_{\theta,c} = \infty$. Thus, for $M_e \leqq 1$ and $t_w = 0(S_w = -1)$, the critical Reynolds number is infinite.

Now consider the condition that $(tf''' - 2t'f'')_c = 0$. When $M_e > 1$, the relation $f'_c \geqq 1 - \frac{1}{M_e}$ must be satisfied. Therefore, η_c cannot be equal to zero and is in fact far from zero for large M_e. The quantities f' and t in equation (22) are then not zero. Therefore, in order that $\phi = 0.58$ when $M_e > 1$ and $t_w = 0$, it is necessary that

$$(tf''' - 2t'f'')_c = 0 \tag{28}$$

The substitution of equation (13) for t and of equation (17) for t' results in a form of equation (28) that contains M_e explicitly, namely,

$$f'''\left[\left(1 + \frac{\gamma - 1}{2} M_e^2\right)(1 + S) - \frac{\gamma - 1}{2} M_e^2 (f')^2\right] -$$
$$2f''\left[\left(1 + \frac{\gamma - 1}{2} M_e^2\right)S' - \frac{\gamma - 1}{2} M_e^2 2f'f''\right] = 0 \qquad (\eta = \eta_c) \tag{29}$$

Equation (29) can also be written as

$$\frac{1 + \frac{\gamma - 1}{2} M_e^2}{\frac{\gamma - 1}{2} M_e^2} = \left[\frac{f'''(f')^2 - 4f'(f'')^2}{f'''(1 + S) - 2f''S'}\right]_c \tag{30}$$

When a value of η_c is chosen arbitrarily, equation (30) gives the value of M_e at which equations (28) and (29) are satisfied.

Calculations of M_e by means of equation (30) for a range of values of η show that equation (28) or (29) is satisfied at two values of η for each value of M_e above a minimum value that depends on β. The minimum values of M_e are found to be greater than unity so that the condition $(tf''' - 2t'f'')_c = 0$ cannot be satisfied for $M_e < 1$. At the smaller value of η the relation $f'_c \geqq 1 - \frac{1}{M_e}$ is not satisfied; at the larger value of η this relation is satisfied when M_e is greater than a value of M_e that depends on β and is called $M_{e,\infty}$. When M_e is greater than $M_{e,\infty}$, the larger value of η is thus η_c and is the value of η that is associated with an allowable value of f_c', a value of f_c' for which

$$f_c' \geqq 1 - \frac{1}{M_e}$$

First consider the case $f_c' > 1 - \frac{1}{M_e}$. Calculations show that for $f_c' > 1 - \frac{1}{M_e}$ the value of M_e given by equation (30) increases as f_c' increases. In order to examine the behavior of M_e as f' approaches 1, substitute for f''' in equation (30) its expression given by equation (23). Equation (30) then becomes

$$\frac{1 + \frac{\gamma - 1}{2} M_e^2}{\frac{\gamma - 1}{2} M_e^2} = \left\{ \frac{\beta(f')^2\left[f'^2 - (1 + S)\right] - f'f''(ff' + 4f'')}{\beta(1 + S)\left[(f')^2 - (1 + S)\right] - f''\left[(1 + S)f + 2S'\right]} \right\}_c \tag{31}$$

As f' approaches 1, the quantities f'', S, and S' all approach zero but the quantity f becomes large. Then, considering

$$f = 1$$

$$f' = 1 - \epsilon$$

$$f'' = \epsilon$$

$$S = -\epsilon$$

$$S' = \epsilon$$

and keeping only the largest part of each term results in

$$\beta(f')^2\left[(f')^2 - (1 + S)\right] \rightarrow \beta\left[(f')^2 - 1 - S\right]$$

$$ff' + 4f'' \rightarrow ff'$$

$$\beta(1 + S)\left[(f')^2 - (1 + S)\right] \rightarrow \beta\left[(f')^2 - 1 - S\right]$$

$$(1 + S)f + 2S' \rightarrow f$$

Then for f' approaching 1, equation (30) becomes

$$\frac{1 + \frac{\gamma - 1}{2} M_e^2}{\frac{\gamma - 1}{2} M_e^2} = \frac{\beta\left[(f')^2 - 1 - S\right] - ff''}{\beta\left[(f')^2 - 1 - S\right] - ff''} = 1 \tag{32}$$

Thus, as f' approaches unity, the value of M_e that satisfies equation (28), or its equivalents equations (29), (30), or (31), approaches infinity.

In order to show that the requirement $\phi = 0.58$ is satisfied when equation (28) is satisfied and $t_w = 0$, note that the process used to obtain equation (32) from equation (31) shows that the left-hand side of equation (28) or (29) is negative for f' near unity. Then because f', t, and f'' in equation (22) are positive, the quantity ϕ is positive for f' near unity. For $t_w \neq 0$ the quantity ϕ is thus zero at a value of f' and M_e given by equation (30) and is positive for f' near unity. At the same M_e there is another smaller value of f' at which ϕ is also zero but this value of f' is too small to satisfy the condition $f_c' \geqq 1 - \frac{1}{M_e}$. (See, for example, fig. 2(b) of ref. 7.) This smaller value of f' corresponds to the smaller of the two values of η mentioned in the discussion that follows the presentation of equation (30).

By expanding f', f'' and so forth around the value of η at which $\phi = 0$ and then neglecting terms in $\eta - \eta_{\phi=0}$ of order higher than the first, it can be shown by a procedure similar to that used to obtain equation (24) that ϕ is approximately proportional to $\eta - \eta_{\phi=0}$ near $\eta = \eta_{\phi=0}$. Therefore, because t_w appears in the denominator of equation (22) the slope of the curve of ϕ against f' becomes very large as t_w becomes very small. Consequently, the value of f' at which $\phi = 0.58$ approaches the value of f' at which equation (28) is satisfied. In the limit $t_w = 0$, the quantity ϕ is equal to 0.58 at this value of f'.

Thus for $f'_c > 1 - \frac{1}{M_e}$ there is a range of M_e extending to infinity for which $\phi = 0.58$ at $t_w = 0$. At $M_e = \infty$, $f'_c = 1$ and $\eta_c = 1$. Because Λ, f''_w, and t_c are not zero in this range of M_e but t_w is zero, equation (19) indicates that $R_{\theta,c} = \infty$. Therefore for $t_w = 0(S_w = -1)$ and a range of M_e that extends to infinity, the value of $R_{\theta,c}$ is infinite.

The range of M_e determined in this way has a lower limit that occurs when $f'_c = 1 - \frac{1}{M_e}$; this value of M_e is $M_{e,\infty}$. In order to find $M_{e,\infty}$, equation (30) for M_e must be solved with the condition that $f'_c = 1 - \frac{1}{M_e}$. The results of this calculation are given in table II. Note that both conditions that allow equation (22) to be satisfied when $t_w = 0$ have been accounted for, namely $f'_c = 0$ when $M_e \leqq 1$ and $(tf''' - 2t'f'')_c = 0$ when $M_e > 1$. (See also page 476 of ref. 7 for a discussion of the case $t_w = 0$.)

For each value of β and $S_w = -1$ there is, in the range of M_e between unity and the value on the right-hand side of the last column of table II, no allowable oscillation in the boundary layer because the conditions $\phi = 0.58$ and $f'_c \geqq 1 - \frac{1}{M_e}$ cannot be satisfied. The usual interpretation, however, is that the boundary layer is stable in this region of M_e. Therefore $R_{\theta,c} = \infty$ for all values of M_e for $S_w = -1$.

For values of $S_w \neq -1(t_w > 0)$ there can also be a region of M_e in which there is no allowable oscillation. This region of M_e can be found for each value of S_w and β, when there is such a region, by noting that at the upper and lower boundary of the region the conditions $\phi = 0.58$ (eq. (22)) and $f'_c = 1 - \frac{1}{M_e}$ are both satisfied.

In order to calculate these boundaries the condition

$$f'_c = 1 - \frac{1}{M_e}$$

was rewritten as

$$1 - M_e^2(1 - f_c')^2 = 0 \tag{33}$$

This term appears in the denominator of equation (19) and, when this term is zero, $R_{\theta,c}$ is infinite. Actually, this condition for $R_{\theta,c} = \infty$ is exact and does not depend upon equation (19). (See page 87 of ref. 4 and page 469 of ref. 7.) The calculation was made by choosing a value of M_e and then finding f_c' from equation (22). The left-hand side of equation (33) was then calculated. This procedure was repeated for a range of M_e large enough to allow interpolation for the value of M_e at which equation (33) is satisfied. This value of M_e is $M_{e,\infty}$; values of $M_{e,\infty}$ are presented in table II and figure 2. The values of $M_{e,\infty}$ in table II indicate that the upper branch of the curve of $M_{e,\infty}$ against β in figure 2 is double-valued between $\beta = -0.3884$ and $\beta = -0.3657$ for $S_w = -1$ and probably also for part of the range between $\beta = -0.3285$ and $\beta = -0.3250$ for $S_w = -0.8$. The curve of $M_{e,\infty}$ against β in figure 2 has been drawn without regard for these double-valued regions. It is remarked that, if $M_{e,\infty}$ were plotted against f_w'' instead of β, there would be no double values. (See table II of ref. 2 for values of f_w''.) Note that both conditions that allow $R_{\theta,c}$ to be equal to infinity (eq. 19) have been accounted for; they are $t_w = 0$ and $1 - M_e^2(1 - f_c')^2 = 0$. The condition $f_c' = 0$ occurs together with $t_w = 0$ for $M_e \leqq 1$.

Figure 3 is a cross plot of figure 2 and shows the connection between the wall temperature ratio for $R_{\theta,c} = \infty$ when $f_c' = 1 - \frac{1}{M_e}$ and M_e for a range of values of the pressure gradient parameter β.

Relation Between n, S_w, and β

The present results give $R_{\theta,c}$ as a function of the pressure gradient parameter β and the enthalpy function at the wall S_w. The method of reference 1, however, results in a distribution along the body surface of the correlation number n which is also a pressure gradient parameter but which is not the same as β. In order to find the distribution of $R_{\theta,c}$ over the surface from the calculated distribution of n and the given distribution of S_w, it is thus necessary to be able to find β when n and S_w are known.

In order to find the connection between β, n, and S_w note that (from eq. (22) of ref. 1)

$$n(1 + S_w) = \frac{\bar{\theta}_{tr}^2}{\bar{U}_e}\left(\frac{\partial^2 \bar{U}}{\partial Y^2}\right)_w \tag{34}$$

Also note that from equation (5)

$$\left(\frac{\partial^2 \bar{U}}{\partial Y^2}\right)_w = f_w'''\left(\frac{\partial \eta}{\partial Y}\right)^3 \sqrt{\frac{2\bar{\nu}_0 \bar{U}_e X}{m + 1}}$$

or, upon using equation (10),

$$\left(\frac{\partial^2 \bar{U}}{\partial Y^2}\right)_w = f_w'''\left(\frac{m + 1}{2}\frac{\bar{U}_e}{\bar{\nu}_0 X}\right)\bar{U}_e$$

Also note that

$$\bar{\theta}_{tr} = \int_0^\infty \frac{\bar{U}}{\bar{U}_e}\left(1 - \frac{\bar{U}}{\bar{U}_e}\right)dY$$

(which is eq. (16) of ref. 1) or, upon making use of equation (6),

$$\bar{\theta}_{tr} = \int_0^\infty f'(1 - f')dY$$

When equation (10) is used, this expression for $\bar{\theta}_{tr}$ becomes

$$\bar{\theta}_{tr} = \frac{1}{\sqrt{\frac{m + 1}{2}\frac{\bar{U}_e}{\bar{\nu}_0 X}}}\int_0^\infty f'(1 - f')d\eta = \frac{\Lambda}{\sqrt{\frac{m + 1}{2}\frac{\bar{U}_e}{\bar{\nu}_0 X}}} \tag{35}$$

Then, equation (34) becomes

$$n(1 + S_w) = \Lambda^2 f_w''' \tag{36}$$

From equation (18a) of reference 2, it follows that

$$f_w''' = -\beta(1 + S_w)$$

Therefore, equation (36) can be written as

$$n = -\beta\Lambda^2 \tag{37}$$

The relation (37) was used to calculate n for all the values of β and Λ given in table II of reference 2; equation (35) shows that the quantity Λ is the same as the quantity $\frac{\bar{\theta}_{tr}}{X}\sqrt{\frac{m+1}{2}\frac{\bar{U}_e X}{\bar{\nu}_0}}$ which is presented in table II of reference 2.

The relation between n, β, and S_w is presented in table III and figure 4.

DISCUSSION

Accuracy

The values of $R_{\theta,c}$ have been calculated by means of equations (19) and (22) which are both approximate. Equation (19) in particular is highly approximate and probably is a useful approximation in a range of M_e whose upper boundary is only slightly greater than unity. (See page 84 of ref. 4.) Moreover, even the more exact method of calculation is believed to be adequate only up to a Mach number of about 2. (See page 473 of ref. 7.) It is consequently apparent that the present calculations of $R_{\theta,c}$ cannot be expected to show more than trends with β and S_w when M_e exceeds unity.

The accuracy of equation (22) and especially that of equation (19) decreases as f_c' increases. The quantity f_c' increases as the ratio of wall temperature to stagnation temperature increases (S_w increases) and as β decreases, except for cold walls $(S_w = -0.8)$. Consequently for hot walls and small β the present calculations of $R_{\theta,c}$ probably can only show trends even when M_e is less than unity.

In order to obtain more direct evidence concerning the accuracy of the calculated values of $R_{\theta,c}$, three comparisons were made. The first is shown in figure 5 and is a comparison of the values of $R_{\theta,c}$ calculated by equations (19) and (22) for the Falkner and Skan profiles (ref. 3) with the values of $R_{\theta,c}$ calculated by Pretsch by an "exact" method (ref. 8); the Mach number is zero and the wall is insulated $(M_e = 0;\ S_w = 0)$. The accuracy of the present results is believed to be adequate.

The second comparison is shown in figure 6 and is a comparison of the variation of $R_{\theta,c}$ with M_e for a strong favorable pressure gradient and an insulated wall $(\beta = 0.6;\ S_w = 0)$ calculated by Laurmann (ref. 9) by an "exact" method with the variation calculated by equations (19) and (22). For this case the accuracy of the present calculations seem to be adequate up to about $M_e = 1.3$. It is remarked that the theory used by Laurmann has been improved by Dunn and Lin. (See ref. 7.)

The third comparison is the variation with M_e of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$, the ratio of wall temperature to the temperature outside the boundary layer required for $R_{\theta,c} = \infty$ when $f_c' = 1 - \frac{1}{M_e}$, when the pressure gradient is zero $(\beta = 0)$. For M_e up to about 2, figure 5.4 of reference 4 shows that the variation of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ with M_e is insensitive to the value of the Prandtl number and the variation of viscosity with temperature. Therefore the accuracy of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ computed by equation (22) can be tested in this range of M_e by comparing these values of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ with more accurate values even though the Prandtl number is different. Equation (33) is also used in the computation of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ but is merely a statement of the condition $f_c' = 1 - \frac{1}{M_e}$. Such a comparison is shown in figure 7; this figure shows that for M_e up to about 2.8 the variation of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ calculated by the use of equation (22) agrees fairly well with the variation given in reference 7.

It is noted that the indication from figure 7 is that, for values of M_e greater than about 2.0, the values of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ are too low. The inference is that this result is caused by the use of a Prandtl number of unity in the calculations of the velocity and temperature profiles of reference 2. This comparison thus shows that formula (22) is adequate for the calculation of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ and $M_{e,\infty}$ up to at least $M_e = 2$. Moreover, the discussions on page 84 of reference 4 and on page 469 of reference 7 indicate that formula (22) is much more accurate for the calculation of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ and $M_{e,\infty}$ than is formula (19) for the calculation of $R_{\theta,c}$. It is remarked that formula (22) is approximate because the number 0.58 is used on the right-hand side instead of a function of f_c' and of the velocity and temperature profiles. This function is close to 0.58 when f_c' is small. (See figs. 2(a), 2(b), and table 8 of ref. 7.)

The approximate connection between $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$ and M_e is shown in figure 3 for constant values of the pressure gradient parameter β. An increase in β, which means an increase in the favorable pressure gradient, causes the temperature ratio necessary for $R_{\theta,c} = \infty$ to rise and also increases the range of M_e in which it is possible to make $R_{\theta,c} = \infty$. Figure 3 also indicates that an insulated surface can be completely stabilized at M_e equal to about 1.6 if $\beta = 0.4$ and for a range of M_e for $\beta > 0.4$. For values of β greater than 0.4, surfaces that are hotter than the insulated surface can also be completely stabilized for a range of M_e that is centered in the M_e region between about 1.6 and 2.0 and that decreases as the surface becomes hotter.

Because figure 3 is a crossplot of figure 2, it is not as accurate as figure 2. The points of intersection of the curves for β constant and the curves for S_w constant are accurately known but the other portions of the curves for β constant depend on the crossplot.

Anomalous Results

The calculations of $R_{\theta,c}$ resulted in two cases in which $R_{\theta,c}$ decreased as β increased, an unexpected result. The first case is that for $S_w = 1$ (fig. 1(a)) when β increased from 1.5 to 2.0, a large increase in favorable pressure gradient.

The reason for this result seems to be that the length θ upon which $R_{\theta,c}$ is based is sufficiently smaller for $\beta = 2.0$ than for $\beta = 1.5$ to cause the decrease in $R_{\theta,c}$. Thus, from table II of reference 2, the value of $\frac{\bar{\theta}_{tr}}{X}\sqrt{\frac{m+1}{2}\frac{\bar{U}_e X}{\bar{\nu}_0}}$, the quantity to which θ is proportional, decreases from 0.1113 at $\beta = 1.5$ to 0.06683 at $\beta = 2.0$, a decrease of 40 percent. If the reference length had been the displacement thickness, the value of $R_{\delta^*,c}$ at $M_e = 0$ would be 14,460 for $\beta = 1.5$ and would be 18,290 for $\beta = 2.0$. The critical Reynolds number $R_{\delta^*,c}$ would thus increase with β, as expected.

The second case is that for the highly cooled wall, $S_w = -0.8$. (See fig. 1(d).) In this case $R_{\theta,c}$ decreases with an increase in β for all M_e below $M_{e,\infty}$. The two values of β that seem to be inconsistent are $\beta = -0.3285$ $(f_w'' = 0.0693)$ and $\beta = -0.325$ $(f_w'' = 0.0493)$. This decrease of $R_{\theta,c}$ with increase in β has previously been encountered and discussed (ref. 10) in the comparison between a highly cooled two-dimensional stagnation-point flow ($\beta = 1$) and a flat-plate flow ($\beta = 0$) with zero or small rates of mass-flow injection. Note, however, that the smallest value of $R_{\theta,c}$, that for $\beta = 2.0$ and $M_e = 0$, is 2.461×10^6, a value that is larger than any value of R_θ likely to be reached. The conclusion therefore seems to be that, for very highly cooled walls with values of $R_{\theta,c}$ larger than any value of the boundary-layer Reynolds number likely to be met, the effect of a favorable pressure gradient is destabilizing when $M_e < M_{e,\infty}$. Calculations for values of M_e up to 8 show that, for values of M_e greater than $M_{e,\infty}$, an increase in β increases $R_{\theta,c}$, the usual effect. The values of $R_{\theta,c}$

decrease rapidly from $R_{\theta,c} = \infty$ to $R_{\theta,c} < 100$ for M_e greater than $M_{e,\infty}$. (See table I.)

Because figure 1(d), which is for $S_w = -0.8$, indicates that for highly cooled walls at $M_e < M_{e,\infty}$ the critical Reynolds number $R_{\theta,c}$ increases as β decreases, the question arises as to what happens as the separation point is approached; at the separation point β is negative and $R_{\theta,c}$ is usually near zero. In the solutions of reference 2 for $S_w = -0.8$, as the pressure gradient parameter β decreases from 2.0 to its maximum negative value, -0.3285, the quantity f''_w to which the skin friction is directly proportional also decreases. A further decrease in f''_w, however, is associated with an increase rather than a decrease in β. (See table II of ref. 2.) In the region between the value of β for separation $\left(f''_w = 0\right)$, namely, -0.3088, and the value -0.3285, there are two positive values of f''_w for each value of β. Because the skin friction is directly proportional to f''_w, it is thus f''_w rather than β which must be used to measure the nearness to separation. Therefore $R_{\theta,c}$ has been plotted against f''_w in figure 8. The two values of β that previously seemed to be inconsistent with the increase in $R_{\theta,c}$ as β decreases, namely, $\beta = -0.3285$ $\left(f''_w = 0.0693\right)$ and $\beta = -0.325$ $\left(f''_w = 0.0493\right)$ are now seen to be consistent. The conclusion from this figure is that, although $R_{\theta,c}$ increases as β and f''_w decrease, a value of f''_w is eventually reached beyond which $R_{\theta,c}$ decreases rapidly with a further decrease in f''_w. The behavior of $R_{\theta,c}$ for highly cooled walls consequently agrees with the usual behavior, namely, that $R_{\theta,c}$ approaches zero as f''_w approaches zero at the separation point.

L
2
2
6

The data in table I indicate that $R_{\theta,c}$ for the case $\beta = -0.325$, $S_w = -0.8$ $\left(f''_w = 0.0493\right)$ behaves in an unusual manner for M_e between about 1.0133 and 1.116. For M_e between 1.0133 and 1.016 the present method of computation results in three values of $R_{\theta,c}$ at the same M_e. (See table I.) The largest values of $R_{\theta,c}$ belong to the set that increases to infinity at M_e equal to 1.016; the other two sets of values of $R_{\theta,c}$ coalesce at a value of $1{,}035 \times 10$ at M_e equal to 1.0133. If all three values of $R_{\theta,c}$ were physically significant, instability would occur at the lowest value of $R_{\theta,c}$. Therefore, the physically significant value of $R_{\theta,c}$ would reach a maximum of $1{,}248 \times 10^5$ at $M_e = 1.0133$, decrease discontinuously to $1{,}035 \times 10$ at this value of M_e, and then decrease as shown in table I. Each of the two values of $R_{\theta,c}$

that appears at $M_e = 1.0133$ belongs to a different set of values of $R_{\theta,c}$. One set increases with M_e to infinity at M_e equal to 1.116. This variation is unlike that encountered for any other case and is probably physically unimportant because the values of $R_{\theta,c}$ in the other set are smaller; this set decreases continuously with M_e in the usual manner and is probably the physically significant set.

CONCLUDING REMARKS

The minimum critical Reynolds numbers for the similar solutions of the compressible laminar boundary layer computed by Cohen and Reshotko and also for the Falkner and Skan solutions as recomputed by Smith have been calculated by Lin's rapid approximate method for two-dimensional disturbances. These results enable the stability of the compressible laminar boundary layer with heat transfer and pressure gradient to be easily estimated after the behavior of the boundary layer has been computed by the approximate method of Cohen and Reshotko.

The previously reported unusual result (NACA Technical Note 4037) that a highly cooled stagnation point flow is more unstable than a highly cooled flat-plate flow is again encountered. Moreover, this result is found to be part of the more general result that a favorable pressure gradient is destabilizing for very cool walls when the Mach number is less than that for complete stability. The minimum critical Reynolds numbers for these wall temperature ratios are, however, all larger than any value of the boundary-layer Reynolds number likely to be encountered. For Mach numbers greater than those for which complete stability occurs a favorable pressure gradient is stabilizing, even for very cool walls.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., February 13, 1959.

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TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS

OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER

(a) $S_w = 1.0$

M_e	η_c	f_c'	S_c	$R_{\theta,c}$
		$\beta = 2.0$		
0	0.1244	0.2793	0.9178	1075
.2	.1252	.2808	.9173	1068
.4	.1275	.2854	.9157	1048
.6	.1314	.2933	.9131	1019
.8	.1371	.3045	.9094	985.2
1.0	.1448	.3195	.9043	956.0
1.2	.1546	.3379	.8977	959.5
1.4	.1677	.3624	.8891	1017
1.6	.1849	.3938	.8777	1402
1.727	------	------	------	∞
2.096	------	------	------	∞
2.2	.2861	.5571	.8112	390.2
2.4	.3449	.6375	.7728	99.45
2.6	.4057	.7095	.7333	45.75
2.8	.5190	.8186	.6608	26.58
		$\beta = 1.5$		
0	0.1513	0.2901	0.9028	1305
.2	.1523	.2918	.9022	1294
.4	.1553	.2969	.9003	1263
.6	.1605	.3056	.8970	1215
.8	.1680	.3181	.8921	1155
1.0	.1782	.3350	.8855	1091
1.2	.1919	.3571	.8768	1029
1.4	.2100	.3856	.8652	977.7
1.6	.2345	.4226	.8495	929.0
1.8	.2676	.4699	.8284	792.5
2.0	.3144	.5326	.7985	402.3
2.2	.3749	.6056	.7601	166.5
2.4	.4440	.6796	.7167	76.98
2.6	.5098	.7404	.6757	44.22
2.8	.5692	.7883	.6393	29.45
		$\beta = 1.0$		
0	0.2039	0.3136	0.8746	1197
.2	.2054	.3157	.8737	1182
.4	.2101	.3220	.8708	1138
.6	.2182	.3328	.8658	1069
.8	.2303	.3486	.8584	979.2
1.0	.2470	.3701	.8482	871.3
1.2	.2698	.3983	.8342	749.2
1.4	.3011	.4360	.8150	601.7
1.6	.3444	.4853	.7887	428.4
1.8	.4027	.5466	.7533	258.2
2.0	.4761	.6164	.7091	138.4
2.2	.5566	.6838	.6612	76.72
2.4	.6351	.7412	.6153	47.37
2.6	.7058	.7859	.5748	32.84
2.8	.7684	.8209	.5397	24.70
		$\beta = 0.5$		
0	0.3748	0.3952	0.7859	481.2
.2	.3789	.3988	.7836	468.6
.4	.3915	.4098	.7765	432.3
.6	.4137	.4289	.7640	376.1
.8	.4477	.4573	.7449	306.0
1.0	.4963	.4961	.7176	230.6
1.2	.5615	.5447	.6811	161.6
1.4	.6462	.6032	.6347	104.7
1.6	.7412	.6618	.5832	67.80
1.8	.8425	.7175	.5303	44.63
2.0	.9371	.7626	.4821	31.70
2.2	1.028	.8010	.4380	23.42
2.4	1.198	.8600	.3599	18.31
2.6	1.183	.8552	.3667	14.83
2.8	1.252	.8756	.3371	12.28
		$\beta = 0.3$		
0	0.6368	0.5096	0.6559	147.5
.2	.6450	.5148	.6515	142.3
.4	.6702	.5302	.6384	127.7
.6	.7131	.5558	.6161	106.8
.8	.7741	.5905	.5848	84.04
1.0	.8522	.6322	.5453	63.04
1.2	.9429	.6769	.5007	46.28
1.4	1.040	.7202	.4546	34.16
1.6	1.138	.7596	.4099	25.70
1.8	1.232	.7936	.3688	19.88
2.0	1.321	.8222	.3320	15.84
2.2	1.404	.8461	.2995	12.92
2.4	1.481	.8661	.2710	10.76
2.6	1.553	.8827	.2460	9.122
2.8	1.620	.8968	.2240	7.840
		$\beta = 0$		
0	1.692	0.7273	0.2727	15.06
.2	1.700	.7298	.2702	14.83
.4	1.722	.7371	.2629	14.17
.6	1.759	.7484	.2516	13.18
.8	1.806	.7630	.2370	11.99
1.0	1.863	.7795	.2205	10.72
1.2	1.926	.7971	.2029	9.468
1.4	1.992	.8147	.1853	8.305
1.6	2.061	.8318	.1682	7.260
1.8	2.129	.8478	.1522	6.344
2.0	2.196	.8624	.1376	5.556
2.2	2.261	.8757	.1243	4.881
2.4	2.323	.8876	.1124	4.307
2.6	2.383	.8982	.1018	3.817
2.8	2.440	.9075	.09246	3.399
		$\beta = -0.1$		
0	2.425	0.7741	0.1512	4.368
.2	2.431	.7758	.1500	4.319
.4	2.450	.7812	.1461	4.171
.6	2.481	.7897	.1398	3.942
.8	2.522	.8007	.1319	3.657
1.0	2.571	.8134	.1228	3.342
1.2	2.625	.8267	.1133	3.024
1.4	2.683	.8406	.1035	2.710
1.6	2.744	.8543	.09393	2.415
1.8	2.806	.8673	.08491	2.147
2.0	2.865	.8790	.07705	1.916
2.2	2.925	.8901	.06950	1.705
2.4	2.983	.9002	.06269	1.520
2.6	3.039	.9092	.05662	1.359
2.8	3.092	.9172	.05139	1.221

TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS

OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER - Continued

(b) $S_w = 0$

M_e	η_c	f_c'	S_c	$R_{\theta,c}$
		$\beta = 2.0$		
0	0.1119	0.1763	0	1007 × 10
.2	.1124	.1771		1008 × 10
.4	.1141	.1796		1012 × 10
.6	.1170	.1838		1028 × 10
.8	.1210	.1897		1075 × 10
1.0	.1264	.1973		1222 × 10
1.2	.1332	.2070		2053 × 10
1.265	------	------		∞
2.887	------	------		∞
		$\beta = 1.6$		
0	0.1260	0.1790	0	9270
.2	.1266	.1799		9270
.4	.1286	.1825		9289
.6	.1319	.1869		9397
.8	.1367	.1931		9767
1.0	.1429	.2012		1097 × 10
1.2	.1509	.2115		1750 × 10
1.274	------	------		∞
2.784	------	------		∞
2.8	.6061	.6488		359.8
		$\beta = 1.2$		
0	0.1472	0.1837	0	8102
.2	.1480	.1846		8093
.4	.1504	.1874		8081
.6	.1545	.1921		8122
.8	.1604	.1989		8352
1.0	.1682	.2078		9193
1.2	.1783	.2192		1359 × 10
1.290	------	------		∞
2.643	------	------		∞
2.8	.7428	.6843		101.9
		$\beta = 1.0$		
0	0.1627	0.1874	0	7306
.2	.1637	.1884		7291
.4	.1664	.1913		7259
.6	.1711	.1964		7258
.8	.1779	.2034		7401
1.0	1871	.2131		8017
1.2	1990	.2256		1120 × 10
1.303	------	------		∞
2.545	------	------		∞
2.6	.7094	.6355		208.4
2.8	.8371	.7050		70.14
		$\beta = 0.8$		
0	0.1843	0.1930	0	6298
.2	.1855	.1940		6276
.4	.1888	.1973		6222
.6	.1945	.2028		6171
.8	.2027	.2107		6208
1.0	.2139	.2214		6562
1.2	.2287	.2354		8490
1.327	------	------		∞
2.395	------	------		∞
2.4	.6840	.5851		1004
2.6	.8351	.6695		96.30
2.8	.9575	.7278		49.54
		$\beta = 0.60$		
0	0 2173	0.2023	0	4997
.2	2187	.2035		4969
.4	2230	.2072		4890
.6	2304	.2135		4786
.8	2412	.2227		4708
1.0	2561	.2354		4779
1.2	2763	.2523		5525
1.374	------	------		∞
2.189	------	------		∞
2.2	6939	.5487		971.1
2.4	.8610	.6403		114.0
2.6	.9992	.7053		55.68
2.8	1 111	.7526		35.06

TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS

OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER - Continued

(b) $S_w = 0$ - Continued

M_e	η_c	f_c'	S_c	$R_{\theta,c}$	M_e	η_c	f_c'	S_c	$R_{\theta,c}$
$\beta = 0.50$					$\beta = 0.40$				
0	0.2416	0.2096	0	4212	0	0.2756	0.2203	0	3328
.2	.2433	.2109	↓	4180	.2	.2777	.2218	↓	3294
.4	.2484	.2150		4090	.4	.2840	.2265		3196
.6	.2572	.2220		3960	.6	.2950	.2346		3047
.8	.2702	.2324		3826	.8	.3115	.2467		2868
1.0	.2883	.2467		3762	1.0	.3350	.2637		2692
1.2	.3135	.2663		4018	1.2	.3684	.2875		2592
1.4	.3491	.2934		9201	1.4	.4172	.3215		2860
1.427	------	------		∞	1.585	------	------		∞
2.037	------	------		∞	1.657	------	------		∞
2.2	.8130	.5902		186.0	1.8	.6159	.4499		1729
2.4	.9697	.6674		75.31	2.0	.7833	.5455		258.2
2.6	1.101	.7236		43.44	2.2	.9507	.6299		97.86
2.8	1.212	.7657		29.41	2.4	1.095	.6938		52.80
					2.6	1.219	.7420		34.18
					2.8	1.325	.7790		24.56
$\beta = 0.30$					$\beta = 0.20$				
0	0.3268	0.2371	0	2365	0	0.4128	0.2662	0	1397
.2	.3296	.2389	↓	2332	.2	.4169	.2686	↓	1368
.4	.3381	.2447		2233	.4	.4296	.2762		1284
.6	.3530	.2546		2079	.6	.4520	.2895		1151
.8	.3756	.2697		1879	.8	.4866	.3100		979.9
1.0	.4089	.2913		1646	1.0	.5377	.3395		779.8
1.2	.4569	.3223		1383	1.2	.6124	.3817		563.3
1.4	.5294	.3675		1059	1.4	.7189	.4395		353.1
1.6	.6400	.4332		614.5	1.6	.8575	.5105		193.2
1.8	.7922	.5170		254.5	1.8	1.011	.5833		103.9
2.0	.9579	.5994		113.3	2.0	1.160	.6477		60.99
2.2	1.111	.6668		60.17	2.2	1.295	.7006		39.76
2.4	1.242	.7192		38.24	2.4	1.415	.7432		28.17
2.6	1.357	.7600		26.97	2.6	1.521	.7776		21.22
2.8	1.458	.7923		20.38	2.8	1.616	.8056		16.71
$\beta = 0.10$					$\beta = 0.05$				
0	0.5771	0.3210	0	602.1	0	0.7129	0.3634	0	344.0
.2	.5838	.3244	↓	584.8	.2	.7211	.3673	↓	333.5
.4	.6043	.3351		535.2	.4	.7464	.3794		303.6
.6	.6405	.3537		460.0	.6	.7899	.4001		259.5
.8	.6953	.3816		369.2	.8	.8535	.4299		208.2
1.0	.7723	.4199		274.9	1.0	.9383	.4689		157.5
1.2	.8738	.4690		190.0	1.2	1.043	.5155		113.6
1.4	.9969	.5260		124.4	1.4	1.162	.5666		79.70
1.6	1.132	.5852		80.29	1.6	1.288	.6178		55.84
1.8	1.268	.6408		53.29	1.8	1.414	.6657		39.92
2.0	1.397	.6896		37.11	2.0	1.533	.7082		29.45
2.2	1.515	.7310		27.16	2.2	1.644	.7449		22.47
2.4	1.623	.7656		20.77	2.4	1.746	.7761		17.69
2.6	1.721	.7944		16.45	2.6	1.839	.8025		14.31
2.8	1.810	.8185		13.41	2.8	1.925	.8248		11.84

TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER - Continued

(b) $S_w = 0$ - Concluded

M_e	η_c	f_c'	S_c	$R_{\theta,c}$	M_e	η_c	f_c'	S_c	$R_{\theta,c}$
		$\beta = 0$					$\beta = -0.05$		
0	0.8950	0.4145	0	186.8	0	1.123	0.4686	0	101.8
.2	.9043	.4186		181.4	.2	1.133	.4725		99.30
.4	.9326	.4312		166.1	.4	1.162	.4843		92.19
.6	.9802	.4520		143.9	.6	1.209	.5037		81.73
.8	1.047	.4810		118.4	.8	1.274	.5299		69.55
1.0	1.133	.5173		93.22	1.0	1.355	.5619		57.21
1.2	1.233	.5589		71.08	1.2	1.499	.5979		45.90
1.4	1.345	.6031		53.27	1.4	1.551	.6359		36.28
1.6	1.461	.6471		39.87	1.6	1.657	.6735		28.55
1.8	1.576	.6883		30.17	1.8	1.763	.7093		22.58
2.0	1.686	.7255		23.28	2.0	1.865	.7419		18.05
2.2	1.790	.7581		18.37	2.2	1.963	.7709		14.64
2.4	1.887	.7863		14.83	2.4	2.054	.7964		12.07
2.6	1.976	.8105		12.21	2.6	2.139	.8185		10.09
2.8	2.058	.8314		10.24	2.8	2.217	.8376		8.565
		$\beta = -0.10$					$\beta = -0.14$		
0	1.404	0.5211	0	55.75	0	1.695	0.5638	0	31.94
.2	1.413	.5247		54.62	.2	1.704	.5670		31.39
.4	1.440	.5352		51.42	.4	1.730	.5765		29.81
.6	1.485	.5523		46.62	.6	1.772	.5919		27.43
.8	1.546	.5753		40.89	.8	1.829	.6123		24.53
1.0	1.621	.6029		34.87	1.0	1.899	.6368		21.40
1.2	1.707	.6339		29.11	1.2	1.978	.6640		18.32
1.4	1.800	.6664		23.96	1.4	2.064	.6926		15.48
1.6	1.897	.6988		19.60	1.6	2.153	.7212		12.98
1.8	1.994	.7298		16.03	1.8	2.243	.7486		10.87
2.0	2.088	.7584		13.19	2.0	2.331	.7742		9.128
2.2	2.179	.7843		10.95	2.2	2.417	.7974		7.710
2.4	2.265	.8072		9.192	2.4	2.498	.8182		6.563
2.6	2.345	.8273		7.801	2.6	2.574	.8366		5.636
2.8	2.421	.8449		6.694	2.8	2.646	.8528		4.881
		$\beta = -0.16$					$\beta = -0.18$		
0	1.893	0.5895	0	21.78	0	2.1864	0.6280	0	11.74
.2	1.901	.5925		21.44	.2	2.195	.6308		11.57
.4	1.926	.6015		20.44	.4	2.218	.6390		11.08
.6	1.967	.6159		18.93	.6	2.257	.6523		10.33
.8	2.022	.6350		17.07	.8	2.309	.6697		9.408
1.0	2.088	.6578		15.05	1.0	2.372	.6903		8.394
1.2	2.164	.6831		13.04	1.2	2.443	.7131		7.372
1.4	2.246	.7096		11.15	1.4	2.520	.7367		6.402
1.6	2.331	.7361		9.474	1.6	2.599	.7602		5.523
1.8	2.417	.7615		8.028	1.8	2.679	.7828		4.753
2.0	2.502	.7852		6.815	2.0	2.759	.8039		4.093
2.2	2.584	.8069		5.812	2.2	2.835	.8232		3.536
2.4	2.662	.8263		4.989	2.4	2.909	.8406		3.070
2.6	2.736	.8436		4.313	2.6	2.979	.8561		2.681
2.8	2.805	.8589		3.757	2.8	3.045	.8699		2.355
		$\beta = -0.19$					$\beta = -0.195$		
0	2.448	0.6673	0	6.231	0	2.699	0.7109	0	3.139
.2	2.456	.6698		6.148	.2	2.706	.7132		3.100
.4	2.479	.6774		5.908	.4	2.728	.7200		2.989
.6	2.516	.6895		5.540	.6	2.763	.7307		2.818
.8	2.565	.7053		5.083	.8	2.809	.7445		2.605
1.0	2.624	.7238		4.579	1.0	2.864	.7607		2.369
1.2	2.691	.7441		4.067	1.2	2.926	.7783		2.127
1.4	2.762	.7650		3.577	1.4	2.992	.7963		1.892
1.6	2.837	.7857		3.126	1.6	3.061	.8140		1.674
1.8	2.912	.8056		2.725	1.8	3.131	.8310		1.477
2.0	2.986	.8241		2.376	2.0	3.199	.8469		1.303
2.2	3.058	.8411		2.076	2.2	3.266	.8614		1.151
2.4	3.127	.8564		1.821	2.4	3.331	.8745		1.019
2.6	3.193	.8701		1.605	2.6	3.393	.8863		.9056
2.8	3.256	.8823		1.421	2.8	3.452	.8968		.8079

M

TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER - Continued

(c) $S_w = -0.4$

M_e	η_c	f_c'	S_c	$R_{\theta,c}$	M_e	η_c	f_c'	S_c	$R_{\theta,c}$
		$\beta = 2.0$					$\beta = 0.5$		
0	0.08762	0.1122	-0.3797	4450×10	0	0.1421	0.1098	-0.3703	3828×10
.2	.08793	.1126	-.3796	4488×10	.2	.1426	.1102	-.3702	3863×10
.4	.08885	.1137	-.3794	4620×10	.4	.1440	.1113	-.3699	3981×10
.6	.09036	.1156	-.3791	4916×10	.6	.1464	.1131	-.3694	4246×10
.8	.09245	.1181	-.3786	5598×10	.8	.1497	.1155	-.3687	4852×10
1.0	.09509	.1213	-.3780	7818×10	1.0	.1538	.1186	-.3678	6845×10
1.138	-------	------	-------	∞	1.134	------	------	-------	∞
		$\beta = 0$					$\beta = -0.2$		
0	0.2645	0.1242	-0.3503	1810×10	0	1.163	0.3356	-0.2139	260.2
.2	.2657	.1247	-.3501	1817×10	.2	1.173	.3390	-.2124	254.1
.4	.2692	.1264	-.3495	1846×10	.4	1.202	.3492	-.2081	236.7
.6	.2751	.1291	-.3484	1918×10	.6	1.250	.3662	-.2008	211.1
.8	.2834	.1330	-.3468	2098×10	.8	1.316	.3899	-.1910	181.6
1.0	.2944	.1382	-.3447	2690×10	1.0	1.406	.4221	-.1780	148.4
1.166	------	------	-------	∞	1.2	1.517	.4627	-.1622	115.2
2.522	------	------	-------	∞	1.4	1.646	.5099	-.1446	85.55
2.6	1.459	.6464	-.1415	110.1	1.6	1.787	.5608	-.1264	61.54
2.8	1.637	.7093	-.1163	48.55	1.8	1.931	.6119	-.1089	43.67
					2.0	2.071	.6596	-.09329	31.37
					2.2	2.202	.7026	-.07972	23.09
					2.4	2.321	.7394	-.06848	17.60
					2.6	2.430	.7708	-.05916	13.83
					2.8	2.528	.7975	-.05153	11.18
		$\beta = -0.24$					$\beta = -0.2483$		
0	1.937	0.4791	-0.1301	38.62	0	2.461	0.5776	-0.08941	9.524
.2	1.946	.4824	-.1290	37.96	.2	2.469	.5806	-.08860	9.383
.4	1.974	.4922	-.1257	36.04	.4	2.495	.5896	-.08621	8.979
.6	2.019	.5084	-.1205	33.10	.6	2.536	.6041	-.08242	8.347
.8	2.082	.5306	-.1134	29.43	.8	2.593	.6234	-.07742	7.574
1.0	2.160	.5581	-.1049	25.42	1.0	2.662	.6465	-.07155	6.722
1.2	2.251	.5901	-.09549	21.37	1.2	2.739	.6724	-.06519	5.853
1.4	2.351	.6247	-.08556	17.61	1.4	2.824	.6996	-.05867	5.031
1.6	2.457	.6603	-.07567	14.31	1.6	2.912	.7270	-.05236	4.287
1.8	2.564	.6952	-.06635	11.56	1.8	3.002	.7534	-.04640	3.640
2.0	2.669	.7280	-.05799	9.364	2.0	3.089	.7782	-.04099	3.095
2.2	2.770	.7578	-.05059	7.647	2.2	3.173	.8006	-.03625	2.643
2.4	2.865	.7842	-.04423	6.323	2.4	3.254	.8210	-.03201	2.268
2.6	2.954	.8075	-.03874	5.290	2.6	3.330	.8388	-.02840	1.964
2.8	3.035	.8276	-.03419	4.491	2.8	3.402	.8548	-.02522	1.709

TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS

OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER - Continued

(d) $S_w = -0.8$

M_e	η_c	f_c'	S_c	$R_{\theta,c}$
$\beta = 2.0$				
0	0.03470	0.03271	-0.7849	2461×10^3
.2	.03473	.03274	-.7849	2514×10^3
.4	.03483	.03283	-.7848	2694×10^3
.6	.03498	.03297	-.7848	3086×10^3
.8	.03518	.03316	-.7847	4031×10^3
1.0	.03542	.03338	-.7846	1018×10^4
1.039	-------	-------	-------	∞
$\beta = 0.5$				
0	0.04014	0.02622	-0.7838	4782×10^3
.2	.04016	.02624	-.7838	4893×10^3
.4	.04024	.02628	-.7838	5268×10^3
.6	.04035	.02636	-.7837	6085×10^3
.8	.04050	.02645	-.7837	8068×10^3
1.0	.04067	.02657	-.7836	2278×10^4
1.031	-------	-------	-------	∞
$\beta = -0.14$				
0	0.04868	0.01875	-0.7825	1335×10^4
.2	.04870	.01876	-.7825	1368×10^4
.4	.04874	.01877	-.7825	1479×10^4
.6	.04881	.01880	-.7825	1722×10^4
.8	.04891	.01884	-.7824	2318×10^4
1.0	.04901	.01888	-.7824	7719×10^4
1.018	-------	-------	-------	∞
$\beta = -0.325$ $(f_w'' = 0.1354)$				
0	0.07613	0.01035	-0.7778	6393×10^4
.2	.07613	.01035	-.7778	6559×10^4
.4	.07616	.01035	-.7777	7119×10^4
.6	.07619	.01035	-.7778	8348×10^4
.8	.07624	.01036	-.7778	1140×10^5
1.0	.07629	.01037	-.7777	5069×10^5
1.009	-------	-------	-------	∞
2.561	-------	-------	-------	∞
2.58	2.438	.6267	-.1580	203.2
2.6	2.475	.6390	-.1513	145.8
2.62	2.508	.6497	-.1455	118.1
2.64	2.537	.6593	-.1405	100.8
2.8	2.715	.7151	-.1122	50.76

M_e	η_c	f_c'	S_c	$R_{\theta,c}$
$\beta = 1.5$				
0	0.03599	0.03111	-0.7846	2845×10^3
.2	.03602	.03114	-.7846	2907×10^3
.4	.03611	.03121	-.7846	3119×10^3
.6	.03626	.03134	-.7845	3581×10^3
.8	.03644	.03150	-.7844	4696×10^3
1.0	.03666	.03169	-.7844	1217×10^4
1.037	-------	-------	-------	∞
$\beta = 0$				
0	0.04528	0.02127	-0.7830	9160×10^3
.2	.04530	.02127	-.7830	9382×10^3
.4	.04536	.02130	-.7830	1013×10^4
.6	.04544	.02134	-.7830	1177×10^4
.8	.04555	.02139	-.7829	1577×10^4
1.0	.04568	.02145	-.7828	4936×10^4
1.020	-------	-------	-------	∞
$\beta = -0.3$				
0	0.06167	0.01294	-0.7805	3755×10^4
.2	.06168	.01294	-.7805	3851×10^4
.4	.06171	.01295	-.7805	4176×10^4
.6	.06175	.01295	-.7805	4888×10^4
.8	.06181	.01297	-.7805	6648×10^4
1.0	.06187	.01298	-.7805	2651×10^5
1.012	-------	-------	-------	∞

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TABLE I.- MINIMUM CRITICAL REYNOLDS NUMBERS FOR SIMILAR SOLUTIONS

OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER - Concluded

(d) S_w = -0.8 - Concluded

M_e	η_c	f_c'	S_c	$R_{\theta,c}$	M_e	η_c	f_c'	S_c	$R_{\theta,c}$
$\beta = -0.3285\ (f_w'' = 0.1100)$					$\beta = -0.3285\ (f_w'' = 0.0693)$				
0	0.08526	0.009559	-0.7759	7331×10^4	0	0.1270	0.009330	-0.7664	5560×10^4
.2	.08526	.009560	-.7759	7522×10^4	.2	.1270	.009331	-.7664	5704×10^4
.4	.08528	.009563	-.7759	8166×10^4	.4	.1270	.009333	-.7664	6193×10^4
.6	.08532	.009566	-.7759	9581×10^4	.6	.1271	.009337	-:7664	7265×10^4
.8	.08536	.009572	-.7759	1311×10^5	.8	.1271	.009343	-.7664	9937×10^4
1.0	.08541	.009577	-.7759	6054×10^5	1.0	.1272	.009349	-.7663	4644×10^5
1.009	-------	--------	-------	∞	1.009	------	--------	-------	∞
2.385	-------	--------	-------	∞	1.995	------	--------	-------	∞
2.4	2.450	.5888	-.1694	361.6	2.0	2.425	.5024	-.1996	709.6
2.6	2.737	.6834	-.1205	55.36	2.2	2.728	.6056	-.1453	68.06
2.8	2.907	.7356	-.09625	33.29	2.4	2.932	.6730	-.1139	35.30
					2.6	3.091	.7225	-.09261	22.99
					2.8	3.222	.7609	-.07711	16.63
$\beta = -0.325\ (f_w'' = 0.0493)$									
0	0.2565	0.01507	-0.7347	7320×10^3					
.2	.2566	.01508	-.7347	7499×10^3					
.4	.2569	.01510	-.7346	8104×10^3					
.6	.2574	.01513	-.7345	9431×10^3					
.8	.2579	.01517	-.7344	1272×10^4					
1.0	.2586	.01522	-.7342	4668×10^4					
1.0133	.2587	.01522	-.7342	1248×10^5	1.0133	1.200	0.1287	-0.4961	1035×10
1.014	.2587	.01522	-.7342	1518×10^5	1.014	1.196	.1281	-.4970	1058×10
1.016	------	-------	-------	∞	1.016	1.188	.1265	-.4991	1118×10
1.0133	1.200	.1287	-.4961	1035×10	1.018	1.181	.1253	-.5008	1172×10
1.014	1.232	.1346	-.4881	8697	1.02	1.175	.1242	-.5023	1223×10
1.016	1.251	.1382	-.4833	7894	1.04	1.134	.1171	-.5125	1705×10
1.018	1.265	.1408	-.4800	7403	1.06	1.108	.1126	-.5191	2286×10
1.02	1.275	.1428	-.4773	7035	1.08	1.088	.1093	-.5240	3150×10
1.04	1.345	.1567	-.4600	5206	1.10	1.071	.1064	-.5284	5150×10
1.06	1.394	.1668	-.4480	4331	1.116	------	-------	-------	∞
1.08	1.435	.1757	-.4380	3760					
1.1	1.472	.1838	-.4289	3345					
1.15	1.553	.2023	-.4093	2700					
1.2	1.634	.2218	-.3899	2221					
1.4	1.937	.3035	-.3191	1268					
1.45	2.016	.3265	-.3015	1020					
1.6	2.248	.3991	-.2513	403.1					
1.8	2.531	.4936	-.1953	127.6					
2.0	2.772	.5760	-.1527	55.84					
2.2	2.970	.6424	-.1220	31.10					
2.4	3.134	.6954	-.09935	20.06					
2.6	3.273	.7374	-.08250	14.28					
2.8	3.391	.7713	-.06971	10.83					

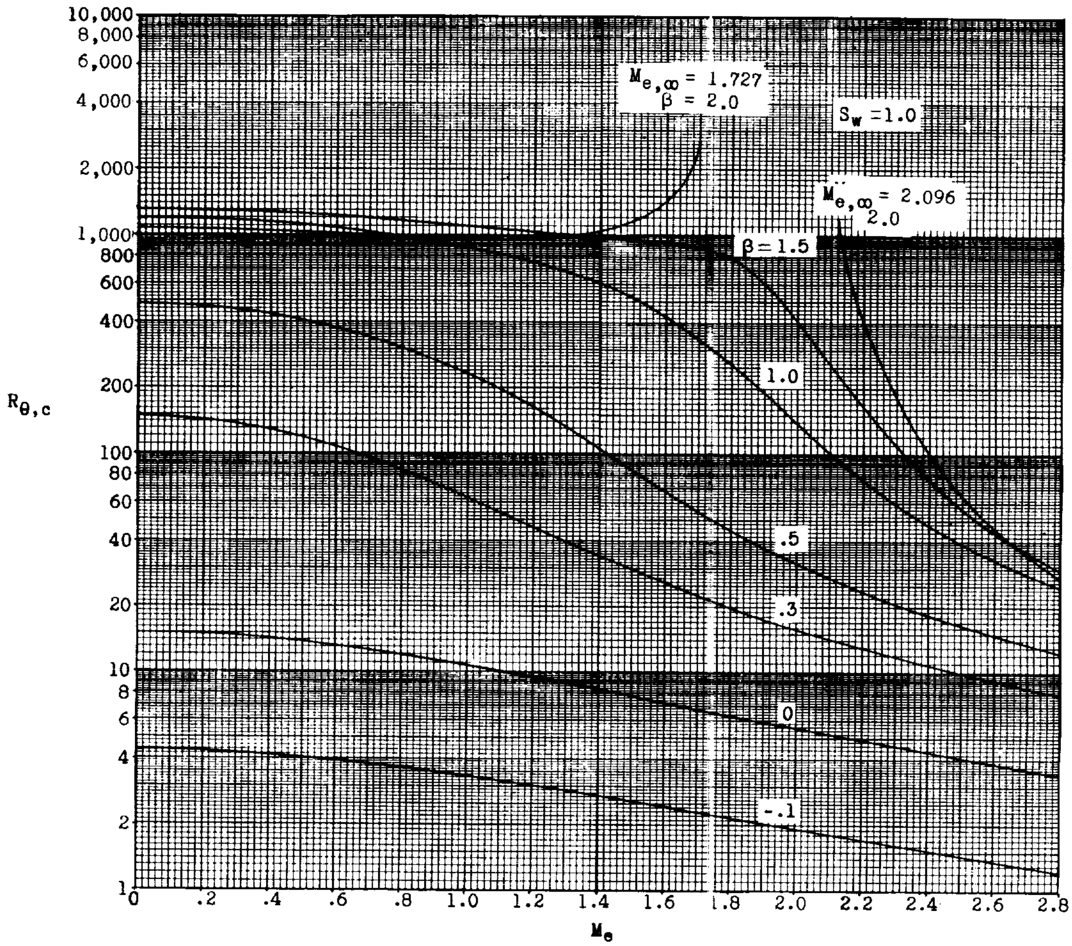
TABLE II.- VALUES OF $M_{e,\infty}$, THE MACH NUMBER AT WHICH

$R_{\theta,c} = \infty$ WHEN $f_c' = 1 - \frac{1}{M_e}$

β	$M_{e,\infty}$	
$S_w = 1.0$		
2.0	1.727	2.096
$S_w = 0$		
2.0	1.265	2.887
1.6	1.274	2.784
1.2	1.290	2.643
1.0	1.303	2.545
.8	1.327	2.395
.6	1.374	2.189
.5	1.427	2.037
.4	1.585	1.657
$S_w = -0.4$		
2.0	1.138	3.673
.5	1.134	3.385
0	1.166	2.522
$S_w = -0.8$		
2.0	1.039	6.814
1.5	1.037	6.307
.5	1.031	5.283
0	1.020	4.478
-.14	1.018	4.042
-.30	1.012	3.054
-.3250 ($f_w'' = .1354$)	1.009	2.561
-.3285 ($f_w'' = .1100$)	1.009	2.385
-.3285 ($f_w'' = .0693$)	1.009	1.995
-.3250 ($f_w'' = .0493$)	1.016	-----
$S_w = -1.0$		
2.0	1	10.50
.5	1	6.981
0	1	5.728
-.14	1	5.209
-.30	1	4.284
-.360	1	3.647
-.3884	1	2.884
-.3657	1	2.304

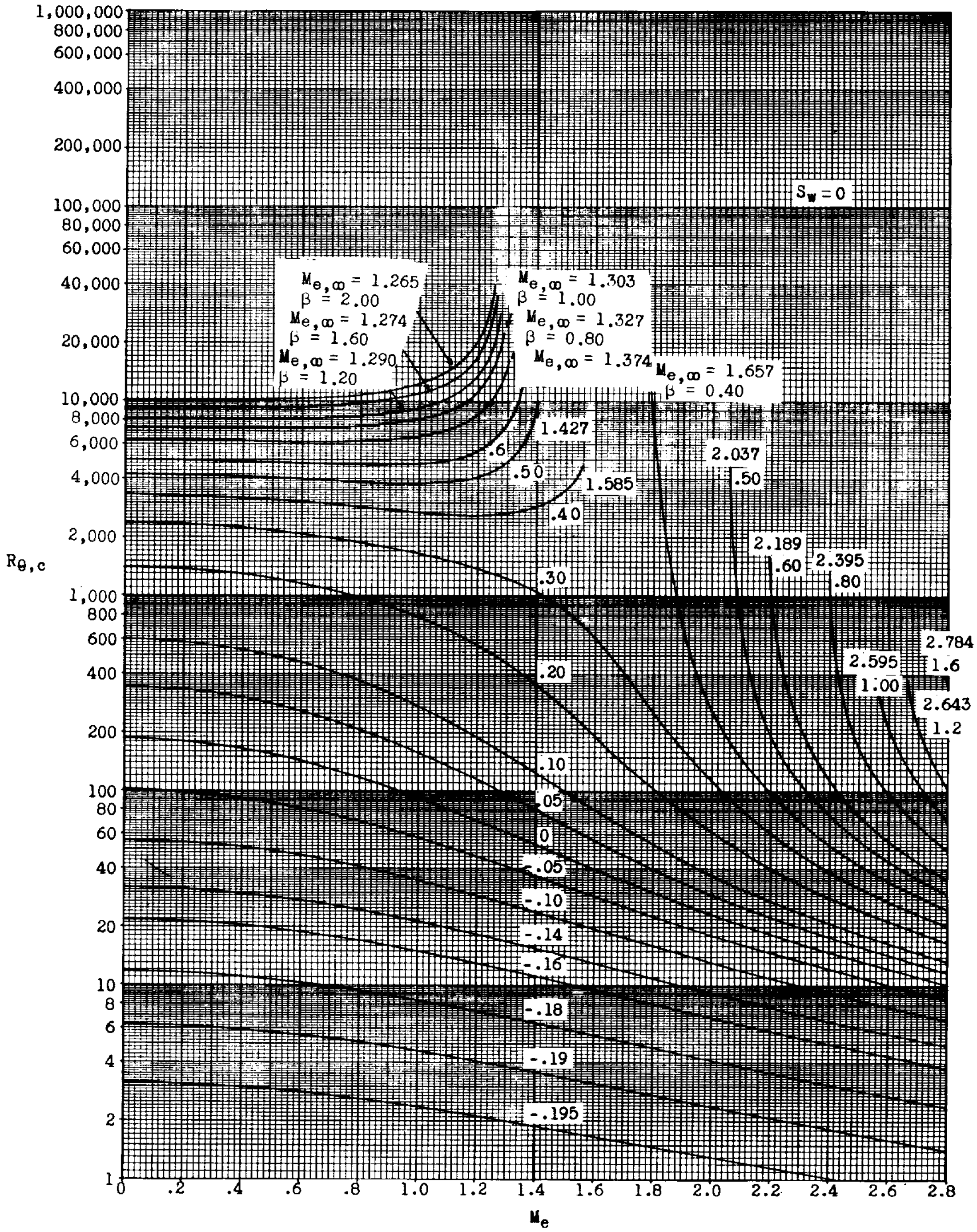
TABLE III.- RELATION BETWEEN n, β, Λ, AND S_w

β	Λ	n
	$S_w = 1.0$	
2.0	0.06683	-0.008932
1.5	.1113	-.01858
1.0	.1765	-.03115
.5	.2740	-.03754
.3	.3334	-.03336
0	.4696	0
-.10	.5425	.02943
-.1295	.5677	.04174
	$S_w = 0$	
2.0	0.2308	-0.1065
1.6	.2504	-.1003
1.2	.2761	-.09148
1.0	.2923	-.08544
.8	.3118	-.07778
.6	.3359	-.06768
.5	.3503	-.06135
.4	.3667	-.05380
.3	.3857	-.04464
.2	.4082	-.03332
.1	.4355	-.01897
.05	.4517	-.01020
0	.4696	0
-.05	.4905	.01203
-.10	.5150	.02652
-.14	.5386	.04061
-.16	.5522	.04878
-.18	.5677	.05801
-.19	.5765	.06316
-.195	.5814	.06591
-.1988	.5854	.06813
	$S_w = -.4$	
2.0	0.2944	-0.1733
.5	.3799	-.07215
0	.4696	0
-.20	.5544	.06148
-.24	.5868	.08263
-.2483	.6001	.08941
-.246	.6045	.08989
	$S_w = -.8$	
2.0	0.3551	-0.2522
1.5	.3659	-.2008
.5	.4091	-.0837
0	.4696	0
-.14	.5037	.03552
-.30	.5821	.1016
-.325	.6107	.1212
-.3285	.6193	.1260
-.3285	.6286	.1298
-.325	.6335	.1304
-.3088	.6274	.1215
	$S_w = -1.0$	
2.0	0.3833	-0.2938
.5	.4235	-.0897
0	.4696	0
-.14	.4952	.03433
-.30	.5498	.09069
-.36	.5908	.1256
-.3884	.6400	.1591
-.3657	.6571	.1579
-.326	.6400	.1335



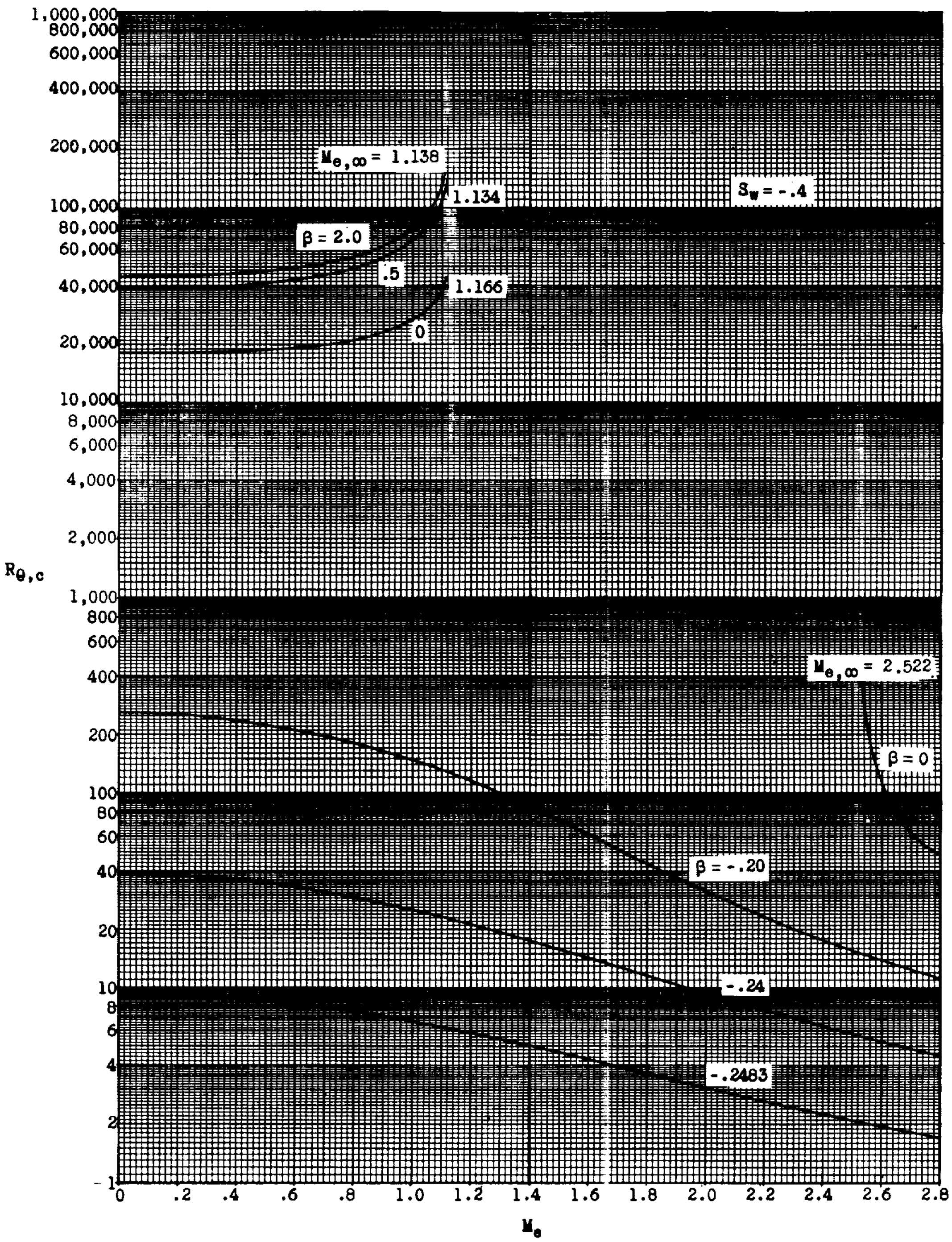
(a) Enthalpy function at the wall. $S_w = 1.0$.

Figure 1.- Variation of boundary-layer critical Reynolds number $R_{\theta,c}$ with Mach number at edge of boundary layer M_e for constant values of the pressure gradient parameter β.



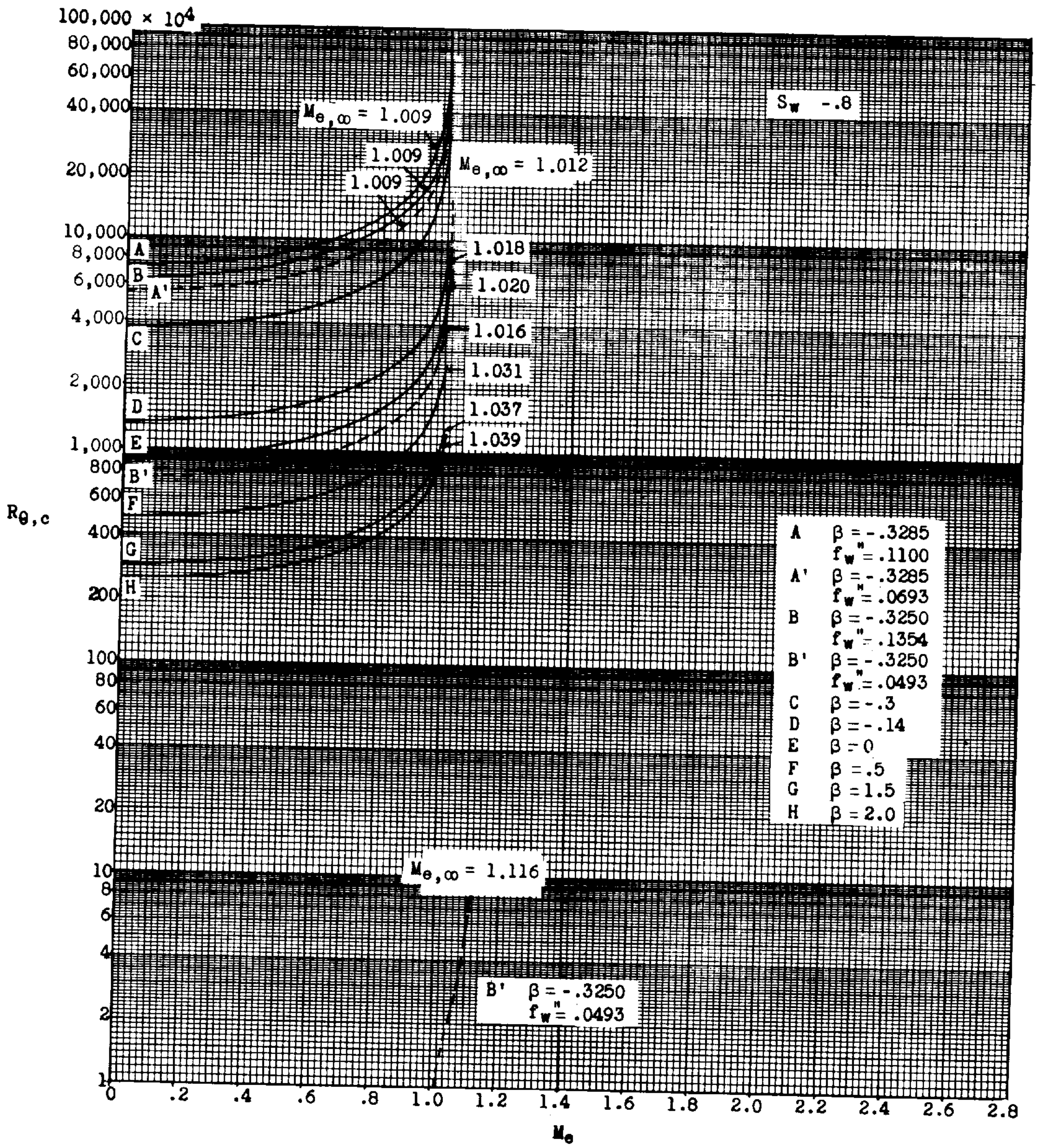
(b) Enthalpy function at the wall. $S_w = 0$.

Figure 1.- Continued.



(c) Enthalpy function at the wall. S_w = -0.4.

Figure 1.- Continued.



(d) Enthalpy function at the wall. $S_w = -0.8$.

Figure 1.- Concluded.

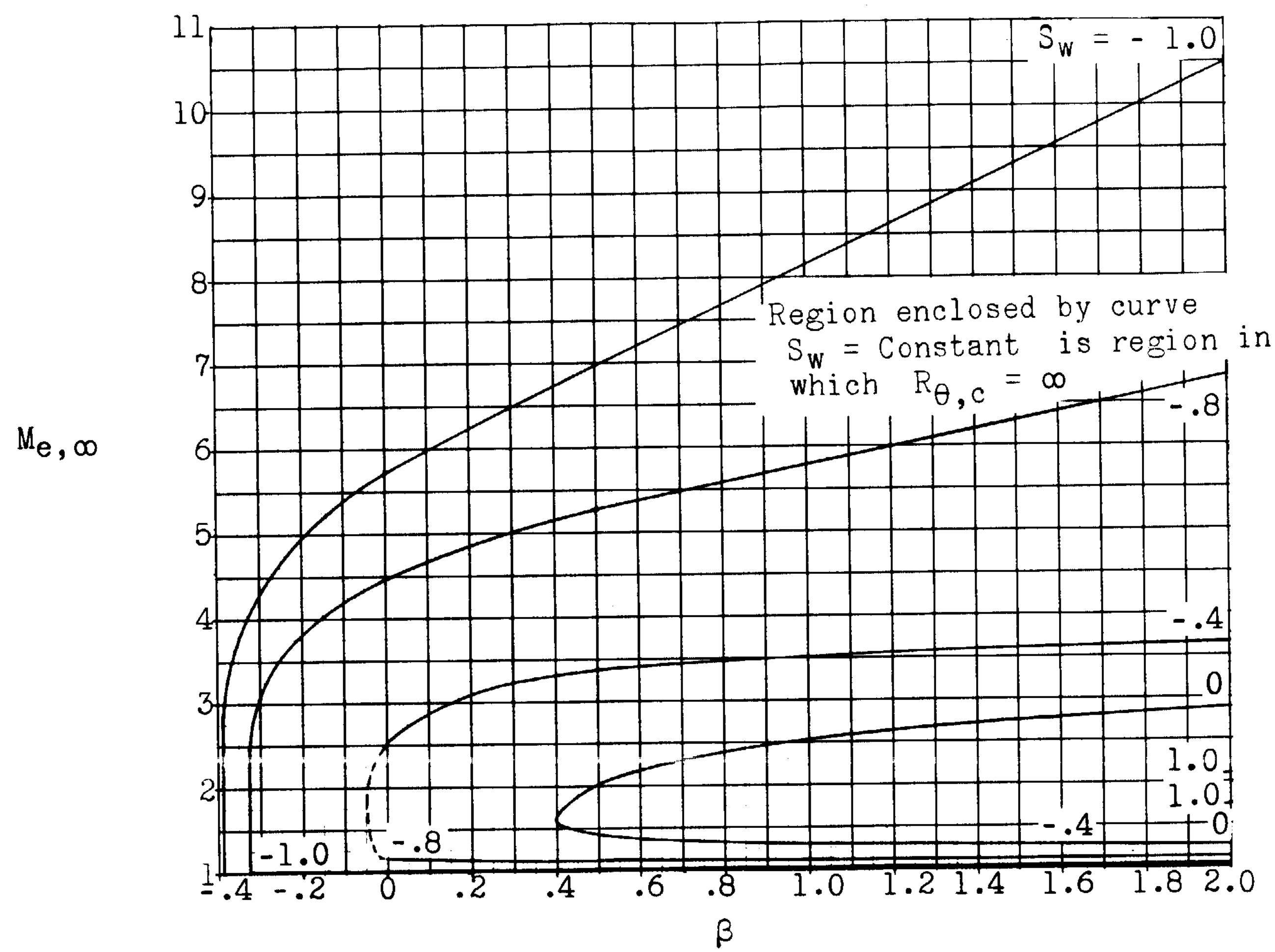


Figure 2.- Dependence of $M_{e,\infty}$ the Mach number for $R_{\theta,c} = \infty$ when $f_c' = 1 - \frac{1}{M_e}$, on the pressure gradient parameter β for fixed values of the surface enthalpy parameter S_w. $\left(R_{\theta,c} = \infty \text{ for all } M_e \text{ for } S_w = -1.\right)$

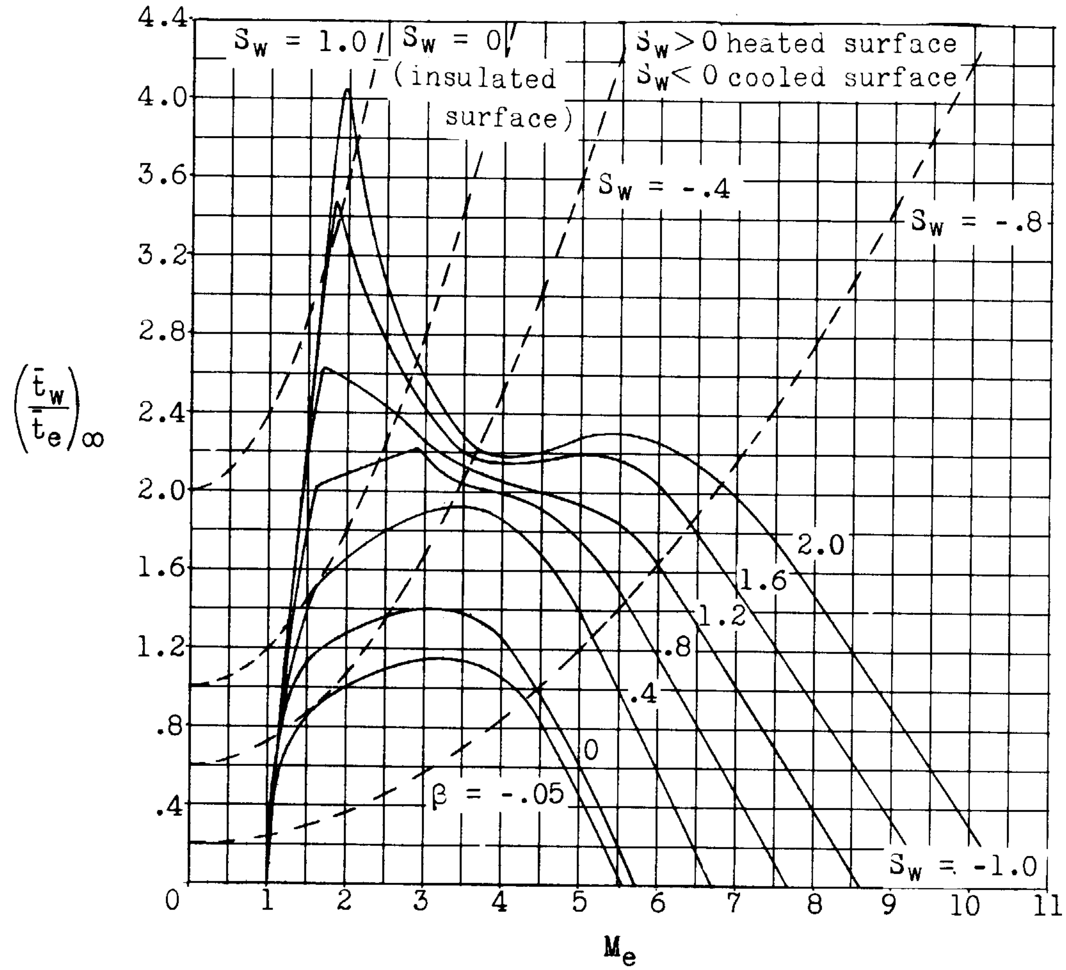


Figure 3.- Dependence of $\left(\frac{\bar{t}_w}{\bar{t}_e}\right)_\infty$, the temperature ratio for $R_{\theta,c} = \infty$ when $f_c' = 1 - \frac{1}{M_e}$, on M_e, the Mach number at the edge of the boundary layer, for constant values of β, the pressure gradient parameter.

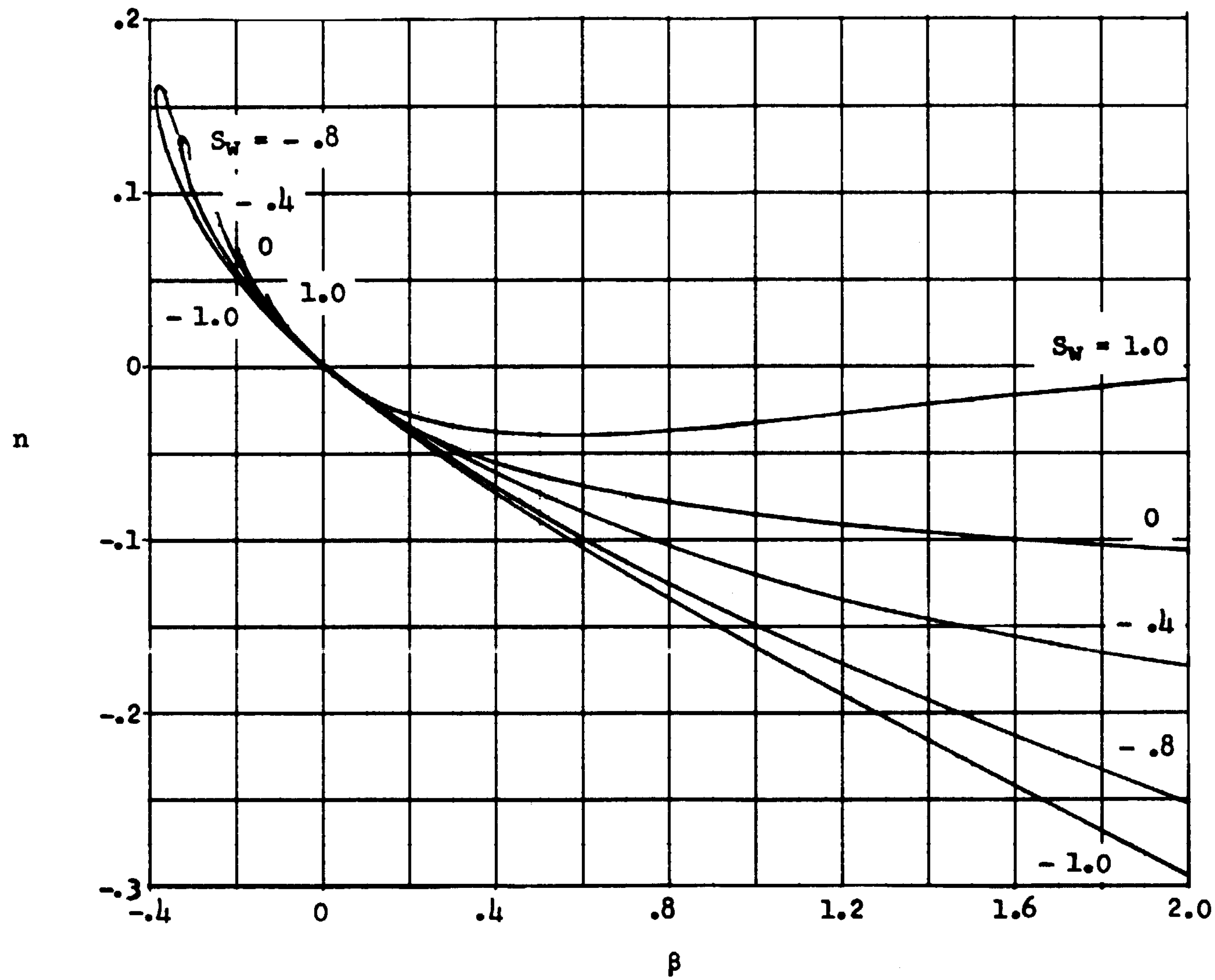


Figure 4.- Relation between correlation number n and pressure gradient parameter β for constant values of the enthalpy function at the wall S_w.

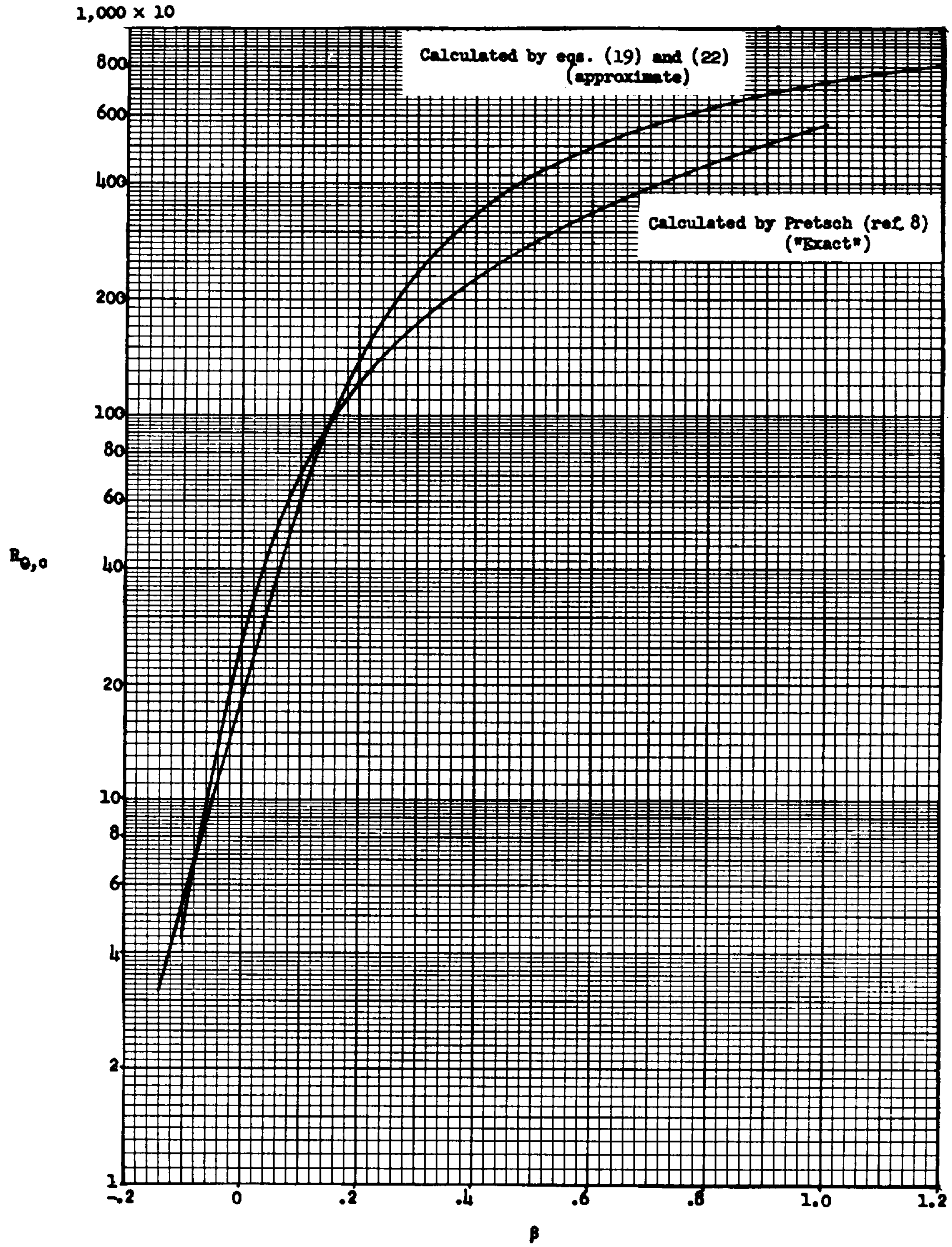


Figure 5.- Dependence of $R_{\theta,c}$ on the pressure gradient parameter β for $M_e = 0$, $S_w = 0$.

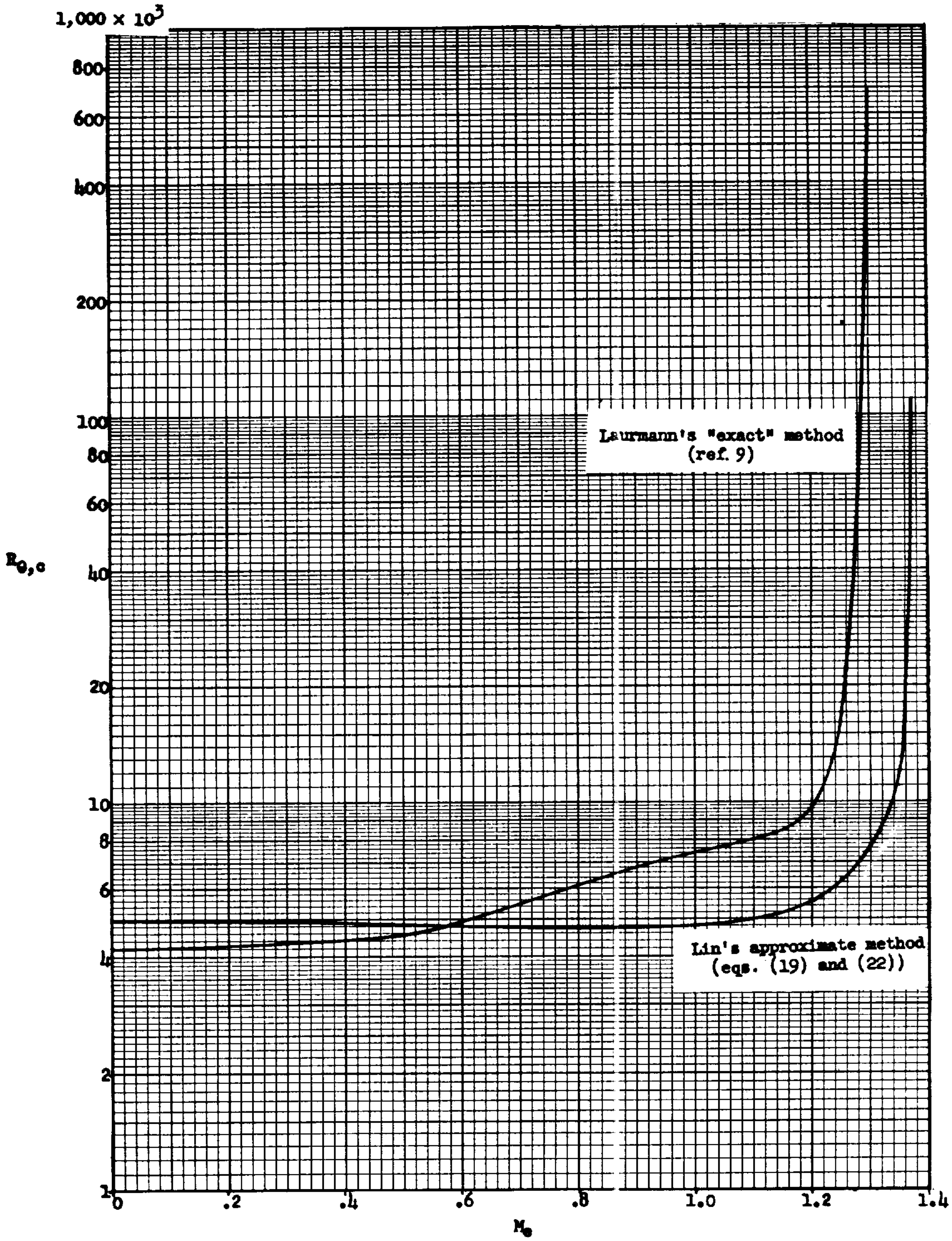


Figure 6.- Variation of critical Reynolds number with Mach number for pressure gradient parameter $\beta = 0.6$ and insulated surface. $S_w = 0$.

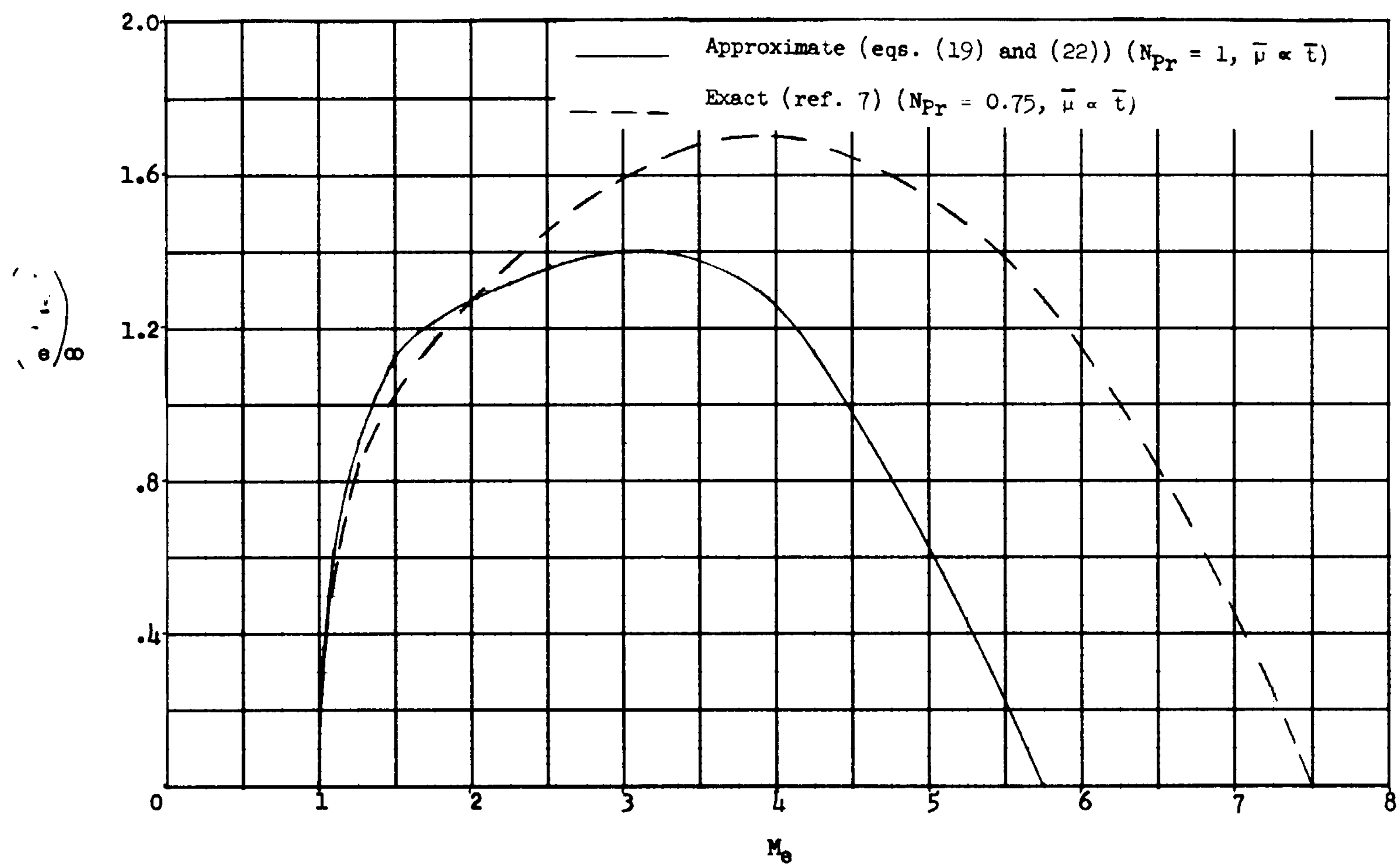


Figure 7.- Dependence of wall temperature ratio for $R_{\theta,c} = \infty$ on Mach number for flow over a flat plate calculated by two methods.

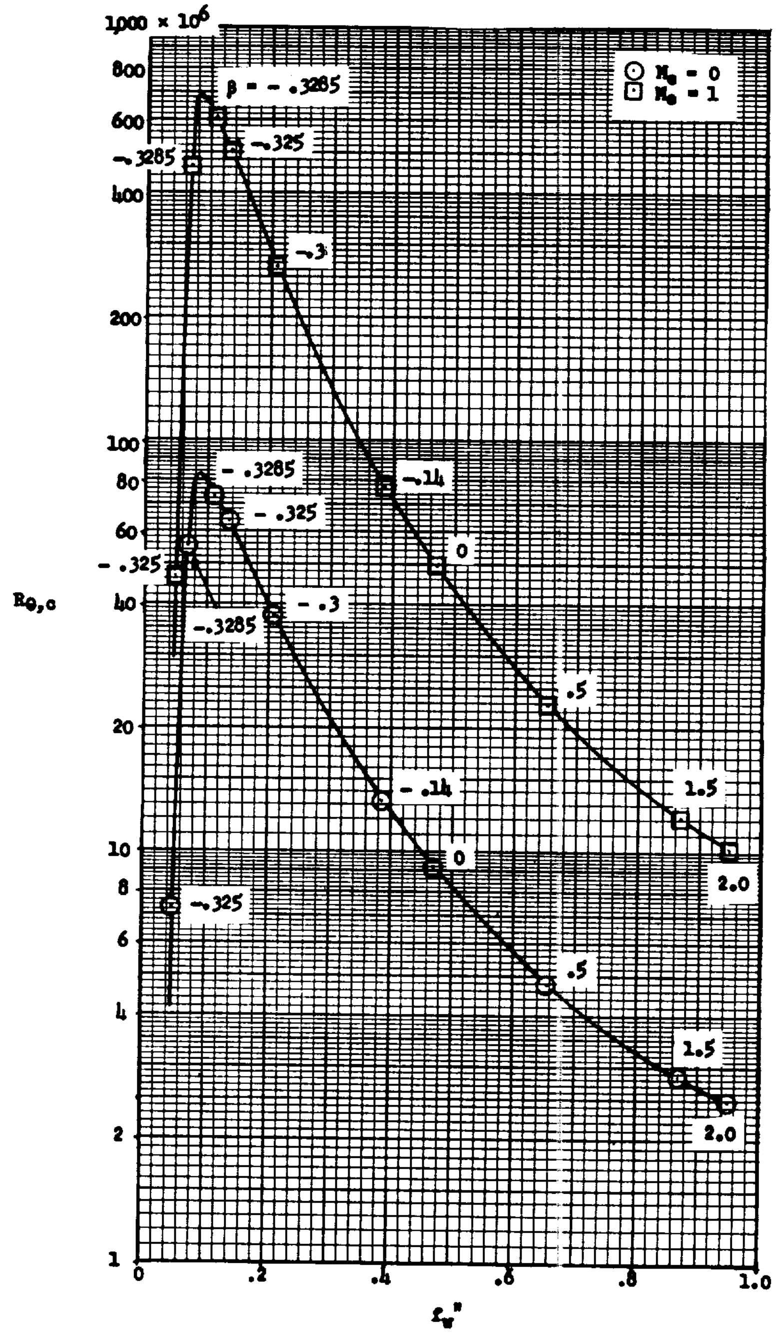


Figure 8.- Dependence of the minimum critical Reynolds number $R_{\theta,c}$ on the skin-friction parameter f_w'' for the surface enthalpy parameter $S_w = -0.8$.

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May 1959. 48p. OTS price, $1.25.
(NASA MEMORANDUM 5-4-59L)

The minimum critical Reynolds numbers for the similar solutions of the compressible laminar boundary layer computed by Cohen and Reshotko and also for the Falkner and Skan solutions as recomputed by Smith have been calculated by Lin's rapid approximate method. These results enable the stability of the compressible laminar boundary layer with heat transfer and pressure gradient to be easily estimated after the behavior of the boundary layer has been computed by the approximate method of Cohen and Reshotko. A favorable pressure gradient is found to be destabilizing for very cool walls.

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