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VELOCITY REQUIREMENTS FOR ABORT FROM THE BOOST
TRAJECTORY OF A MANNED LUNAR MISSION

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An investigation is made of the abort velocity requirements associated with failure of a propulsion system for a manned lunar mission. Two cases are considered: abort at less than satellite speed, which results in maximum decelerations in the following entry, and abort at greater than satellite speed with immediate return to earth. The velocity requirements associated with the latter problem are found to be substantial (several thousand feet per second) and are found to be even more severe if boost trajectories which lead to burnout at high altitudes or large flight-path angles are used. The velocity requirements associated with abort at less than satellite speed are found to be less severe than those for abort at greater than satellite speed except for nonlifting vehicles. It is found that abort rockets sufficient for abort at greater than satellite speed can be used to reduce maximum decelerations in entries following an abort at lower speeds. This reduction is accomplished by use of the abort rockets to decrease entry angle immediately prior to entry into the atmosphere.

INTRODUCTION

One of the missions which has recently generated considerable interest is manned flight to the vicinity of the moon. This mission requires that a vehicle be accelerated to near escape speed. Since the booster systems required are large, and since they burn for extended periods of time, the possibilities for malfunction are numerous. For this reason, some attention has been focused on provisions for crew safety in the event of booster failure.

The abort problems encountered for a manned lunar mission are both difficult and varied; however, many of these problems are similar to those encountered in launch trajectories for satellite missions. For the satellite case, considerable work has been done in studies of abort problems associated with specific launch and entry vehicles. In addition, abort problems for satellite ascent trajectories were treated in a general way in reference 1.
In the present report, a general treatment of some of the abort problems for lunar missions will be given. Attention will be restricted to those problems that occur after the vehicle has left the atmosphere. Two problems appear to warrant specific consideration. The first of these occurs before the vehicle has achieved circular satellite speed. If the booster fails in this case, the vehicle may be placed on a trajectory that leads to entry with prohibitively large decelerations or heating rates. The second problem occurs when the vehicle has achieved greater than circular satellite speed. If the booster fails in this case, the vehicle may be placed in a highly elliptical orbit with subsequent long exposure time in the radiation belts. Both of these conditions may be alleviated by the use of abort rockets to alter the trajectory. It is the particular purpose of this report to investigate the velocity increments required of the rocket system used in these abort maneuvers.

TYPICAL BOOST TRAJECTORY

In the present investigation, an attempt has been made to treat the abort problems considered in a general way applicable to all boost trajectories. For this reason, no particular boost trajectory will be used. It is still informative, however, to examine one typical boost trajectory in order to demonstrate types of abort problems encountered. A three-stage gravity turn boost trajectory suitable for placing a manned expedition on a lunar orbit is shown in figure 1. In this trajectory the altitude reached is higher before burnout than it is at burnout. This lofted trajectory is rather typical of current booster systems which tend to have relatively long burning times. In a launch trajectory such as this for a manned vehicle there are at least five abort situations which usually warrant attention. These situations are, in chronological order, (1) escape from the launch pad in the event of booster misfire, (2) abort at maximum dynamic pressure in the boost trajectory, (3) abort that results in maximum deceleration in the subsequent entry, (4) abort that results in the maximum heating in the subsequent entry, and (5) abort at greater than satellite speed with immediate return to earth.

The first four of these situations are also common to launch trajectories where the end objective is a near-earth satellite orbit and they have been treated in some detail in the literature. For a lunar mission, the fourth case of abort (maximum heating) does not appear to be a serious problem since the mission would appear to make ablation cooling a necessity at the present time. If this is the case, the return entry phase of the mission appears to be more severe in terms of the heat shield requirement than any abort trajectory. Accordingly, the maximum heating type of abort problem will not be considered herein.

Some attention will be directed to the problem of abort at supersatellite speed. It will be seen that this problem imposes relatively large abort velocity requirements. Since this is the case, it appears
worthy to examine again the third problem of abort with maximum
deceleration, since if large abort rockets are available, this problem
may be reduced. In this event, some of the constraints normally placed
on the launch trajectory may be relaxed, perhaps easing some of the
booster problems. Abort at less than satellite speed, which has associated
with it large entry decelerations, will be considered first.

ABORT AT LESS THAN SATELLITE SPEED

During the initial portion of the boost trajectory failure of the
propulsion system will leave the vehicle on an elliptical path with
subsequent entry into the atmosphere. In typical boost trajectories
it is found that if abort occurs at velocities from 14,000 to 18,000
feet per second where the vehicle is substantially above the atmosphere,
the resulting entry angle at lower altitude may be in excess of -20°
and, hence, the entry decelerations may be severe if no corrective
thrust is used.

In order to determine the effects of entry velocity, entry angle,
and vehicle aerodynamics on entry decelerations, calculations were made
on a high-speed digital computer using the complete equations of motion
for a nonrotating spherical earth. Calculations were made for an entry
altitude of 300,000 feet and covered entry velocities from 15,000 to
18,000 feet per second, entry angles to -20°, and vehicle lift-drag ratios
from 0 to 2 for an \( \text{m/C}_{\text{DA}} \) of 5 slugs per square foot. The results, which
are shown in figure 2, demonstrate that for entry angles to -10°,
the entry decelerations are more strongly dependent on the flight-path angle than on
velocity over the velocity range of interest. For example, for \( \text{L/D} = 0.5 \)
(fig. 2(b)) the decelerations are virtually independent of velocity for
an entry angle of -6.5°. This result indicates that the abort-rocket
thrust should be used in such a manner as to reduce the entry flight-path
angle in order to be most advantageous in reducing the entry deceleration.
Furthermore, it has been found that the abort rocket thrust should be
applied immediately prior to entry (considered here to be at 300,000 ft).
If the thrust is applied at higher altitudes, the entry angle cannot always
be made small. This trend results because the velocity after the abort
rocket is fired is normally still subcircular and the trajectory generally
has a nonzero eccentricity. After apogee of an elliptical trajectory,
the flight-path angle decreases (increases negatively) as the vehicle
approaches the atmosphere. For this reason, a steep entry may still result
if the velocity impulse is added at too high an altitude. If the use of
corrective thrust is delayed until shortly before entry, the entry angle
may be reduced directly by applying the abort rocket thrust in a direction
normal to the desired flight path after the velocity impulse.

A list of the symbols used is given in appendix A. A typical value
of \( \text{m/C}_{\text{DA}} \) of 5 slugs per square foot was chosen since the entry decelerations are relatively insensitive to variations in \( \text{m/C}_{\text{DA}} \).
The velocity increments, $\Delta V$, required to reduce the entry decelerations to given limits are determined as follows. For various burnout or abort conditions, the velocity and flight-path angle at 300,000 feet can be calculated with the simple satellite equations of motion. With the velocity vector so determined and with the results given in figure 2, the velocity increment required of the abort rocket system to reduce decelerations to a given level can be determined. These results are presented in figure 3 where the velocity increments required to restrict the maximum decelerations to 8 g for vehicles having lift-drag ratios of 0.5, 1, and 2 are presented. For a vehicle with $L/D = 0$, the decelerations cannot be reduced to 8 g in this manner. Therefore, in this case, the velocity increment required to reduce the flight-path angle to 0° at 300,000 feet is shown. For a vehicle with $L/D = 0$, entry decelerations will then be limited to approximately 12 g. In general, these results show that the velocity increment required increases with increasing flight-path angle and altitude at burnout, and decreases with increasing $L/D$.

**ABORT AT GREATER THAN SATELLITE SPEED**

As noted earlier, failure of the propulsion system during the portion of the boost trajectory where the speed is greater than satellite speed presents a somewhat different abort problem than the one just described. In this case, an immediate return to earth is usually desired in order to avoid, among other things, extended exposure to the Van Allen radiation belts. If this return is accomplished by direct retrothrust, it is expensive in terms of the velocity increment required. For example, if an abort occurs at near escape speed, a velocity impulse of some 10,000 feet per second is required for reduction to satellite velocity alone. A more attractive procedure appears to be to apply rocket thrust in such a manner as to deflect the trajectory so that it lies within the normal entry corridor as defined in reference 2. Entry along the overshoot boundary using negative lift (ref. 2) is generally the least expensive in terms of the velocity increment required. With the trajectory so altered, a direct entry can be accomplished in a manner similar to a normal entry from a lunar mission. For this type of maneuver, the velocity increment required depends not only on the burnout conditions at the abort point but also on the factors which influence the overshoot trajectory. These factors are vehicle lift-drag ratio and $m/CDA$, entry velocity, and of course, the altitude at which trajectory is altered to coincide with the overshoot trajectory.

One of the easiest ways to determine the velocity increment required of an abort rocket system applied in the manner just described is to employ a graphical procedure. Velocity diagrams appropriate for this purpose are shown in figure 4. The upper portion of the figure gives the burnout conditions at the abort represented as a vector with the origin to the left. The range of burnout conditions presented includes velocities from 1.2 to $\sqrt{2}$ local satellite speed, flight-path angles from $-2^\circ$ to $+6^\circ$, and altitudes
A study of typical rocket boost trajectories has indicated that these ranges of conditions should include those of current interest. The lower portion of the figure shows the entry flight conditions necessary for reduction of the velocity to satellite speed in a single pass using negative lift (the overshoot boundary). The curves shown correspond to vehicles with lift-drag ratios from 0 to -2, \( m/C_{D}A \) from 1 to 10 slugs per square foot, and entry velocities from 1.2 to \( \sqrt{2} \) local satellite speed. The curves were computed from the results given in references 2 and 3 for vehicles entering with constant \( m/C_{D}A \) and \( L/D \). The velocity increment required of the abort system is represented by the vector between the appropriate burnout point and the curve representing the appropriate vehicle aerodynamics. Since the lowest velocity increment is desired, the vector normal to the appropriate vehicle curve is the one of interest.

The velocity increments required of the abort-rocket system have been determined in the manner just described and the results are summarized in figures 5 through 8. This series of figures shows the effect on velocity requirements of vehicle \( m/C_{D}A \) and \( L/D \), and of burnout velocity and flight-path angle, respectively. In each case, the velocity requirement is shown as a function of burnout altitude. It is noted that the entry vehicle characteristics \( m/C_{D}A \) and \( L/D \) are of secondary importance, except possibly at the lowest altitudes shown. Far more important are the flight conditions at the point of abort. In this connection it is noted that the abort velocity requirement increases rather markedly with increasing burnout flight-path angle and altitude. For a nominal entry vehicle of \( m/C_{D}A = 5 \) slugs per square foot and \( L/D = -0.5 \), abort at escape speed with zero flight-path angle requires a \( \Delta V \) of approximately 2,000 feet per second at 300,000 feet altitude; 4,000 feet per second at 500,000 feet; and 8,000 feet per second at 1,500,000 feet.

The results also indicate that generally the abort velocity requirement increases by over 600 feet per second for every 1° increase in the burnout flight-path angle at near escape speed. Furthermore, for a given propulsion system with a fixed payload, an increase in the burnout flight-path angle is usually associated with an increase in the burnout altitude. For example, in some of the trajectories studied in the present investigation for a fixed payload, the burnout flight-path angle increased about 1/4° for each 100,000 feet increase in the burnout altitude. In particular, a nominal boost trajectory resulted in burnout at 500,000 feet at zero flight-path angle. When the launch trajectory was altered to give burnout at 1,500,000 feet, the flight-path angle at burnout increased to 2.5°. From the results presented in figure 8, then, the abort velocity requirement is increased from approximately 4,000 feet per second to over 9,000 feet per second. The latter figure is more than 1,000 feet per second higher than the value previously cited for zero flight-path angle at 1,500,000 feet.

From the results presented in figures 5 through 8, then, it would appear that the abort velocity requirement will be minimized if the burnout
altitude and flight-path angle are held to minimum values. Fortunately, these conditions may be desirable in boost trajectories for other reasons. For example, the payload capability of most booster systems increases as the burnout altitude decreases.

**ILLUSTRATIVE EXAMPLES**

In order to demonstrate the combined implications of the results presented in the previous sections, use will again be made of the boost trajectory presented in figure 1. Burnout occurs at escape speed and zero flight-path angle, at an altitude of 500,000 feet. For an entry vehicle with $L/D = -0.5$ and $m/C_pA = 5$ slugs per square foot, the abort velocity requirement is 4,000 feet per second (fig. 6). Application of this velocity increment in the optimum direction is sufficient to place the vehicle on the overshoot boundary where entry is accomplished with negative lift. If $L/D$ is zero, the velocity requirement is increased to 4,300 feet per second and if it is -2, it is decreased to 3,700 feet per second. If the burnout altitude could be decreased to 300,000 feet, the three values of the velocity requirement decrease to 2,000 feet per second for $L/D = -0.5$; 2,600 feet per second for $L/D = 0$; and 1,300 feet per second for $L/D = -2$. It is noted that entry vehicle aerodynamics have a larger effect at the lower altitude. Boost trajectories with burnout altitudes less than 300,000 feet appear to be impractical because of drag effects on the booster itself.

The velocity requirement for abort at burnout of stage 2 of the trajectory of figure 1 will next be considered. In order to relate the flight conditions (velocity, flight-path angle, and altitude) in the boost trajectory to the desired flight conditions at burnout near escape speed, an approximate analysis has been made of the equations of motion for the boost trajectory and is presented in appendix B. The flight-path angle is given by equation (10) and the altitude by equation (14) (figs. 9 and 10, respectively). The trajectory presented in figure 1 may be closely approximated with the results of this analysis, at least for the final stage of propulsion, by use of a thrust-to-weight parameter $\sigma$ of 1/6. With this value of the parameter, the flight-path angle at 16,000 feet per second is 9.9° (fig. 9), and the altitude is 680,000 feet (fig. 10). If an abort occurs at this point in the boost trajectory, the entry flight conditions at 300,000 feet are easily calculated using the satellite equations of motion. The entry velocity is 16,700 feet per second and the flight-path angle is -16.3°. For an entry vehicle of $L/D = 0.5$ and $m/C_pA = 5$ slugs per square foot, the entry that follows produces a maximum deceleration of 18 g, provided, of course, no corrective thrust is applied. The maximum entry deceleration may be reduced to 8 g if an abort velocity increment of 2,800 feet per second is applied normal to the desired resultant flight path at 300,000 feet (fig. 3). For an entry vehicle
of \( L/D = 0 \) and \( m/C_pA = 5 \) slugs per square foot, a velocity increment of 4,700 feet per second is required to reduce the flight-path angle to 0° at 300,000 feet. The resulting maximum entry deceleration is then approximately 12 g.

CONCLUDING REMARKS

An investigation has been made of the abort velocity requirements associated with failure of a propulsion system for a manned lunar mission. Two cases are considered: abort at less than satellite speed, which results in maximum decelerations in the following entry, and abort at greater than satellite speed with immediate return to earth. The velocity requirements associated with the latter problem were found to be substantial (several thousand feet per second) and tend to become quite severe if the boost trajectories resulted in burnout at escape speeds at high altitudes or flight-path angles. If the flight-path angle is kept small and the burnout altitude is limited to about 300,000 feet, the required velocity impulse may be as low as 2,000 feet per second, depending, in part, upon the lift and drag characteristics of the entry vehicle.

The velocity requirements associated with abort at velocities less than satellite speed for lifting vehicles tend to be less than those required at greater than satellite speed. If abort rockets sufficient to handle burnout at greater than satellite speed are provided, it appears possible to reduce maximum decelerations in an entry following an abort to 8 g. This reduction in deceleration is accomplished by use of the abort rockets to decrease the entry angle immediately prior to entry. Application of rocket thrust at higher altitudes is less effective in diminishing the entry flight-path angle.

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APPENDIX A

SYMBOLS

A   reference area
C_D drag coefficient
D   drag
\( g \) gravity acceleration
I   specific impulse
L   lift
m   mass
r   radius from center of the earth
r_c radius of curvature
r_o radius of the earth
s   distance along the flight path
t   time
T   thrust
V   velocity
\( \bar{V} \) ratio of velocity to local satellite speed
W   weight
y   altitude
\( \Delta V \) velocity impulse
\( \gamma \) flight-path angle with respect to local horizontal (positive upward)
\( \sigma \) reduced thrust-weight ratio (see eq. (11))
Subscripts

bo  burnout
E  entry
i  initial
o  sea level
APPENDIX B

ROCKET BOOST EQUATIONS

In this analysis a rocket booster is considered which develops a constant thrust tangent to the flight path. For a two-dimensional trajectory, the equations of motion parallel and normal to the flight path are, respectively,

\[ \frac{m}{dt} \frac{dV}{dt} + mg \sin \gamma + D = T \]  \hspace{1cm} (1)

\[ \frac{mV^2}{r_0} = mg \cos \gamma \]  \hspace{1cm} (2)

For trajectories sufficiently near the earth (i.e., for \( \gamma \ll r_0 \)), \( r \approx r_0 \) and \( g \approx g_0 \). The radius of curvature may then be written as

\[ \frac{1}{r_c} = \frac{\cos \gamma}{r_0} - \frac{dy}{ds} \]  \hspace{1cm} (3)

and equation (2) becomes

\[ \frac{dy}{\cos \gamma} = \left( \frac{1}{r_0} - \frac{g_0}{V^2} \right) ds \]  \hspace{1cm} (4)

For trajectories for which

\[ mg \sin \gamma + D << T \]  \hspace{1cm} (5)

equation (1) simplifies to

\[ T - m \frac{dV}{dt} = 0 \]  \hspace{1cm} (6)

and it follows that the mass of the vehicle is

\[ m = m_i e^{-\frac{V-V_i}{Ig_0}} \]  \hspace{1cm} (7)

where \( m_i \) and \( V_i \) are the initial mass and velocity, respectively, of the stage and \( I \) is the specific impulse. Combination of equations (4), (6), and (7), for a booster stage initially of weight \( W_i \) gives
\[
\frac{d\gamma}{\cos \gamma} = \frac{V_i}{T} e^{-\frac{V}{Ig_0}} \left( \frac{1}{V} - \frac{V}{Ig_0} - \frac{1}{V} e^{-\frac{V}{Ig_0}} \right) dV
\]  

which integrates to

\[
\tan \left( \frac{\gamma}{2} + \frac{\pi}{4} \right) = \frac{V_i}{T} e^{-\frac{V}{Ig_0}} \left\{ \ln \left[ -\frac{Ig_0}{r_0} \left( -\frac{V}{Ig_0} + 1 \right) + e^{-\frac{V}{Ig_0}} \left( \frac{V}{Ig_0} + 1 \right) \right] \right. \\
+ Ei \left( -\frac{V_{bo}}{Ig_0} \right) - Ei \left( -\frac{V}{Ig_0} \right) \right\}
\]

where \( Ei \) denotes the exponential integral defined as

\[
- Ei(-x) = \int_x^\infty \frac{e^{-u}}{u} du
\]

The boundary conditions in equation (9) are applied by specifying the desired conditions at burnout (subscript \( bo \)) rather than initial conditions.

For the portion of the boost trajectory of interest where the flight-path angles are small, the approximation \( \sin \gamma \approx \gamma \) may be used and equation (9) may be written as

\[
(\gamma - \gamma_{bo}) \sigma = f(V)
\]

where \( f(V) \) is the expression in brackets in equation (9) and \( \sigma \) is defined as

\[
\sigma = \frac{T}{W_i} e^{-\frac{V_i}{Ig_0}}
\]

The parameter \( \sigma \) characterizes the booster stage and is what may be termed a reduced thrust-weight ratio. The flight-path angle (eq. (10)) is shown in figure 9 as a function of velocity for a specific impulse of 400 seconds and a burnout velocity of 36,000 feet per second. The altitude is given approximately by

\[
y = \int y \, ds
\]
where

\[ ds = \frac{1}{\sigma_0} e^{-\frac{V}{I_g}} V \, dV \]  

(13)

Integration along the flight path gives

\[ (\gamma_{bo} - y)\sigma^2 = \frac{1}{\sigma_0} \int_{V}^{V_{bo}} f(V)e^{-\frac{V}{I_g}} V \, dV \]

\[ -\gamma_{bo} I_g \sigma^2 \left[ e^{-\frac{V_{bo}}{I_g}} \left( \frac{V_{bo}}{I_g} + 1 \right) - e^{-\frac{V}{I_g}} \left( \frac{V}{I_g} + 1 \right) \right] \]  

(14)

The first term on the right-hand side of equation (14) is not analytic but is a unique function of velocity given the specific impulse and the burnout velocity and may be computed numerically. The term \((\gamma_{bo} - y)\sigma^2\) is shown in figure 10 as a function of velocity for a specific impulse of 400 seconds and a burnout velocity of 36,000 feet per second.
REFERENCES


Figure 1.- Typical boost trajectory.
Figure 2.- Entry deceleration.

(a) L/D = 0
Figure 2.- Continued.

(b) \( L/D = 0.5 \)
Figure 2.- Continued.

(c) \( L/D = 1 \)

Entry deceleration, \( g \)

\(-\gamma_E, \text{ deg}\)

\( V_E = \) 18,000 ft/sec

17,000

16,000

15,000
Figure 2.- Concluded.
(a) $V_{bo} = 14,000 \text{ ft/sec}, \ L/D = 0$

Figure 3.- Abort velocity requirement for burnout at 14,000 and 16,000 ft/sec.
(c) $V_{bo} = 14,000$ ft/sec, $L/D = 1$

Figure 3.- Continued.
Figure 3.—Continued.
(e) $V_{bo} = 16,000 \text{ ft/sec}, \quad L/D = 0$

Figure 3.- Continued.
(f) $V_{bo} = 16,000$ ft/sec, $L/D = 0.5$

Figure 3.- Continued.
Figure 3.- Continued.
$V_{bo} = 16,000 \text{ ft/sec}$
$L/D = 2$

Entry deceleration, $8g$

$V_{bo} = 1,000,000 \text{ ft}$
$800,000$
$600,000$
$500,000$
$400,000$
$300,000$

$h) V_{bo} = 16,000 \text{ ft/sec}, L/D = 2$

Figure 3.- Concluded.
$y_{bo} = 300,000 \text{ ft}$

Figure 4: Burnout conditions at supersatellite speed and entry conditions for the overshoot boundary.

(a) $y_{bo} = 300,000 \text{ feet.}$
(b) $y_{ba} = 400,000$ feet

Figure 4.- Continued.
(c) \( y_{bo} = 500,000 \text{ feet} \)

Figure 4. - Continued.
\( y_{bo} = 600,000 \text{ ft} \)

Figure 4. - Continued.

(d) \( y_{bo} = 600,000 \text{ feet} \)
(e) $y_{bo} = 800,000$ feet

Figure 4.- Continued.
(f) $y_{bo} = 1,000,000$ feet

Figure 4.- Continued.
Horizontal velocity, ft/sec

Vertical velocity, ft/sec

$V_{bo} = 1.2$ 

$\sqrt{y_{bo}} = 6^\circ$

$y_{bo} = 1,200,000$ feet

(g) $y_{bo} = 1,200,000$ feet

Figure 4.- Continued.
Figure 4.- Concluded.

(h) \( y_{bo} = 1,500,000 \) feet
Figure 5.- Effect of $m/C_D A$ on abort velocity requirement for burnout at escape speed.
Figure 6.- Effect of \( \frac{L}{D} \) on abort velocity requirement for burnout at escape speed.
Figure 7.- Effect of burnout velocity on abort velocity requirement.
Figure 8.- Effect of burnout flight-path angle on abort velocity requirement for burnout at escape speed.
Figure 9.— Boost trajectory flight-path angle.
Figure 10.- Boost trajectory altitude.