NASA TR R-62

IN-18
39 357

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL REPORT
R-62

AN ANALYSIS OF ABLATION-SHIELD REQUIREMENTS FOR MANNED REENTRY VEHICLES

By LEONARD ROBERTS

1960

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C. Yearly subscription, $15; foreign, $19; single copy price varies according to size - - - - - - - - - Price 40 cents

TECHNICAL REPORT R-62

AN ANALYSIS OF ABLATION-SHIELD REQUIREMENTS FOR MANNED REENTRY VEHICLES

By LEONARD ROBERTS

Langley Research Center
Langley Field, Va.

CONTENTS

TECHNICAL REPORT R-62

AN ANALYSIS OF ABLATION-SHIELD REQUIREMENTS FOR MANNED REENTRY VEHICLES

By Leonard Roberts

SUMMARY

The problem of sublimation of material and accumulation of heat in an ablation shield is analyzed and the results are applied to the reentry of manned vehicles into the earth's atmosphere. The parameters which control the amount of sublimation and the temperature distribution within the ablation shield are determined and presented in a manner useful for engineering calculation. It is shown that the total mass loss from the shield during reentry and the insulation requirements may be given very simply in terms of the maximum deceleration of the vehicle or the total reentry time.

INTRODUCTION

The successful return of a vehicle through the earth's atmosphere depends largely on the provision that is made for reducing aerodynamic heat transfer to the structure of the vehicle. Analyses of the heating experienced during reentry have been made for both ballistic vehicles (ref. 1) and manned vehicles (ref. 2).

For the purpose of the present report the types of reentry are categorized as follows:

(a) Lifting vehicles of constant lift-drag ratio which reenter the atmosphere at very small angles (so that skipping does not occur) and which experience maximum decelerations less than about $8g$

(b) Nonlifting vehicles which reenter at small angles and which experience maximum decelerations between $8g$ and $14g$

(c) Ballistic vehicles which enter at larger angles and experience maximum decelerations greater than $14g$

The vehicles considered in (a) and (b) are suitable for manned reentry, whereas the decelerations associated with higher entry angles (type (c)) generally exceed human tolerances.

The heating experience of the manned vehicles also differs from that of the higher-entry-angle ballistic vehicle. For the ballistic vehicle the maximum heating rates are such that surface temperatures may exceed the melting temperatures of metals which have been considered for heat-sink shields. For manned vehicles the flight duration is much longer and, although the maximum heating rate is much lower, the total heat input exceeds that of the ballistic vehicle and the use of a metal heat-sink shield becomes inefficient from a weight standpoint.

As an alternative to the heat sink, consideration has been given to the use of ablation materials. The term ablation applies when there is a removal of material (and an associated removal of heat) caused by aerodynamic heating, and therefore embraces melting, sublimation, melting and subsequent vaporization of the liquid film, burning, and depolymerization.

Several approximate analyses have been made of the steady-state shielding effects which result from this removal of material. Aerodynamic melting has been considered in references 3, 4, and 5, sublimation in reference 6, and simultaneous melting and vaporization in references 7 and 8. A general treatment of the boundary layer with mass addition has been given in reference 9. The problem of keeping the vehicle structure at a suitably low temperature cannot be answered by investigating a steady-state situation, however;

consideration must be given to the problem of insulation, and the conduction of heat to the structure is a transient phenomenon.

The suitability of a heat shield (whether heat sink or ablation material) depends on the weight required to keep the structure below a given temperature, and a simple quantity of merit such as the effective heat capacity or effective heat of ablation gives no indication of the severity of the insulation problem. The use of high-ablation-temperature materials such as graphite, for example, which has high thermal conductivity, could lead to an intolerable heating condition although the effective heat capacity is higher than that of most materials. It is possible that the high-ablation-temperature materials experience less mass loss because of radiation cooling, but this is not necessarily desirable since the high surface temperature makes the insulation problem more severe.

It is seen, therefore, that the problem of maintaining a cool vehicle structure is twofold: there must be adequate provision of material for ablation, and there must be sufficient insulation to prevent the structure from becoming hot.

The effectiveness of an ablation material depends on its capability to dispose of heat by convection in the liquid film, as latent heat, and by convection in gaseous form in the boundary layer. It has been shown (refs. 6 and 7) that in general an ablation shield is most effective when a large fraction of the mass loss undergoes vaporization. When sublimation takes place there is no liquid film; all the mass lost from the shield undergoes vaporization and subsequent convection in the high-temperature boundary layer and thereby removes a large amount of heat. For this reason a material which undergoes sublimation rather than melting is generally more efficient (apart from considerations of latent heat).

Naturally, the choice of ablation material will be dictated by the type of vehicle and its heating history during reentry. For a nonlifting vehicle whose dimensions are such that the heating rates experienced are too high to be balanced by radiative cooling, it is advantageous to use a material with a low ablation temperature in order to reduce the insulation problem. For lifting vehicles, which experience lower heating rates over most of the surface, the primary means of cooling would be radiative except at the leading edges where the limited use of a material with a high ablation temperature would seem more appropriate; ablation would then take place only near the peak heating condition.

The materials here considered most suitable for the reentry of a manned nonlifting capsule are therefore those which undergo sublimation at a low temperature (say less than 1,500° R) and have low conductivity so that no further insulation is required. The absence of a liquid phase insures that the material is removed in gaseous form and therefore convects a large amount of heat from the shield, and also precludes the possibility of liquid-film instability.

The purpose of the report is to develop an approximate method of solution for such a shield from which may be determined the total sublimation of material during reentry and the temperature distribution within the remaining shield; the ablation temperature is such that radiation may be neglected. The analysis is directed toward obtaining results useful for engineering purposes.

SYMBOLS

A	reference area for drag and lift, sq ft
a	horizontal acceleration, g units
C_D	drag coefficient
c	specific heat, Btu/(lb)(°R)
c_p	specific heat at constant pressure, Btu/(lb)(°R)
g	gravitational acceleration, ft/sec^2
H_{eff}	effective heat capacity, Btu/lb
h	altitude, ft
J	mechanical equivalent of heat, 778 ft-lb/Btu
k	thermal conductivity, Btu/(ft)(sec)(°R)
k_{I}	ratio of local heat flux to that at stagnation point, $\frac{q_0}{q_{sp,0}}$
k_{II}	average value of heat flux relative to stagnation-point value, $\frac{1}{S}\int_S \frac{q_0}{q_{sp,0}}\,dS$
L	latent heat of sublimation, Btu/lb
L/D	ratio of lift force to drag force
l	characteristic length of vehicle, ft
M	mass of vehicle, slugs
m	mass ablated per unit area, lb/sq ft
N_{Pr}	Prandtl number

N_{Sc}	Schmidt number
Q	total convective heat absorbed per unit area, Btu/sq ft
$\overline{Q}$	dimensionless heat absorbed per unit area
q	local convective heat-transfer rate per unit area, Btu/(sq ft)(sec)
$\overline{q}$	dimensionless heat-transfer rate
R	radius of curvature of nose, ft
R_∞	Reynolds number, $\rho_\infty V l/\mu_\infty$
r	distance from center of earth to orbit, ft
S	surface area wetted by boundary layer, sq ft
T	temperature, °R
t	time, sec
$\bar{t}=t(\beta g)^{1/2}=\dfrac{t}{27}$	
u	tangential velocity component, ft/sec
u_c	circular orbital velocity, $(gr)^{1/2}=26{,}000$ ft/sec
$\overline{u}=\dfrac{u}{u_c}$	
V	total velocity, ft/sec
v	temperature ratio, $\dfrac{T-T_\infty}{T_a-T_\infty}$
W	weight of vehicle at earth's surface, lb
y	outward normal distance from initial position of ablation surface, ft
Z	dimensionless function of $\overline{u}$ determined by equation (4)
z	outward normal distance from ablation surface, ft
α	fractional temperature rise of gaseous material
β	atmospheric density decay parameter, ft^{-1}
ϵ	dimensionless ablation rate
ζ	dimensionless heat content
η	fractional decrease in mass loss due to latent heat
θ	integral thickness of heated layer in solid shield, ft
λ	latent heat parameter, equation (33) or (34)
μ	coefficient of dynamic viscosity, slugs/ft-sec
ξ	dimensionless distance, $\left(\dfrac{\rho_b c_b}{k_b t_f}\right)^{1/2} z$
ρ	density, slugs/cu ft
ρ^*	value of reference density for exponential approximation to density-altitude relationship, 0.0027 slug/cu ft
τ	dimensionless time, t/t_f
φ	flight-path angle relative to local horizontal direction; negative for descent, deg
χ	conduction parameter, equation (64)

Subscripts:

0	no sublimation
∞	free stream
1	gas produced by sublimation
2	air behind shock wave
a	sublimation condition
b	solid shield condition
e	external to boundary layer at stagnation point
f	final conditions
i	initial condition
s	surface condition
sp	stagnation point

Superscripts:

$'$	differentiation with respect to $\overline{u}$
$\bar{\ }$	dimensionless quantity
$\sim$	mean value

ANALYSIS

ASSUMPTIONS AND APPROXIMATIONS

The analysis of unsteady sublimation of material from the nose of a body during reentry into the earth's atmosphere requires a knowledge of the heating experienced by the vehicle. Throughout this report the equations which describe the motion and heating history of the vehicle are those developed in reference 2; the present analysis therefore contains all the assumptions stated in reference 2 in addition to those stated now which concern the properties of the material and its behavior during sublimation:

(1) The density ρ_b, thermal conductivity k_b, specific heat c_b, and latent heat of sublimation L have constant values

(2) The Prandtl number N_{Pr} and the Schmidt number N_{Sc} of the gas mixture in the boundary layer and the specific heats $c_{p,1}$ and $c_{p,2}$ are constant

(3) The sublimation temperature T_a is constant

(4) Sublimation leaves a smooth surface and causes negligible change in the shape of the shield compared with the scale of the nose

(5) The boundary layer remains laminar for most of the reentry

To facilitate a simplification of the conduction problem the following approximations are made:

(6) The material is sufficiently thick to allow use of the "infinitely thick slab" approximation.

(7) Conduction of heat takes place along lines normal to the surface.

A few remarks in justification of the assumptions and approximations seem relevant. Assumption (1) of constant properties may not always be appropriate, but it is hoped that the analysis will give useful results when mean values of ρ_b, c_b, and k_b are used. The effect on the gas-layer shielding of variations in N_{Pr} and N_{Sc} is secondary compared with variations in $c_{p,1}$ and $c_{p,2}$ (see assumption (2)); however these specific heats appear only in the ratio $c_{p,1}/c_{p,2}$, which should have little variation with temperature.

The sublimation temperature T_a is correctly determined by consideration of the phase equilibrium between the solid material and its vapor; in practice, however, T_a varies within a limited range only and assumption (3) is justified. The assumption of smooth sublimation is actually a required property of the material if it is to be a successful shield.

The Reynolds numbers experienced during reentry depend on the parameter W/C_DA for the vehicle. For manned-capsule reentry, typical values of the Reynolds number indicate laminar flow throughout the flight through the upper atmosphere where the most severe heating is experienced.

MOTION AND HEATING DURING REENTRY

An analysis of shallow reentry into the earth's atmosphere, for both lifting and nonlifting vehicles, has been made in reference 2. The results of interest for the present application are included here for completeness.

The variation of density with altitude is assumed to be exponential in form; that is,

$$\frac{\rho_\infty}{\rho^*}=e^{-\beta h} \tag{1}$$

where ρ^* and β have the numerical values

$$\rho^*=0.0027 \text{ slug/cu ft} \tag{2a}$$

$$\beta=\frac{1}{23500}\text{ ft}^{-1} \tag{2b}$$

The equations of motion are combined and written in terms of a dimensionless tangential velocity component $\overline{u}=\frac{u}{u_c}=\frac{u}{(gr)^{1/2}}$ and a new independent variable

$$Z=\frac{\rho^*}{2\frac{M}{C_DA}}\left(\frac{r}{\beta}\right)^{1/2}\overline{u}e^{-\beta h} \tag{3}$$

in the following form:

$$\overline{u}Z''-\left(Z'-\frac{Z}{\overline{u}}\right)=\frac{1-\overline{u}^2}{\overline{u}Z}\cos^4\varphi-(\beta r)^{1/2}\frac{L}{D}\cos^3\varphi \tag{4}$$

For reentry at small angles the tangential component of velocity u is approximately equal to the total velocity, and $\cos\varphi\approx 1$.

The initial conditions used herein are those appropriate to entries starting from a circular or near-circular orbit at high altitude:

$$Z_i=0 \tag{5a}$$

$$Z_i'=(\beta r)^{1/2}\sin\varphi_i \tag{5b}$$

at $\overline{u}=1$.

All quantities of interest—for example, deceleration, heating rate, and Reynolds number—can be determined from the solution of equation (4). The deceleration is given by

$$-a=-\frac{1}{g}\frac{du}{dt}=(\beta r)^{1/2}\overline{u}Z=30\,\overline{u}Z \tag{6}$$

and the elapsed time by

$$t=(\beta g)^{-1/2}\bar{t}=27\int_{\overline{u}_2}^{\overline{u}_1}(\overline{u}Z)^{-1}d\overline{u} \tag{7}$$

The free-stream Reynolds number per unit length is

$$\frac{R_\infty}{l}=\frac{V\rho_\infty}{\mu_\infty}=7100\frac{W}{C_DA}Z \tag{8}$$

As indicated in reference 2, the Reynolds numbers experienced during satellite reentry are such that laminar flow may be expected to occur for the greater part of the reentry $\left(\text{typical values } \frac{W}{C_DA}=40 \text{ and } Z=0.5 \text{ give } \frac{R_\infty}{l}=1.4\times10^5\right)$.

The stagnation temperature of the stream is given by

$$c_{p,2}T_\infty+\frac{1}{2}\frac{u^2}{Jg}=c_{p,2}T_t$$

where $T_\infty = 432°$ R. Thus

$$c_{p,2}(T_e - T_\infty) = \frac{1}{2}\,\bar{u}^2\,\frac{u_c^2}{Jg} \tag{9}$$

The laminar convective heating rate which would be experienced by a nonablating body is given in terms of $\frac{M}{C_D A R}$, $\bar{u}$, and Z as

$$q_0 = K k_I \left(\frac{M}{C_D A R}\right)^{1/2} \bar{q}_0 \tag{10}$$

where

$$\bar{q}_0 = \left[\bar{u}^2 - \frac{c_{p,2}(T_s - T_\infty)}{\frac{1}{2}\frac{u_c^2}{Jg}}\right](\bar{u}Z)^{1/2}$$

$k_I = \frac{q_0}{q_{sp,0}}$ is a factor determined by the shape of the nose of the body, and the viscosity law $\mu_e \sim T_e^{1/2} \sim \bar{u}$ is used. The term $\frac{c_{p,2}(T_s - T_\infty)}{\frac{1}{2}\frac{u_c^2}{Jg}}$ is small compared with $\bar{u}^2$ and is neglected; thus

$$\bar{q}_0 = \bar{u}^2(\bar{u}Z)^{1/2} \tag{11}$$

The constant K in equation (10) is that used in reference 2; that is, $K = 590$. Then

$$q_0 = 590 k_I \left(\frac{M}{C_D A R}\right)^{1/2} \bar{u}^2(\bar{u}Z)^{1/2} \text{ Btu/(sq ft)(sec)} \tag{12}$$

The total heat which would be accumulated by a nonablating shield is found by integration of equation (12) to be

$$Q_0 = 15{,}900 k_{II} \left(\frac{M}{C_D A R}\right)^{1/2} \bar{Q}_0 \text{ Btu/sq ft} \tag{13}$$

where

$$\bar{Q}_0 = \int_{\bar{u}}^{\bar{u}_i} \bar{u}^2(\bar{u}Z)^{-1/2} d\bar{u} \tag{14}$$

and

$$k_{II} = \frac{1}{S}\int_S \frac{q_0}{q_{sp,0}}\, dS \tag{15}$$

The foregoing relations (eqs. (6) to (15)) are used in the determination of the mass required for sublimation and the accumulation of heat within the solid shield.

GENERAL EQUATIONS FOR SUBLIMATION AND HEAT ACCUMULATION

Before the vehicle reenters the atmosphere the ablation shield is assumed to have uniform temperature T_∞. In the early part of reentry the shield is heated until the surface temperature reaches the ablation temperature T_a. During this preablation heating period the problem of conduction of heat through the shield can be treated without difficulty since the heating rate is known (eq. (10)). When sublimation of material occurs, however, the conduction of heat within the material depends on the rate of mass loss from the surface, the problem becomes nonlinear, and the exact solution involves lengthy numerical procedures. (See, for example, ref. 10.)

It is not the purpose of this report to obtain exact solutions of the nonlinear equations; rather, approximate results are obtained which show all the important parameters that enter into the problem and give estimates of the material required for sublimation and for absorbing the heat conducted to the interior. Figure 1 is a diagram of the heat shield under consideration.

The development of the differential equation and appropriate boundary conditions is given in the appendix. For the present it is more useful to consider a heat energy balance as follows:

$$Q(t) \quad = \quad [c_b(T_a - T_\infty) + L]m + \rho_b c_b \int_{-\infty}^{0} (T - T_\infty)\, dz \tag{16}$$

Net heat input at surface	Heat absorbed by sublimated material	Heat accumulated by remaining material

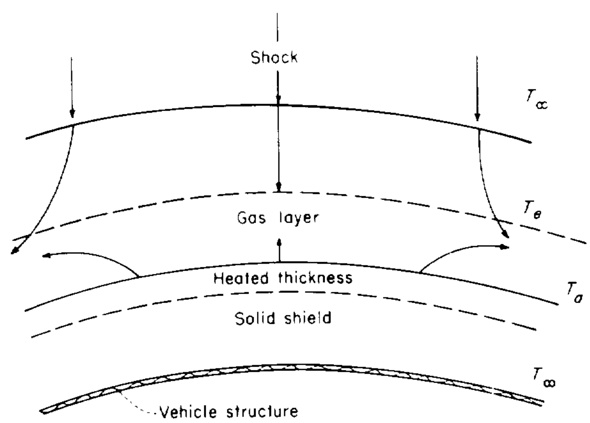

FIGURE 1. Sublimation at the nose.

When an integral thickness θ defined as

$$\theta(t)=\int_{-\infty}^{0}\frac{T-T_{\infty}}{T_s-T_{\infty}}\,dz \tag{17}$$

is introduced into equation (16) the following equation results:

$$Q(t)=[c_b(T_a-T_{\infty})+L]m+\rho_b c_b(T_s-T_{\infty})\theta \tag{18}$$

Alternatively, equation (18) may be written in differential form as

$$q(t)=[c_b(T_a-T_{\infty})+L]\frac{dm}{dt}+\rho_b c_b\frac{d}{dt}[(T_s-T_{\infty})\theta] \tag{19}$$

An additional equation (a boundary condition at the surface of sublimation) is written

$$q(t)=L\frac{dm}{dt}+\left(k_b\frac{\partial T}{\partial z}\right)_{z=0} \tag{20}$$

Net heat-transfer rate to surface	Rate of heat absorbed in phase change	Heat-transfer rate to interior

It is important to note that in equations (19) and (20) the heat-transfer rate $q(t)$ is that which the shield actually experiences and is itself a function of the rate of sublimation. Throughout this report the quasi-steady relation for the reduction in heat-transfer rate due to the introduction of mass into the boundary layer is used (see ref. 6):

$$q_0(t)-q(t)=\alpha\tilde{c}_p(T_e-T_a)\frac{dm}{dt} \tag{21}$$

where $q_0(t)$ is the heat-transfer rate experienced by a nonablating body at the surface temperature T_a.

In equation (21) $\tilde{c}_p$ is the effective mean specific heat and $\alpha(T_e-T_a)$ the effective temperature rise of the mass convected in the boundary layer. The expressions for α and $\tilde{c}_p$ derived in reference 6 for a laminar boundary layer are

$$\alpha=1-\frac{1}{3}N_{Pr}^{-0.6} \tag{22}$$

and

$$\tilde{c}_p=c_{p,1}\tilde{w}+c_{p,2}(1-\tilde{w}) \tag{23}$$

where $\tilde{w}$ is the effective concentration of the shield material in gaseous form in the boundary layer and is given as a function of N_{Sc} in reference 6.

The unknown heating rate $q(t)$ is eliminated from equations (19) and (20) by use of equation (21) to give

$$q_0(t)=[c_b(T_a-T_{\infty})+L+\alpha\tilde{c}_p(T_e-T_a)]\frac{dm}{dt}+\rho_b c_b\frac{d}{dt}[(T_s-T_{\infty})\theta] \tag{24}$$

and

$$q_0(t)=[L+\alpha\tilde{c}_p(T_e-T_a)]\frac{dm}{dt}+\left(k_b\frac{\partial T}{\partial z}\right)_{z=0} \tag{25}$$

A comparison of equations (24) and (25) shows that

$$\left(k_b\frac{\partial T}{\partial z}\right)_{z=0}=c_b(T_a-T_{\infty})\frac{dm}{dt}+\rho_b c_b\frac{d}{dt}[(T_s-T_{\infty})\theta] \tag{26}$$

which, when integrated, gives

$$\int_0^t\left(k_b\frac{\partial T}{\partial z}\right)_{z=0}dt=c_b(T_a-T_{\infty})m+\rho_b c_b(T_s-T_{\infty})\theta \tag{27}$$

Equation (27) shows that only part of the heat transferred from the surface toward the interior is accumulated (an amount $\rho_b c_b(T_s-T_{\infty})\theta$), the remainder being required to raise the mass m to the ablation temperature.

In order to solve equations (24) and (25) the relationship between $\left(k_b\frac{\partial T}{\partial z}\right)_{z=0}$ and θ must be known; the exact determination of this relationship would involve the solution of the nonlinear conduction equation (as shown in the appendix). In this report only approximate relations between $\left(k_b\frac{\partial T}{\partial z}\right)_{z=0}$ and θ are used.

Before attempting to take account of the conduction of heat within the shield it is useful to make a simple analysis of the sublimation to obtain expressions for the total mass loss during reentry.

SUBLIMATION OF MATERIAL FROM THE SHIELD

The purpose of an ablation shield is to reduce the heat-transfer rate at the surface from the aerodynamic rate q_0 to a value $\left(k_b \frac{\partial T}{\partial z}\right)_{z=0}$ by providing material which absorbs heat (through latent heat of sublimation) and convects heat in the gas boundary layer; this situation is reflected in equation (25). If this process is successful, then $\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} \ll q_0$ for most of the reentry and an upper limit to the rate of mass loss can be obtained by neglecting heat conduction in the solid. From equation (25), with $\left(k_b \frac{\partial T}{\partial z}\right)_{z=0}=0$,

$$\frac{dm}{dt}=\frac{q_0}{L+\alpha\tilde{c}_p(T_e-T_a)} \tag{28}$$

Alternatively, if it is assumed that the rate of accumulation of heat

$$\frac{d}{dt}\left[\rho_b c_b(T_s-T_\infty)\theta\right]$$

is small compared with the rate of disposal of heat

$$\left[c_b(T_a-T_\infty)+L+\alpha\tilde{c}_p(T_e-T_a)\right]\frac{dm}{dt}$$

equation (24) becomes

$$\frac{dm}{dt}=\frac{q_0}{c_b(T_a-T_\infty)+L+\alpha\tilde{c}_p(T_e-T_a)} \tag{29}$$

Equation (29) is the quasi-steady expression for the rate of mass loss but is not, in general, an upper limit. Substitution for q_0 and $T_e-T_a\approx T_e-T_\infty$ from equations (12) and (9), respectively, and use of the relation

$$\frac{dm}{dt}=\frac{dm}{d\bar{u}}\frac{d\bar{u}}{dt}=-\frac{1}{27}\bar{u}Z\frac{dm}{d\bar{u}} \tag{30}$$

gives

$$\frac{dm}{d\bar{u}}=1.18k_{\mathrm{I}}\frac{\left(\frac{M}{C_DAR}\right)^{1/2}}{\alpha\frac{\tilde{c}_p}{c_{p,2}}}\frac{d\overline{m}}{d\bar{u}}\text{ lb/sq ft} \tag{31}$$

where

$$\frac{d\overline{m}}{d\bar{u}}=-\left(1-\frac{\lambda}{\bar{u}^2+\lambda}\right)(\bar{u}Z)^{-1/2} \tag{32}$$

and

$$\lambda=\frac{L}{\frac{1}{2}\frac{u_c^2}{Jg}\alpha\frac{\tilde{c}_p}{c_{p,2}}} \tag{33}$$

when equation (28) is used, or

$$\lambda=\frac{L+c_b(T_a-T_\infty)}{\frac{1}{2}\frac{u_c^2}{Jg}\alpha\frac{\tilde{c}_p}{c_{p,2}}}=\frac{\text{Maximum internal shielding enthalpy}}{\text{Maximum external shielding enthalpy}} \tag{34}$$

when equation (29) is used.

Equation (31) shows immediately the importance of the parameter

$$\frac{\left(\frac{M}{C_DAR}\right)^{1/2}}{\alpha\frac{\tilde{c}_p}{c_{p,2}}}\approx\frac{\text{Heating coefficient}}{\text{Gas shielding coefficient}}$$

which depends on the vehicle size and shape through $\left(\frac{M}{C_DAR}\right)^{1/2}$ and the properties of the gas boundary layer. It is seen that the mass loss can be reduced by designing the vehicle so that M/C_DAR is small (low mass, high drag, blunt nose) and by choosing an ablation material having a high shielding coefficient $\alpha\frac{\tilde{c}_p}{c_{p,2}}$. The factor α depends on the nature of the boundary layer and is given in equation (22) for a laminar boundary layer. The specific-heat ratio $\tilde{c}_p/c_{p,2}$ is given in equation (23) in terms of $c_{p,1}/c_{p,2}$ and it is seen that this ratio should be large in order to reduce the mass loss.

A second parameter, the enthalpy ratio

$$\lambda=\frac{L}{\frac{1}{2}\frac{u_c^2}{Jg}\alpha\frac{\tilde{c}_p}{c_{p,2}}}=\frac{\text{Shielding due to latent heat}}{\text{Gas-layer convective shielding}}$$

shows the effect of the latent heat L in reducing the mass loss; when $\alpha\frac{\tilde{c}_p}{c_{p,2}}$ is small the effect of latent heat becomes more important.

Mass loss.—An upper limit to the total mass loss is obtained by integration of equation (31) as

$$m=1.18k_{\mathrm{II}}\frac{\left(\frac{M}{C_DAR}\right)^{1/2}}{\alpha\frac{\tilde{c}_p}{c_{p,2}}}\overline{m}\text{ lb/sq ft} \tag{35}$$

where

$$\overline{m}=\int_{\bar{u}_f}^{\bar{u}_0}\left(1-\frac{\lambda}{\bar{u}^2+\lambda}\right)(\bar{u}Z)^{-1/2}d\bar{u} \tag{36}$$

and $\bar{u}_a$ and $\bar{u}_f$ are, respectively, the values of $\bar{u}$ when sublimation begins and ends.

Equation (35) with $k_{II}=1$ gives the mass loss at the stagnation point, and the factor $k_{II}=\frac{1}{S}\int\frac{q_0}{q_{sp,0}}dS$ modifies this mass loss according to the variation of heating rate over the surface of the shield. The analysis of reference 2 does not apply at the condition $\bar{u}=1$ and it is necessary to assume a value $\bar{u}<1$ as the upper limit of $\bar{u}$. The nominal value $\bar{u}=0.995$, used in reference 2, is also used here as the value at which sublimation begins. The lower limit depends on the ablation temperature of the material, since ablation will cease before the stagnation temperature of the stream falls below the ablation temperature. The value $\bar{u}_f=0.05$ (which corresponds to a stream temperature of about 200° F) is used throughout this report; this value is considered sufficiently low to include any material now under consideration.

Thus $\bar{m}$ can be written

$$\bar{m}=\bar{m}_{\lambda=0}(1-\eta) \tag{37}$$

where

$$\bar{m}_{\lambda=0}=\int_{0.05}^{0.995}(\bar{u}Z)^{-1/2}d\bar{u} \tag{38}$$

and

$$\eta=\frac{\int_{0.05}^{0.995}\frac{\lambda}{\bar{u}^2+\lambda}(\bar{u}Z)^{-1/2}d\bar{u}}{\int_{0.05}^{0.995}(\bar{u}Z)^{-1/2}d\bar{u}} \tag{39}$$

Equation (38) shows that, even when $\lambda=0$ (no latent heat), there is a limiting value of the total mass loss, whereas equation (39) gives $\eta(\lambda)$, the fractional decrease in mass loss due to latent heat. The evaluation of the integrals in equations (38) and (39) can be carried out when the appropriate Z-functions are inserted. The dependence of these functions on L/D and $-\varphi_i$ has been discussed in detail in reference 2.

Effective heat capacity.—When comparing an ablation shield with a solid "heat-sink" shield (for example, copper or beryllium) it is convenient to introduce an effective heat capacity defined here by the following ratio:

$$H_{eff}=\frac{\text{Total heat which would be absorbed by a nonablating shield}}{\text{Total mass loss from ablation shield}}$$

The total heat absorbed Q_0 is given by equation (13) as

$$Q_0=15{,}900k_{II}\left(\frac{M}{C_DAR}\right)^{1/2}\bar{Q}_0\text{ Btu/sq ft}$$

where, from equation (14),

$$\bar{Q}_0=\int_{0.05}^{0.995}\bar{u}^2(\bar{u}Z)^{-1/2}d\bar{u}$$

and the mass loss is given by equations (35) and (36).

When these expressions are used the following result is obtained:

$$H_{eff}=13{,}500\alpha\frac{\tilde{c}_p}{c_{p,2}}\bar{H}_{eff}\text{ Btu/lb} \tag{40}$$

where

$$\bar{H}_{eff}=(\bar{H}_{eff})_{\lambda=0}(1-\eta)^{-1} \tag{41a}$$

and

$$(\bar{H}_{eff})_{\lambda=0}=\frac{\int_{0.05}^{0.995}\bar{u}^2(\bar{u}Z)^{-1/2}d\bar{u}}{\int_{0.05}^{0.995}(\bar{u}Z)^{-1/2}d\bar{u}} \tag{41b}$$

Equations (40) and (41) show that H_{eff} depends only on the properties of the ablation material (through $\tilde{c}_p/c_{p,2}$ and λ) and on the vehicle trajectory (since $(\bar{H}_{eff})_{\lambda=0}$ is a function only of trajectory).

Relations between mass loss, deceleration, and time of reentry.—It is seen from equations (30) and (32) that the maximum sublimation rate depends on the value of λ, since

$$\frac{dm}{dt}\sim\frac{\bar{u}^2}{\bar{u}^2+\lambda}(\bar{u}Z)^{1/2}$$

For large values of λ this maximum occurs when $\bar{u}^2(\bar{u}Z)^{1/2}$ is greatest (that is, at peak heating (see eq. (11))) and when $\lambda=0$, it occurs when $(\bar{u}Z)^{1/2}$ is greatest (that is, at peak deceleration (as seen from eq. (6))). Thus, in general, the maximum sublimation rate occurs between peak heating and peak deceleration.

In general, the total mass loss will depend on the total time taken to complete reentry since $\bar{m}_{\lambda=0}$ is a function of the trajectory. It is of extreme interest, therefore, to determine how the mass loss may be reduced by allowing the vehicle to complete reentry in a short period of time but with the reservation that the maximum deceleration be kept to a tolerable level.

The relationship between the total mass loss, horizontal deceleration, and time of reentry becomes apparent when equations (6), (7), and (38) are recalled:

$$-a=(\beta r)^{1/2}\bar{u}Z$$

$$\bar{t}=\int_{0.05}^{0.995}(\bar{u}Z)^{-1}d\bar{u}$$

and

$$\bar{m}_{\lambda=0}=\int_{0.05}^{0.995}(\bar{u}Z)^{-1/2}d\bar{u}$$

Since $\bar{u}Z\leqq(\bar{u}Z)_{max}$, it is seen that

$$\bar{m}_{\lambda=0}\geqq\left[\frac{-a_{max}}{(\beta r)^{1/2}}\right]^{-1/2}$$

and since

$$\left[\int_0^1(\bar{u}Z)^{-1/2}d\bar{u}\right]^2\leqq\int_0^1(\bar{u}Z)^{-1}d\bar{u}$$

(by a simple application of the Schwartz integral inequality), then

$$\bar{m}_{\lambda=0}\leqq\bar{t}^{1/2}$$

Thus, in general, $\bar{m}_{\lambda=0}$ satisfies

$$\left[\frac{1}{30}(-a_{max})\right]^{-1/2}\leqq\bar{m}_{\lambda=0}\leqq\left(\frac{t}{27}\right)^{1/2}$$

(where the numerical values of $(\beta r)^{1/2}$ and $(\beta g)^{1/2}$ have been inserted), a result independent of vehicle characteristics or trajectory. When an average value of $\bar{u}Z$ equal to $\frac{1}{2}(\bar{u}Z)_{max}$ is used, the following simple rule may be expected to hold:

$$\bar{m}_{\lambda=0}\approx\left[\frac{1}{30}\left(\frac{-a_{max}}{2}\right)\right]^{-1/2} \tag{42}$$

That is,

Rule A: *The total mass loss varies inversely as the square root of the maximum horizontal deceleration.*

Alternatively, comparing the integral expressions for t and $\bar{m}_{\lambda=0}$ leads to the following relation:

$$\bar{m}_{\lambda=0}\sim\left(\frac{t}{27}\right)^{1/2}$$

That is,

Rule B: *The total mass loss varies as the square root of time for reentry.*

The foregoing relations between mass loss, deceleration, and reentry time provide very simple expressions for the sublimation during reentry.

Application to particular vehicles.—The simple approximate rule A is investigated by comparing exact results for $\bar{m}_{\lambda=0}$ (both analytic and numerical) with equation (42). The integrals that appear in equations (38), (39), and (41b) are evaluated for the following types of vehicle:

(a) Lifting vehicle at zero reentry angle
(b) Nonlifting capsule at small entry angle
(c) Ballistic vehicle

Although the decelerations experienced by type (c) vehicles usually exceed human tolerances, it is nevertheless of interest to compare the results with those for types (a) and (b).

(a) Lifting vehicle $\left(\frac{L}{D}>1;-\varphi_i=0\right)$

The appropriate Z-function is written

$$Z=\frac{1-\bar{u}^2}{30\frac{L}{D}\bar{u}} \tag{43}$$

From equation (38),

$$\bar{m}_{\lambda=0}=\left(30\frac{L}{D}\right)^{1/2}\int_{0.05}^{0.995}(1-\bar{u}^2)^{-1/2}d\bar{u}$$
$$=1.418\left(30\frac{L}{D}\right)^{1/2} \tag{44}$$

and from equation (39),

$$\eta=\frac{\int_{0.05}^{0.995}\frac{\lambda}{\bar{u}^2+\lambda}(1-\bar{u}^2)^{-1/2}d\bar{u}}{\int_{0.05}^{0.995}(1-\bar{u}^2)^{-1/2}d\bar{u}}\approx\left(\frac{\lambda}{1+\lambda}\right)^{1/2} \tag{45}$$

(When the limits of integration are 0 and 1, $\eta\equiv\left(\frac{\lambda}{1+\lambda}\right)^{1/2}$.) From equation (41b),

$$(\bar{H}_{eff})_{\lambda=0}=\frac{\int_{0.05}^{0.995}\bar{u}^2(1-\bar{u}^2)^{-1/2}d\bar{u}}{\int_{0.05}^{0.995}(1-\bar{u}^2)^{-1/2}d\bar{u}}\approx\frac{1}{2} \tag{46}$$

(Again, $(\bar{H}_{eff})_{\lambda=0}\equiv\frac{1}{2}$ when the limits are 0 and 1.)

The maximum deceleration is

$$-a_{max}=30(\overline{u}Z)_{max}=\left(\frac{L}{D}\right)^{-1} \tag{47}$$

and equation (42) gives the approximate result

$$\overline{m}_{\lambda=0}=2^{1/2}\left(30\frac{L}{D}\right)^{1/2} \tag{48}$$

A comparison with equation (44) shows that the error is less than one-half of 1 percent.

(b) Nonlifting capsule $\left(\frac{L}{D}=0;-\varphi_i<5^\circ\right)$

The Z-functions for nonlifting vehicles at small reentry angle $0\leqq-\varphi_i<5^\circ$ are tabulated in reference 2, together with associated functions of Z. These functions, which are included in table I of the present paper for easy reference, are used to evaluate $\overline{m}_{\lambda=0}$, η, and $(\overline{H}_{eff})_{\lambda=0}$ by numerical integration and the results are given in table II, together with those for cases (a) and (c). For case (b) the agreement between the results for $\overline{m}_{\lambda=0}$ given by rule A and the exact values is again good except for $-\varphi_i\leqq\frac{1}{2}^\circ$, where the error is 10 percent.

The quantities $(\overline{H}_{eff})_{\lambda=0}$ and η for case (b) agree almost exactly at $-\varphi_i=0$ with the corresponding values for the lifting vehicle. As $-\varphi_i$ increases from zero, however, both $(\overline{H}_{eff})_{\lambda=0}$ and η change, but approach constant values independent of φ_i when $-\varphi_i>5^\circ$ as shown by the following consideration of case (c).

(c) Ballistic vehicle $\left(\frac{L}{D}=0;-\varphi_i>5^\circ\right)$

The required Z-function is now

$$Z=30\sin(-\varphi_i)\overline{u}(-\log_e\overline{u}) \tag{49}$$

which is inserted into equation (38) to give

$$\overline{m}_{\lambda=0}=[30\sin(-\varphi_i)]^{-1/2}\int_{0.05}^{0.995}(-\overline{u}^2\log_e\overline{u})^{-1/2}d\overline{u}$$
$$=3.32\,[30\sin(-\varphi_i)]^{-1/2} \tag{50}$$

The maximum deceleration is also easily obtained as

$$-a_{max}=\left(-\frac{1}{g}\frac{du}{dt}\right)_{max}=\frac{30^2\sin(-\varphi_i)}{2e} \tag{51}$$

and equation (42) gives

$$\overline{m}_{\lambda=0}=3.30[30\sin(-\varphi_i)]^{-1/2} \tag{52}$$

A comparison of equations (50) and (52) shows that the error incurred by the use of the approximate relation (42) is less than 1 percent.

Evaluation of the integrals in equation (41b) gives

$$(\overline{H}_{eff})_{\lambda=0}=\frac{\left(\frac{\pi}{2}\right)^{1/2}}{3.32}=0.35 \tag{53}$$

which is independent of $-\varphi_i$.

The function $\eta(\lambda)$ is also independent of $-\varphi_i$ for $-\varphi_i>5^\circ$ and is evaluated numerically from equation (39). Table II shows that $\eta\rightarrow1$ as $\lambda\rightarrow\infty$ for all cases. Also, as $\lambda\rightarrow\infty$, the effective heat capacity H_{eff} must tend to a limit L, the latent heat, or

$$\overline{H}_{eff}=(\overline{H}_{eff})_{\lambda=0}(1-\eta)^{-1}\rightarrow\lambda$$

This behavior may be verified for case (a) where

$$(\overline{H}_{eff})_{\lambda=0}(1-\eta)^{-1}=\frac{1}{2}\left[1-\left(\frac{\lambda}{1+\lambda}\right)^{1/2}\right]^{-1}\rightarrow\lambda$$

It is seen that $\overline{H}_{eff}\rightarrow\lambda$ because the exponent of $\frac{\lambda}{1+\lambda}$ is equal to $\overline{H}_{eff}\left(=\frac{1}{2}\right)$. This behavior suggests that η, which is equal to $\left(\frac{\lambda}{1+\lambda}\right)^{1/2}$ for case (a), should be more generally

$$\eta=\left(\frac{\lambda}{1+\lambda}\right)^{(\overline{H}_{eff})_{\lambda=0}} \tag{54}$$

and in fact a comparison shows that this result agrees with those in table II to within 1 percent.

The foregoing results for $\overline{m}_{\lambda=0}$, $(\overline{H}_{eff})_{\lambda=0}$, and η are shown in figures 2, 3, 4, and 5; in figure 2 the values of the maximum deceleration required for equation (42) were obtained from figure 6(a) of reference 2 for $0\leqq-\varphi_i<6^\circ$, and from equation (51) of the present report for $-\varphi_i>6^\circ$.

In figure 3, the values of the maximum deceleration were taken from figure 10 of reference 2 for $0\leqq\frac{L}{D}\leqq1$, and from equation (47) of the present report for $\frac{L}{D}>1$.

TABLE I.—VALUES OF Z AND RELATED QUANTITIES FOR $L/D=0$ AND $0 \leqq -\varphi_i \leqq 5°$

$\bar{u}$	Z	$\bar{q}_0$	$\bar{Q}_0$	t, sec	Z	$\bar{q}_0$	$\bar{Q}_0$	t, sec	Z	$\bar{q}_0$	$\bar{Q}_0$	t, sec
	$-\varphi_i=0°$				$-\varphi_i=\frac{1}{2}°$				$-\varphi_i=1°$			
0.995	0.00058	0.0238	0	0	0.00131	0.0357	0	0	0.0026	0.0504	0	0
.99	.00165	.0396	.156	140	.00270	.0507	.113	72	.0053	.0710	.080	37
.98	.00467	.0650	.338	238	.00603	.0738	.265	140	.0108	.0988	.191	73
.96	.01315	.1035	.550	309	.01457	.1090	.460	199	.0229	.1366	.342	108
.94	.0241	.1330	.688	341	.0253	.1363	.593	229	.0361	.1628	.451	128
.92	.0369	.1559	.792	360	.0378	.1578	.695	248	.0505	.1824	.537	141
.90	.0515	.1744	.875	374	.0519	.1751	.777	262	.0660	.1974	.609	152
.85	.0939	.2041	1.030	396	.0934	.2036	.932	284	.1092	.2201	.750	170
.80	.1435	.2168	1.140	411	.1421	.2158	1.042	298	.1580	.2275	.853	183
.75	.1991	.2174	1.223	421	.1970	.2162	1.126	309	.212	.2243	.933	192
.70	.260	.2090	1.288	429	.257	.2078	1.191	317	.270	.2130	.996	200
.65	.324	.1939	1.340	436	.321	.1930	1.243	324	.333	.1966	1.047	207
.60	.392	.1746	1.381	442	.389	.1740	1.285	330	.398	.1759	1.088	213
.55	.463	.1526	1.415	448	.460	.1521	1.318	336	.466	.1531	1.122	218
.50	.536	.1294	1.442	453	.533	.1291	1.346	341	.537	.1295	1.149	223
.45	.610	.1061	1.464	458	.607	.1058	1.367	346	.608	.1060	1.170	228
.40	.684	.0837	1.481	463	.681	.0835	1.385	351	.680	.0834	1.188	233
.35	.757	.0631	1.495	468	.754	.0629	1.399	356	.752	.0628	1.202	238
.30	.827	.0448	1.505	473	.824	.0447	1.409	361	.821	.0447	1.212	244
.25	.892	.0295	1.513	479	.890	.0295	1.417	367	.885	.0294	1.220	249
.20	.949	.0174	1.519	486	.947	.0174	1.423	374	.942	.0174	1.226	256
.15	.992	.0087	1.523	493	.991	.0087	1.427	382	.986	.0086	1.230	264
.10	1.009	.0032	1.525	504	1.009	.0032	1.429	392	1.005	.0032	1.232	275
.05	.958	.0005	1.527	521	.958	.0005	1.431	409	.956	.0005	1.234	292
.025	.825	.0001	1.527	537	.825	.0001	1.431	425	.824	.0001	1.234	307
	$-\varphi_i=2°$				$-\varphi_i=3°$				$-\varphi_i=4°$			
0.995	0.0052	0.0712	0	0	0.0078	0.0872	0	0	0.0105	0.1012	0	0
.99	.0105	.0999	.057	18	.0157	.1222	.047	12	.0209	.1410	.040	9
.98	.0210	.1378	.136	37	.0313	.1682	.111	24	.0416	.1939	.097	18
.96	.0422	.1855	.246	55	.0624	.2256	.201	37	.0827	.2597	.175	28
.94	.0638	.2164	.327	66	.0934	.2618	.268	44	.1233	.3008	.233	33
.92	.0858	.2378	.393	74	.1243	.2862	.323	50	.1633	.3289	.281	38
.90	.1080	.2525	.449	80	.1550	.3025	.370	54	.203	.3462	.322	41
.85	.1651	.2706	.560	92	.231	.3201	.463	62	.300	.3648	.404	47
.80	.224	.2709	.646	100	.306	.3166	.536	68	.393	.3588	.469	52
.75	.284	.2596	.714	107	.380	.3003	.595	74	.482	.3382	.521	56
.70	.346	.2411	.769	113	.452	.2756	.643	78	.568	.3089	.564	59
.65	.409	.2178	.815	119	.523	.2464	.683	82	.649	.2744	.599	63
.60	.474	.1920	.852	124	.591	.2144	.717	86	.726	.2376	.630	66
.55	.538	.1645	.883	128	.657	.1818	.744	90	.797	.2003	.655	69
.50	.603	.1373	.908	133	.721	.1501	.767	94	.863	.1642	.676	72
.45	.668	.1110	.929	137	.781	.1200	.787	98	.922	.1304	.693	75
.40	.732	.0866	.946	142	.838	.0926	.802	102	.974	.0999	.708	79
.35	.794	.0646	.959	147	.890	.0684	.815	106	1.018	.0731	.719	82
.30	.853	.0455	.969	152	.936	.0477	.824	110	1.052	.0506	.729	86
.25	.907	.0298	.977	157	.975	.0309	.832	115	1.076	.0324	.736	91
.20	.954	.0175	.983	164	1.005	.0179	.838	121	1.086	.0186	.741	97
.15	.989	.0087	.987	172	1.021	.0088	.841	129	1.079	.0091	.745	104
.10	1.002	.0032	.989	182	1.016	.0032	.844	139	1.048	.0032	.747	114
.05	.952	.0005	.991	200	.952	.0005	.845	157	.960	.0005	.748	131
.025	.822	.0001	.991	215	.820	.0001	.845	172	.821	.0001	.749	146

TABLE II. TABULATED VALUES OF THE MASS-LOSS PARAMETERS

Vehicle	Entry angle, $-\varphi_i$, deg	Dimensionless mass loss, $\overline{m}_{\lambda=0}$: Exact	Rule A, $\left[\frac{1}{30}\left(\frac{-a_{max}}{2}\right)\right]^{-1/2}$	Rule B, $0.84(\bar{t})^{1/2}$	$(\overline{H}_{eff})_{\lambda=0}$	η: $\lambda=0$	$\lambda=0.05$	$\lambda=0.1$	$\lambda=0.25$	$\lambda=0.50$	$\lambda=1.0$	$\lambda=\infty$
(a) Lifting, $\frac{L}{D}>1$	0	$1.418\left(30\frac{L}{D}\right)^{1/2}$	$1.414\left(30\frac{L}{D}\right)^{1/2}$	$1.44\left(30\frac{L}{D}\right)^{1/2}$	0.5	*0	*0.218	*0.302	*0.447	*0.577	*0.707	*1
(b) Nonlifting capsule, $\frac{L}{D}=0$	0	3.05	2.70	3.69	0.500	0	0.223	0.305	0.450	0.580	0.707	1
	$\frac{1}{2}$	2.95	2.72	3.27	.484	0	.234	.319	.463	.589	.715	1
	1	2.73	2.70	2.76	.453	0	.249	.337	.482	.608	.730	1
	2	2.40	2.57	2.29	.412	0	.273	.364	.511	.633	.750	1
	3	2.17	2.30	2.03	.390	0	.289	.383	.530	.650	.762	1
	4	1.99	2.13	1.85	.380	0	.302	.397	.542	.660	.770	1
(c) Ballistic, $\frac{L}{D}=0$	$-\varphi_i>5$	$3.32[30\sin(-\varphi_i)]^{-1/2}$	$3.30[30\sin(-\varphi_i)]^{-1/2}$	$3.22[30\sin(-\varphi_i)]^{-1/2}$	0.351	0	0.340	0.431	0.572	0.680	0.785	1

* $\eta=\left(\frac{\lambda}{1+\lambda}\right)^{1/2}$ for case (a); $\eta\approx\left(\frac{\lambda}{1+\lambda}\right)^{(\overline{H}_{eff})_{\lambda=0}}$ more generally.

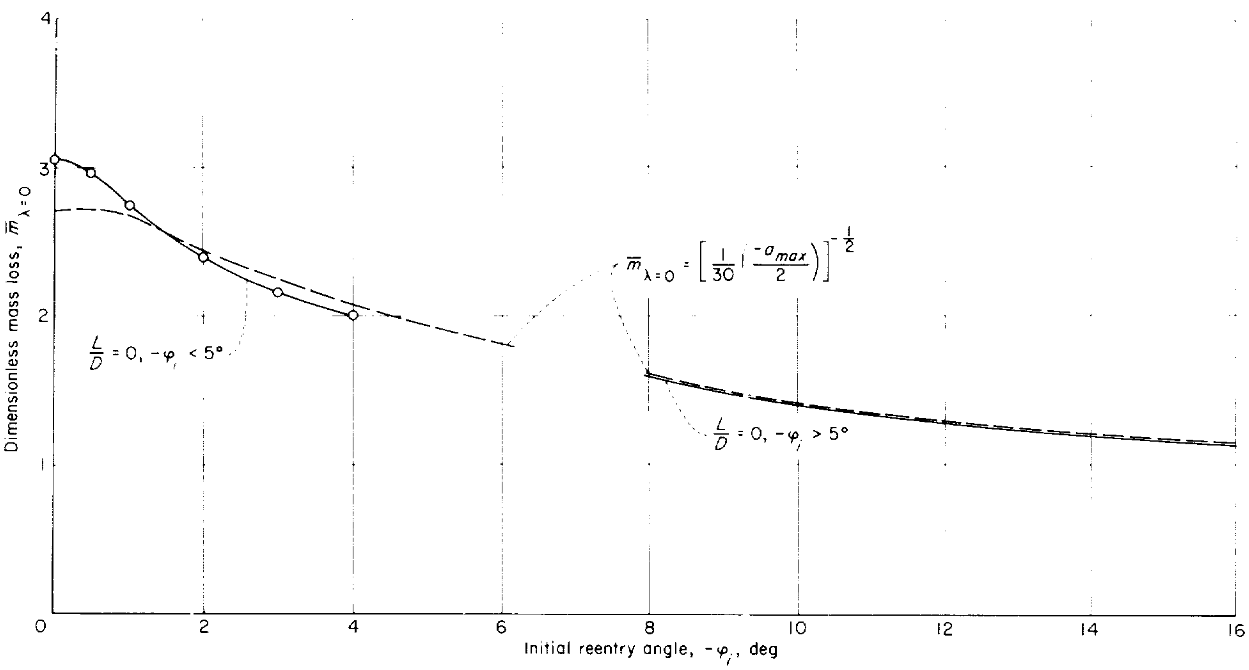

FIGURE 2.—Dimensionless total mass loss for nonlifting vehicles.

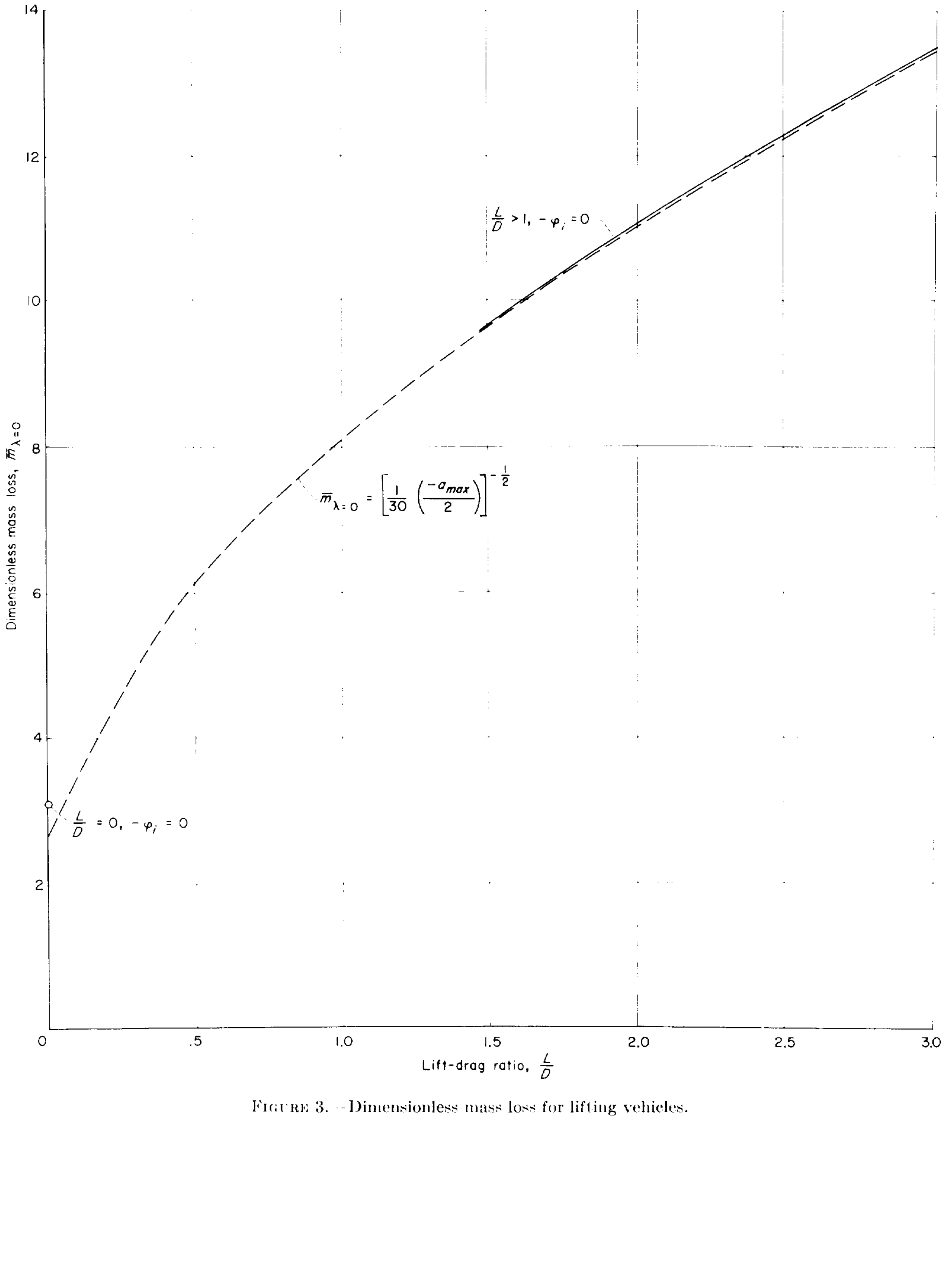

FIGURE 3.—Dimensionless mass loss for lifting vehicles.

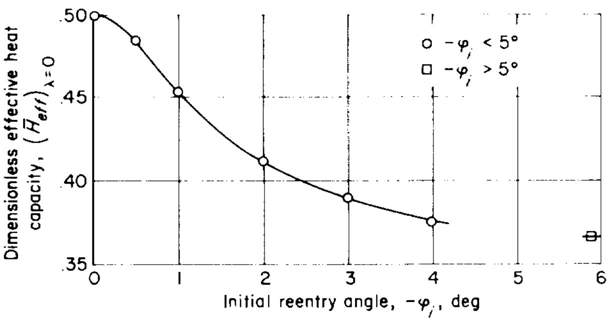

FIGURE 4.—Dimensionless effective heat capacity for nonlifting vehicles.

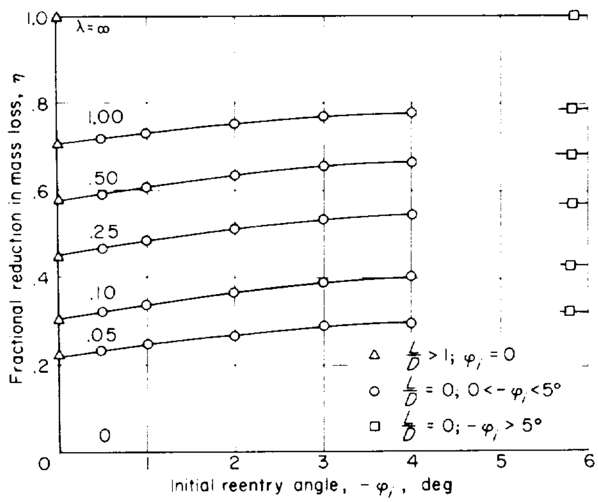

FIGURE 5.—Fractional reduction in mass loss due to latent heat, for lifting and nonlifting vehicles.

The values of $(\overline{H}_{eff})_{\lambda=0}$ and η found numerically for the case of $\frac{L}{D}=0$ and $-\varphi_i=0$ are in good agreement with the analytical results for $\frac{L}{D}>1$ and $-\varphi_i=0$:

$$(\overline{H}_{eff})_{\lambda=0}=\frac{1}{2}$$

and

$$\eta=\left(\frac{\lambda}{1+\lambda}\right)^{1/2}$$

as seen in table II, and it is inferred therefore that these values are valid for all cases where $\frac{L}{D}>0$ and $\varphi_i=0$.

In general it is seen from figure 4 that $0.35 \leqq (\overline{H}_{eff})_{\lambda=0} \leqq 0.5$ for nonlifting vehicles, the lower limit corresponding to $-\varphi_i>5°$. It is to be expected that $(\overline{H}_{eff})_{\lambda=0}$ decreases as $-\varphi_i$ increases, since a greater part of the reentry is spent in the lower atmosphere where the gas-layer shielding effect is less.

A check on the approximate rule B is made as follows:

For (a) lifting vehicle $\left(\frac{L}{D}>1;\ \varphi_i=0\right)$,

$$\frac{t_f}{27}=30\frac{L}{D}\int_{0.05}^{0.995}(1-\overline{u}^2)^{-1}d\overline{u}$$

$$=30\frac{L}{D}\left[\frac{1}{2}\log_e\frac{1+\overline{u}}{1-\overline{u}}\right]_{0.05}^{0.995}$$

$$=2.94\left(30\frac{L}{D}\right)$$

Thus

$$\overline{m}_{\lambda=0}=\frac{1.418}{(2.94)^{1/2}}\left(\frac{t_f}{27}\right)^{1/2}$$

$$=0.83\left(\frac{t_f}{27}\right)^{1/2}$$

and is independent of L/D.

For (b) nonlifting capsule $\left(\frac{L}{D}=0;\ 0\leqq\varphi_i<5°\right)$,

Numerical values of $t_f/27$ show that the approximate relation

$$\overline{m}_{\lambda=0}=0.83\left(\frac{t_f}{27}\right)^{1/2}$$

is accurate except for $-\varphi_i<\frac{1°}{2}$, where the errors are more than 10 percent.

For (c) ballistic vehicle $\left(\frac{L}{D}=0;\ -\varphi_i>5°\right)$,

$$\frac{t_f}{27}=30\sin-\varphi_i\int_{0.05}^{0.995}(-\overline{u}^2\log_e\overline{u})^{-1}d\overline{u}$$

$$=14.67\,[30\sin(-\varphi_i)]$$

Thus

$$\overline{m}_{\lambda=0}=\frac{3.32}{(14.67)^{1/2}}\left(\frac{t_f}{27}\right)^{1/2}$$

$$=0.84\left(\frac{t_f}{27}\right)^{1/2}$$

and is independent of $-\varphi_i$. The exact expressions for $\overline{m}_{\lambda=0}$ are compared with the approximate relation

$$\overline{m}_{\lambda=0}=0.84\left(\frac{t_f}{27}\right)^{1/2} \tag{55}$$

in table II and shown in figure 6.

In the foregoing analysis, expressions for the total mass loss and the effective heat capacity of the shield have been obtained by assuming that sublimation starts early during reentry and that the accumulation of heat is negligible compared with the disposal of heat. Both assumptions lead to conservative values for the total mass loss (that is, values that are too large).

An analysis of the heat-conduction problem within the shield is desirable in order to justify these assumptions and also to estimate the amount of insulation required to keep the structure cool.

ACCUMULATION OF HEAT WITHIN THE SHIELD

The effectiveness of an ablation shield in reducing heat transfer to the vehicle structure is measured finally in terms of the mass required to keep the structure below a given temperature. It has been shown in the previous section that the mass loss due to ablation is virtually independent of surface temperature when $T_r \gg T_s$. The mass requirements for insulation, however, depend critically on the ablation temperature, and it is to be expected that the use of materials with low ablation temperatures will reduce considerably the insulation problem with relatively little increase in the total mass loss.

In order to estimate the insulation required, the unknown relation between $\left(k_h \frac{\partial T}{\partial z}\right)_{z=0}$ and θ

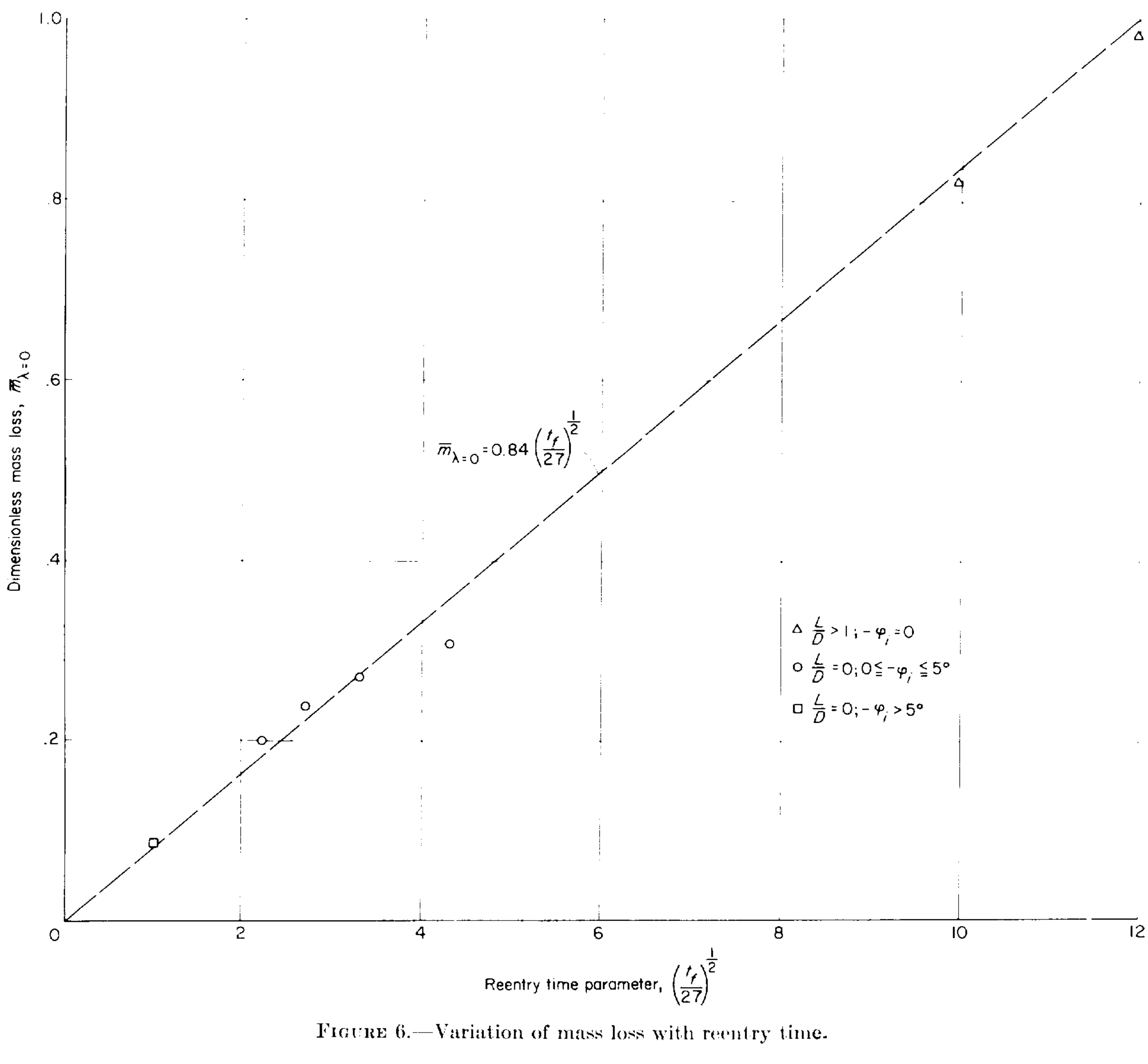

FIGURE 6.—Variation of mass loss with reentry time.

discussed earlier in the report is assumed to be

$$\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} = \frac{k_b(T_s - T_\infty)}{\theta}$$

Although this equation appears at first sight to be at most a crude approximation, one should expect a relation of the form

$$\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} = A(t) \frac{k_b(T_s - T_\infty)}{\theta} \tag{56}$$

where $A(t)$ is a slowly varying dimensionless parameter. It is shown in the appendix that $\frac{2}{\pi} \leqq A(t) < 1$.

Equations (24) and (25) are now written, respectively,

$$q_0(t) = [c_b(T_a - T_\infty) + L + \alpha \bar{c}_p(T_e - T_a)] \frac{dm}{dt} + \rho_b c_b \frac{d}{dt} [(T_s - T_\infty)\theta] \tag{57}$$

and

$$q_0(t) = [L + \alpha \bar{c}_p(T_e - T_a)] \frac{dm}{dt} + \frac{k_b(T_s - T_\infty)}{\theta} \tag{58}$$

which may be solved for the unknown quantities T_s and θ when $\frac{dm}{dt} = 0$ (before ablation) or for m and θ when $T_s = T_a$ (during ablation).

Preablation heating.—Before appreciable ablation occurs (when the mass-loss rate has negligible effect on the heat-transfer rate), equations (57) and (58) reduce to

$$q_0 = \rho_b c_b \frac{d}{dt} [(T_s - T_\infty)\theta] = \frac{k_b(T_s - T_\infty)}{\theta} \tag{59}$$

and elimination of θ gives

$$q_0 Q_0 = \rho_b c_b k_b (T_s - T_\infty)^2 \tag{60}$$

When sublimation begins, $\bar{u} = \bar{u}_a$ and $T_s = T_a$; equation (60) then becomes

$$q_0(\bar{u}_a) Q_0(\bar{u}_a) = \rho_b c_b k_b (T_a - T_\infty)^2 \tag{61}$$

which is written in dimensionless form as

$$\bar{q}_0(\bar{u}_a)\bar{Q}_0(\bar{u}_a) = \frac{1.07}{k_{\rm I} k_{\rm II}} \frac{\rho_b c_b k_b}{\frac{M}{C_D A R}} (T_a - T_\infty)^2 (10^{-7}) \tag{62}$$

(Strictly speaking, sublimation occurs at all values of the surface temperature and T_s is related to dm/dt through the phase relation which describes the equilibrium of the solid material with its vapor. In practice, however, dm/dt is negligible except when T_s lies within a limited range which includes the mean value T_a used here.)

Since $\frac{1.07}{k_{\rm I} k_{\rm II}} \approx 1$, it is seen from equation (62) that

$$\bar{q}_0(\bar{u}_a)\bar{Q}_0(\bar{u}_a) \approx \chi^2 (10^{-7}) \tag{63}$$

where

$$\chi = \frac{(\rho_b c_b k_b)^{1/2}}{\left(\frac{M}{C_D A R}\right)^{1/2}} (T_a - T_\infty) \tag{64}$$

Equation (63) determines whether sublimation will occur during reentry. If

$$(\bar{q}_0 \bar{Q}_0)_{max} < \chi^2 (10^{-7}) \tag{65}$$

sublimation will not occur, since equation (63) cannot be satisfied and the maximum value of T_s will remain below T_a throughout reentry. In terms of $\bar{u}$ and Z, the condition that sublimation will not occur is written

$$\chi^2 > 10^7 \left[\bar{u}^2 (\bar{u}Z)^{1/2} \int_{\bar{u}}^{0.995} \bar{u}^2 (\bar{u}Z)^{-1/2} d\bar{u} \right]_{max} \tag{66}$$

and it will be shown that the bracketed expression in this inequality is virtually constant (approximately 0.2) and is independent of L/D and φ_i.

When χ is written

$$\chi = \frac{\rho_b c_b (T_a - T_\infty) \left(\frac{k_b}{\rho_b c_b}\right)^{1/2}}{\left(\frac{M}{C_D A R}\right)^{1/2}}$$

it is seen that sublimation cannot occur if

(1) the parameter $\left(\frac{M}{C_D A R}\right)^{1/2}$, which determines the level of the heating rate, is too small

(2) the thermal capacity $\rho_b c_b (T_a - T_\infty)$ is too large, or

(3) the thermal diffusivity $\left(\frac{k_b}{\rho_b c_b}\right)^{1/2}$ is too large

The materials under consideration in this report have low ablation temperature and low thermal conductivity; more specifically, the materials under serious consideration have properties with the following orders of magnitude: $\rho_b \approx 0(10^2)$ lb/cu ft, $c_b \approx 0(1)$ Btu/(lb)(°R), $k_b \approx 0(10^{-5})$ Btu/(ft)(sec)(°R), and $T_a - T_\infty \approx 0(10^2, 10^3)$ °R; for vehicles considered here $\frac{M}{C_D A R} > 10^{-1}$. Thus $\chi < 10^2$ and sublimation will occur.

It is of interest to determine what fraction of the total flight time passes before sublimation takes place. The ratio t_a/t_f is found as follows:

If $t=0$ when $\bar{u}=0.995$, then, since $\bar{q}_0$ is an increasing function of $\bar{u}$ during the early part of reentry,

$$\bar{q}_0(\bar{u}_a) > \bar{q}_0(0.995)$$

and

$$\bar{Q}_0(\bar{u}_a) > \bar{t}_a \bar{q}_0(0.995)$$

Therefore

$$\bar{q}_0(\bar{u}_a)\bar{Q}_0(\bar{u}_a) > \bar{q}_0^2(0.995)\bar{t}_a$$

Also

$$\bar{t}_f = \int_{0.05}^{0.995} (\bar{u}Z)^{-1} d\bar{u}$$

so that

$$\frac{t_a}{t_f} = \frac{\bar{t}_a}{\bar{t}_f} < \frac{\bar{q}_0(\bar{u}_a)\bar{Q}_0(\bar{u}_a)}{\bar{q}_0^2(0.995)} \left[\int_{0.05}^{0.995} (\bar{u}Z)^{-1} d\bar{u}\right]^{-1} \quad (67)$$

where the right-hand side of equation (67) is approximately equal to

$$\left[Z(0.995)\int_{0.05}^{0.995} (\bar{u}Z)^{-1} d\bar{u}\right]^{-1} \chi^2(10^{-7})$$

The ratio t_a/t_f is evaluated in a later section by inserting the appropriate Z-functions, and it is shown that $\frac{t_a}{t_f} \ll 1$.

Total accumulation of heat.—In the analysis that follows an upper limit is found for the accumulation of heat during reentry by assuming that the surface of the shield is raised instantaneously to the temperature T_a at $t=0$. (Although this assumption implies an infinite heating rate at $t=0$, it is nevertheless a good approximation except for small values of t/t_f.)

Equations (26) and (56) are first combined to give

$$\rho_b\theta \frac{d}{dt}(\rho_b\theta) = \frac{\rho_b k_b}{c_b} - \rho_b\theta \frac{dm}{dt} \quad (68)$$

and the last term is then neglected to give

$$\rho_b\theta \frac{d}{dt}(\rho_b\theta) < \frac{\rho_b k_b}{c_b} \quad (69)$$

Integration then gives the following upper bound for θ:

$$\rho_b\theta < \left(2\frac{\rho_b k_b}{c_b} t\right)^{1/2} \quad (70)$$

The accumulation of heat at time t is therefore

$$Q = \rho_b c_b (T_a - T_\infty)\theta < (2\rho_b c_b k_b)^{1/2}(T_a - T_\infty) t^{1/2} \quad (71)$$

and, when $t = t_f$,

$$Q_f < (2\rho_b c_b k_b)^{1/2}(T_a - T_\infty) t_f^{1/2}$$

This upper limit is independent of $\frac{M}{C_D A R}$ and for given material properties depends only on the total time of reentry. When Q_f is expressed as a fraction of $Q_{0,f}$ there is obtained

$$\frac{Q_f}{Q_{0,f}} < \frac{0.46}{k_{II}} \frac{\bar{t}_f^{1/2}}{\bar{Q}_{0,f}} \chi(10^{-3}) \quad (72)$$

where

$$\frac{\bar{t}_f^{1/2}}{\bar{Q}_{0,f}} = \frac{\left[\int_{0.05}^{0.995} (\bar{u}Z)^{-1} d\bar{u}\right]^{1/2}}{\int_{0.05}^{0.995} \bar{u}^2(\bar{u}Z)^{-1/2} d\bar{u}} \quad (73)$$

and is evaluated by inserting the appropriate Z-functions.

Application to particular vehicles.—The results of the previous two sections are summarized briefly before specific application is made:

The quantity

$$\chi = \frac{(\rho_b c_b k_b)^{1/2}(T_a - T_\infty)}{\left(\frac{M}{C_D A R}\right)^{1/2}}$$

was shown to be an important parameter of the heat-conduction problem; numerically $\chi < 10^2$ for the low-temperature low-conductivity materials considered herein.

Sublimation will not occur during reentry if

$$\chi^2 > 10^7 F_1$$

where

$$F_1 = \left[\bar{u}^2(\bar{u}Z)^{1/2}\int_{\bar{u}}^{0.995} \bar{u}^2(\bar{u}Z)^{-1/2} d\bar{u}\right]_{max} \quad (74)$$

The ratio t_a/t_f satisfies

$$\frac{t_a}{t_f} < 10^{-7}\chi^2 F_2$$

where

$$F_2=\left[Z(0.995)\int_{0.05}^{0.995}(\bar{u}Z)^{-1}d\bar{u}\right]^{-1} \tag{75}$$

The ratio $Q_f/Q_{0,f}$ satisfies

$$\frac{Q_f}{Q_{0,f}}<10^{-3}\chi F_3$$

where

$$F_3=\frac{\frac{1}{2}\left[\int_{0.05}^{0.995}(\bar{u}Z)^{-1}d\bar{u}\right]^{1/2}}{\int_{0.05}^{0.995}\bar{u}^2(\bar{u}Z)^{-1/2}d\bar{u}} \tag{76}$$

$\left(\text{where it is assumed that } \frac{0.46}{k_{11}}\approx\frac{1}{2} \text{ in eq. (72)}\right)$.

The numerical values of F_1, F_2, and F_3 depend on the particular Z-functions. In each case, however, the combination of functions is such that the total power of Z is zero and the factors L/D and $\sin(-\varphi_i)$ which enter into the evaluation finally cancel, giving results which are independent of L/D and $-\varphi_i$, as shown below.

(a) Lifting vehicle $\left(\frac{L}{D}>1; -\varphi_i=0\right)$

With $\bar{u}Z=\frac{1-\bar{u}^2}{30\frac{L}{D}}$, F_1, F_2, and F_3 are evaluated as $F_1=0.2$, $F_2=34$, and $F_3=1.1$, and are independent of L/D.

(b) Nonlifting capsule $\left(\frac{L}{D}=0; 0\leq-\varphi_i<5°\right)$

Numerical evaluation of the integrals gives the following average values: $F_1=0.2$, $F_2=50$, and $F_3=1.5$.

(c) Ballistic vehicle $\left(\frac{L}{D}=0; -\varphi_i>5°\right)$

Evaluation of the integrals by using

$$Z=30\sin(-\varphi_i)\bar{u}(-\log_e\bar{u})$$

gives $F_1=0.2$, $F_2=14$, and $F_3=1.5$.

It is concluded from these numerical values that, for cases (a), (b), or (c):

(1) Sublimation will not occur during reentry if $\chi^2>2\times10^6$. (This condition does not hold necessarily if the shield is a composite slab of different materials; in such a case the factors $\rho_b c_b(T_a-T_\infty)$ and $\left(\frac{k_b}{\rho_b c_b}\right)^{1/2}$ which appear in the parameter χ would be replaced by the analogous composite quantities.)

(2) The ratio

$$\frac{t_a}{t_f}=\frac{\text{Preablation heating period}}{\text{Total reentry time}}$$

satisfies

$$\frac{t_a}{t_f}<(5)\ (10^{-6})\ \chi^2$$

(3) The ratio

$$\frac{Q_f}{Q_{0,f}}=\frac{\text{Heat accumulated by ablation shield}}{\text{Heat accumulated by heat sink}}$$

satisfies

$$\frac{Q_f}{Q_{0,f}}<(1.5)(10^{-3})\chi$$

The assumptions, made in the analysis of sublimation, that sublimation begins soon after the initiation of reentry (when $\bar{u}=0.995$) and that the accumulation of heat is small compared with the disposal of heat are justified, since $\frac{t_a}{t_f}<0.05$ and $\frac{Q_f}{Q_{0,f}}<0.15$ for $\chi<10^2$.

Insulation requirements.—The method of the preceding section gives an estimate only of the amount of heat accumulated by the solid shield at the completion of reentry. The temperature distribution through the shield is also of interest, however, and an approximate analysis from which this distribution can be obtained is desirable. The nonlinear differential equation and boundary conditions are derived in dimensionless form, together with the approximate method of solution, in the appendix. The results so obtained are summarized briefly and discussed below.

The temperature distribution at the end of reentry is

$$\frac{T-T_\infty}{T_a-T_\infty}=e^{\epsilon_f\xi}\left(1-\frac{1}{2}\operatorname{erfc}\frac{\epsilon_f+\xi}{2}\right)+\frac{1}{2}\operatorname{erfc}\frac{\epsilon_f-\xi}{2} \tag{77}$$

where

$$\xi=\left(\frac{\rho_b c_b}{k_b t_f}\right)^{1/2}z$$

and

$$\epsilon_f = 0.17 k_{II} \frac{\left(\frac{M}{C_{D}AR}\right)^{1/2}}{\alpha \frac{\tilde{c}_p}{c_{p,2}}} (1-\eta)\left(\frac{c_b}{\rho_b k_b}\right)^{1/2} \tag{78}$$

The ratio $\frac{T-T_\infty}{T_a-T_\infty}$ is shown as a function of ξ in figure 7. (The more general expression for temperature distribution during reentry is given in the appendix.)

The accumulation of heat is

$$Q_f = \frac{2}{\pi^{1/2}} (\rho_b c_b k_b t_f)^{1/2} (T_a - T_\infty)\zeta \tag{79}$$

where

$$\zeta = \frac{1}{2} e^{-\epsilon_f^2/4} - \frac{\pi^{1/2}}{4} \epsilon_f \operatorname{erfc} \frac{\epsilon_f}{2} + \frac{\pi^{1/2}}{2\epsilon_f} \operatorname{erf} \frac{\epsilon_f}{2}$$

The function ζ is shown in figure 8; it may be

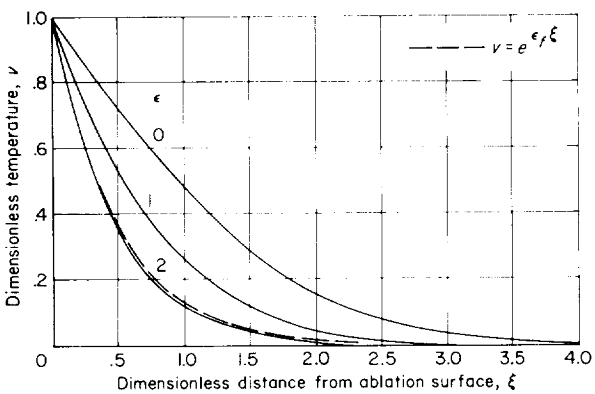

FIGURE 7. Temperature distribution within the solid shield.

$$v = \frac{T-T_\infty}{T_a-T_\infty};\ \xi = \left(\frac{\rho_b c_b}{k_b t_f}\right)^{1/2} z.$$

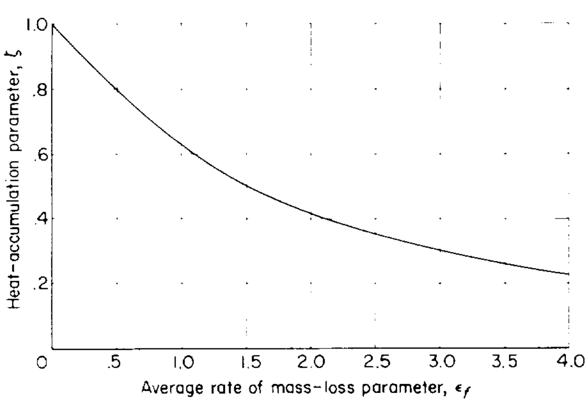

FIGURE 8.—Variation of ζ with ϵ_f.

verified that $\zeta < 1$ and $\zeta \to \frac{\pi^{1/2}}{2\epsilon_f}$ for large values of ϵ_f (in practice, $\epsilon_f > 2$).

The mass loss rate dm/dt will be small throughout reentry if the latent heat is large, since

$$\frac{dm}{dt} < \frac{q_0}{L + \alpha \frac{\tilde{c}_p}{c_{p,2}} (T_s - T_a)}$$

Thus, for a low-temperature shield (for instance, where $\chi < 10^2$) which also has sufficiently large latent heat, most of the aerodynamic heat input is lost in boundary-layer shielding and in the phase change, whereas the amount of heat conducted from the surface to the interior is sufficient only to keep the surface at the ablation temperature T_a. In the limiting case when $\frac{dm}{dt} \to 0$ and $T_s = T_a$ throughout reentry, the conduction problem is described once more by a linear equation which has the solution (as $\epsilon \to 0$)

$$\frac{T-T_\infty}{T_a-T_\infty} = \operatorname{erfc}\left[-\left(\frac{\rho_b c_b}{k_b t}\right)^{1/2} \frac{z}{2}\right] \tag{80}$$

Equation (80) actually furnishes an upper limit to the temperature distribution, since when ablation takes place the heat content of the remaining shield is reduced.

An upper limit to the heat content of the material is obtained as

$$Q = \rho_b c_b (T_a - T_\infty) \int_{-\infty}^{0} \frac{T-T_\infty}{T_a-T_\infty} dz$$

which is simply

$$Q = \frac{2}{\pi^{1/2}} (\rho_b c_b k_b)^{1/2} (T_a - T_\infty) t^{1/2}$$

At the end of reentry, $t = t_f$ and

$$Q_f = \frac{2}{\pi^{1/2}} (\rho_b c_b k_b)^{1/2} (T_a - T_\infty) t_f^{1/2} \tag{81}$$

This exact limit is about 20 percent less than the approximate value given in equation (71).

The behavior of the solution for large values of ϵ_f at the end of reentry is written

$$\frac{T-T_\infty}{T_a-T_\infty} \approx e^{\epsilon_f \xi}$$

$$= \exp\left[0.17 k_{II} \frac{\left(\frac{M}{C_D A R}\right)^{1/2}}{\alpha \frac{\tilde{c}_p}{c_{p,2}}} (1-\eta) \frac{c_b}{k_b} \frac{z}{t_f^{1/2}}\right] \tag{82}$$

Equation (82) shows the effect of the sublimation parameters $\dfrac{\left(\dfrac{M}{C_D A R}\right)^{1/2}}{\alpha \dfrac{\bar{c}_p}{c_{p,2}}}$ and η on the temperature distribution within the shield.

The insulation requirements—that is, the amount of material which should remain when sublimation has ceased in order to maintain the structure at some design temperature below the sublimation temperature—will depend primarily on the thermal diffusivity of the material $k_b/\rho_b c_b$, the total time of reentry t_f, and the ablation temperature T_a. The required insulation thickness can be determined approximately from equation (77) or figure 7.

For engineering purposes the curves of figure 7 are easily approximated by

$$\frac{T-T_\infty}{T_a-T_\infty}=e^{z/\delta} \tag{83}$$

where δ is chosen in such a way that the heat content is correct; that is,

$$\rho_b c_b \int_{-\infty}^{0} (T-T_\infty)dz=\frac{2}{\pi^{1/2}}(\rho_b c_b k_b t_f)^{1/2}(T_a-T_\infty)\zeta(\epsilon_f)$$

Evaluation of the integral using the profile of equation (83) gives

$$\delta=\frac{2}{\pi^{1/2}}\left(\frac{k}{\rho c}\right)^{1/2} t_f^{1/2}\zeta(\epsilon_f) \tag{84}$$

Therefore the amount of insulation $-\rho z$ ahead of a station having temperature T is, from equations (83) and (84),

$$-\rho z=\frac{2}{\pi^{1/2}}\left(\frac{\rho_b k_b}{c_b}\right)^{1/2} t_f^{1/2}\zeta(\epsilon_f) \log_e \frac{T_a-T_\infty}{T-T_\infty} \tag{85}$$

The importance of the shield material parameters $\rho_b k_b/c_b$ and T_a is evident from the foregoing expressions. It is seen also that the amount of insulation varies as $t_f^{1/2}$.

DISCUSSION

In the preceding analysis, the primary objective has been to obtain simple useful expressions to describe the sublimation of material from, and the accumulation of heat by, a low-temperature low-conductivity shield suitable for manned reentry.

For the sake of simplicity several approximations have been made, but they are of such a nature as to give conservative results, since upper limits have been obtained for the total mass loss due to sublimation and for the total heat accumulated during reentry.

It has been shown that the total mass required for sublimation depends primarily on the paramaters

$$\frac{\left(\dfrac{M}{C_D A R}\right)^{1/2}}{\alpha \dfrac{\bar{c}_p}{c_{p,2}}}=\frac{\text{Heating coefficient}}{\text{Gas-shielding coefficient}}$$

and

$$\lambda=\frac{L+c_b(T_a-T_\infty)}{\dfrac{1}{2}\dfrac{u_e^2}{Jg}\alpha\dfrac{\bar{c}_p}{c_{p,2}}}$$

$$=\frac{\text{Maximum internal shielding enthalpy}}{\text{Maximum external shielding enthalpy}}$$

This definition of λ gives the quasi-steady result, whereas conservative results are obtained when

$$\lambda=\frac{L}{\dfrac{1}{2}\dfrac{u_e^2}{Jg}\alpha\dfrac{\bar{c}_p}{c_{p,2}}}$$

It is evident that the correct interpretation of $\alpha \dfrac{\bar{c}_p}{c_{p,2}}$ is important in the use of these parameters; the quantity arises when the convective shielding in the boundary layer is considered and is correctly interpreted as

$$\alpha\frac{\bar{c}_p}{c_{p,2}}=\frac{\text{Enthalpy of gases convected in boundary layer}}{\text{Enthalpy difference across boundary layer}}$$

In reference 7 this ratio has been replaced by an empirical factor $0.68\left(\frac{M_2}{M_1}\right)^{0.26}$, where M is the molecular weight, whereas in reference 6 simple approximate expressions for α and $\bar{c}_p/c_{p,2}$ were derived under the assumption of constant $c_{p,1}$ and $c_{p,2}$ and appear as equations (22) and (23) of the present report. In practice, however, $c_{p,1}$ and $c_{p,2}$ are functions of temperature and the average values used should be consistent with the foregoing physical interpretation.

Furthermore, although the quantity $\alpha=1-\frac{1}{3}N_{Pr}^{-0.6}$ is an approximate constant value, the exact theoretical values for α depart from this when the internal shielding enthalpy $L+c_b(T_a-T_\infty)$ is small compared with the gas-layer shielding enthalpy. (See the discussion of $S\approx 1$ on page 7 of ref. 6.) This situation may occur in the early part of reentry if the latent heat of the material is small enough, and the use of a value of α that is too large would lead to an underestimation of the shield requirements.

Although the simple expressions (22) and (23) are used herein it is understood that an experimental verification of the ratio

$$\frac{\text{Enthalpy of gases convected in boundary layer}}{\text{Enthalpy difference across boundary layer}}$$

for the particular material under consideration is desirable.

The total mass loss during reentry can be written

$$m=1.18k_1\frac{\left(\frac{M}{C_DAR}\right)^{1/2}}{\alpha\frac{\tilde{c}_p}{c_{p,2}}}\,\overline{m}_{\lambda=0}(1-\eta)\text{ lb/sq ft}$$

where $\overline{m}_{\lambda=0}$ depends only on the vehicle trajectory. The exact values are given in table II, although $\overline{m}_{\lambda=0}$ is given approximately by

$$\overline{m}_{\lambda=0}\approx\left[\frac{1}{39}\left(\frac{-a_{max}}{2}\right)\right]^{-1/2}$$

or

$$\overline{m}_{\lambda=0}\approx 0.84\left(\frac{t_f}{27}\right)^{1/2}$$

The quantity η, primarily a function of λ, represents the fractional reduction in mass loss due to latent heat.

The relation between total mass loss and maximum deceleration shows immediately the weight penalty incurred as the price of limiting the maximum deceleration to a low value, and it is concluded that the use of a material with low ablation temperature is not appropriate to vehicles which have high lift-drag ratios. It is seen in figure 3, for example, that the value of $\overline{m}_{\lambda=0}$ for $\frac{L}{D}=0.5$ is about twice that for $\frac{L}{D}=0$.

The effective heat capacity of the ablation material is written

$$H_{eff}=13{,}500\alpha\frac{\tilde{c}_p}{c_{p,2}}(\overline{H}_{eff})_{\lambda=0}(1-\eta)^{-1}\text{ Btu/lb}$$

where $(\overline{H}_{eff})_{\lambda=0}$ and η are dimensionless and do not vary appreciably with trajectory, as seen in figures 4 and 5. Even when $\eta=0$ (negligible latent heat) the effective heat capacity of the material is

$$13{,}500\alpha\frac{\tilde{c}_p}{c_{p,2}}(\overline{H}_{eff})_{\lambda=0}$$

where

$$0.35<(\overline{H}_{eff})_{\lambda=0}<0.5$$

Again the importance of $\alpha\frac{\tilde{c}_p}{c_{p,2}}$ is seen; when $\alpha\frac{\tilde{c}_p}{c_{p,2}}=\frac{1}{2}$, for example, the effective heat capacity is between 2,650 Btu/lb and 3,375 Btu/lb even when the latent heat is negligible. When this range of effective heat capacities is compared with that for heat-sink metals of the order of 1,000 Btu/lb the reduction in shield weight is quickly realized. Moreover, since the foregoing comparison does not depend on the ablation temperature, the advantage is enhanced when materials with low ablation temperatures are considered in view of the attendant reduction in insulation requirements.

For an ablation shield to perform successfully it must dispose of, rather than accumulate, heat energy. The preablation heating period should therefore be small compared with the total reentry time, and the heat accumulated should be a small fraction of that which would be accumulated by a heat sink. Here, the deciding parameter is

$$\chi=\frac{(\rho_bc_bk_b)^{1/2}(T_a-T_\infty)}{\left(\frac{M}{C_DAR}\right)^{1/2}}$$

It has been shown that sublimation will not occur if $\chi^2>2\times10^6$, that the ratio

$$\frac{\text{Preablation heating period}}{\text{Total heating period}}<5(10^{-6})\chi^2$$

and that

$$\frac{\text{Heat accumulated by ablation shield}}{\text{Heat accumulated by heat-sink shield}}<1.5(10^{-3})\chi$$

For the lifting vehicle $\left(\frac{L}{D}>\frac{1}{2}, \text{say}\right)$ the mass loss will be large if ablation is allowed to take place during most of the reentry. Since such a vehicle would be cooled primarily by radiation, the use of a high-ablation-temperature material at the leading edge seems more appropriate. In such a design the ablation temperature should probably be near the mean radiation temperature of the vehicle, and the parameter χ should be near the critical value 2×10^6 if ablation is to take place only near peak heating. The behavior of the ablation material during this long preablation period may be of concern, however.

When the heat conduction problem is considered, an upper limit to the accumulation of heat is found as

$$Q_f<\frac{2}{\pi^{1/2}}(\rho_b c_b k_b t_f)^{1/2}(T_a-T_\infty) \text{ Btu/sq ft}$$

This result is independent of the vehicle characteristics and heating experience except as they affect t_f. The temperature distribution satisfies the relation

$$\frac{T-T_\infty}{T_a-T_\infty}<\operatorname{erfc}\left[-\frac{z}{2}\left(\frac{\rho_b c_b}{k_b t_f}\right)^{1/2}\right]$$

where the right-hand side is the limiting solution for negligible mass loss. From the foregoing expressions for Q and T it is seen that the amount of insulation will vary as $t_f^{1/2}$.

Thus, for a given vehicle the total shield weight required for sublimation and insulation varies approximately as the square root of the reentry time, or inversely as the square root of the maximum deceleration. For ballistic vehicles $(-\varphi_i>5°)$ and manned capsules $(0\leqq-\varphi_i<5°)$, therefore, the ablation shield offers an efficient way to dispose of heat continuously during reentry. For the lifting vehicle a high-ablation-temperature material which would allow radiation from the surface for the greater part of reentry appears to be more appropriate; ablation would then take place for a limited time near the maximum heating condition or in case of an emergency maneuver.

CONCLUDING REMARKS

An approximate analysis has been made of ablation-shield requirements for reentry vehicles. The type of shield considered was one of low ablation temperature and low thermal conductivity which produces no liquid film during ablation. It is shown that

1. The total mass required for sublimation depends primarily on parameters which are functions of the ratios

$$\frac{\text{Heating enthalpy}}{\text{Gas shielding enthalpy}}$$

and

$$\frac{\text{Shielding due to latent heat}}{\text{Gas-layer shielding}}$$

2. For a given vehicle and shield the total mass loss varies as the square root of the total time for reentry or inversely as the square root of the maximum deceleration.

3. The heat accumulated is a small percentage of that accumulated by a heat-sink shield, the percentage being determined by a single parameter which combines the effects of the heating level experienced during reentry, the thermal capacity of the remaining shield, and the diffusivity of the material.

4. The amount of insulation material also varies as the square root of the time or inversely as the square root of the maximum deceleration.

From the foregoing dependence of sublimation and insulation requirements on deceleration and time of reentry it is concluded that the low-ablation-temperature shield should dispose of heat very efficiently for nonlifting vehicles, but the limited use of a high-ablation-temperature shield at the leading edges is more appropriate for lifting vehicles, where the primary means of cooling would be radiative.

LANGLEY RESEARCH CENTER,
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION,
LANGLEY FIELD, VA., *October 1, 1959.*

APPENDIX

FORMAL DEVELOPMENT OF THE DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

The heat conduction problem in a semi-infinite slab is represented by the following equation and boundary conditions. The heat conduction equation is

$$\rho_b c_b \frac{\partial T}{\partial t} = k_b \frac{\partial^2 T}{\partial z^2} \tag{A1}$$

The initial condition is

$$T = T_\infty \qquad (t=0,\ y<0) \tag{A2}$$

The boundary conditions are

$$q(t) = k_b \frac{\partial T}{\partial y} \qquad (y=0,\ t<t_a) \tag{A3a}$$

$$T \rightarrow T_\infty \qquad (y \rightarrow -\infty,\ t<t_a) \tag{A3b}$$

and

$$T = T_a \qquad \left(y = -\frac{m}{\rho_b},\ t>t_a\right) \tag{A4a}$$

$$q(t) = k_b \frac{\partial T}{\partial y} + L\frac{dm}{dt} \qquad \left(y = -\frac{m}{\rho_b},\ t>t_a\right) \tag{A4b}$$

$$T \rightarrow T_\infty \qquad (y \rightarrow -\infty,\ t>t_a) \tag{A4c}$$

The heat-transfer rate $q(t)$ is, in general, unknown and must be determined by an analysis of the aerodynamic heat-transfer problem. As stated in the body of the report the expression used here is

$$q(t) = q_0(t) \qquad (t<t_a) \tag{A5a}$$

$$q(t) = q_0(t) - \alpha \tilde{c}_p (T_e - T_a)\frac{dm}{dt} \qquad (t>t_a) \tag{A5b}$$

where $q_0(t)$ is the aerodynamic heat-transfer rate to a nonablating body and $\alpha \tilde{c}_p (T_e - T_a)\frac{dm}{dt}$ represents the shielding effect of the gas boundary layer.

The moving boundary $y = -\frac{m}{\rho_b}$ is first eliminated by choosing a coordinate $z = y + \frac{m}{\rho_b}$ so that $z=0$ is the sublimating surface. Then the equations are made dimensionless by the introduction of new variables:

$$r = \frac{T - T_\infty}{T_a - T_\infty} \qquad \tau = \frac{t}{t_f} \qquad \xi = \left(\frac{\rho_b c_b}{k_b t_f}\right)^{1/2} z \tag{A6}$$

Equations (A1) to (A4) become

$$\frac{\partial r}{\partial \tau} + \left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2} \frac{dm}{dt}\frac{\partial r}{\partial \xi} = \frac{\partial^2 r}{\partial \xi^2} \tag{A7}$$

$$r = 0 \qquad (\tau = 0,\ \xi \leqq 0) \tag{A8a}$$

$$r \rightarrow 0 \qquad (\xi \rightarrow -\infty,\ \tau > 0) \tag{A8b}$$

$$\frac{q_0(\tau)}{c_b(T_a - T_\infty)}\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2} = \frac{\partial r}{\partial \xi} \qquad \left(\xi = 0,\ \tau \leqq \frac{t_a}{t_f}\right) \tag{A9}$$

$$\frac{q_0(\tau)}{c_b(T_a - T_\infty)}\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2} = \frac{\partial r}{\partial \xi} + \frac{L + \alpha \tilde{c}_p (T_e - T_a)}{c_b (T_a - T_\infty)}\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2}\frac{dm}{dt} \qquad \left(\xi = 0,\ \tau > \frac{t_a}{t_f}\right) \tag{A10}$$

$$r = 1 \qquad \left(\xi = 0,\ \tau > \frac{t_a}{t_f}\right) \tag{A11}$$

For $\tau < \frac{t_a}{t_f}$, dm/dt is equal to zero and the problem is the conventional one of finding the temperature distribution when the heat-transfer rate at the surface is given. For $\tau > \frac{t_a}{t_f}$, however, the differential equation (A7) is nonlinear and there is an additional boundary condition corresponding to the additional unknown quantity $\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2}\frac{dm}{dt}$.

The following approximate method is used to solve the foregoing system of equations. A mean value $\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2} \frac{d\tilde{m}}{dt}=\epsilon$ is used and the linear equation which results from equation (A7) is solved to give $r(\xi,\tau,\epsilon)$, and ϵ is then regarded as a function of τ in this solution.

As a further simplifying approximation it is assumed that the preablation heating period is small compared with the total time of reentry; that is, $\frac{t_a}{t_f}\ll 1$. Then $r=1$ at $\xi=0$ for $0<t<t_f$, where $t=0$ when $\overline{u}=0.995$.

Equation (A7), with ϵ replacing $\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2} \frac{dm}{dt}$, becomes

$$\frac{\partial r}{\partial \tau}+\epsilon\frac{\partial r}{\partial \xi}=\frac{\partial^2 r}{\partial \xi^2} \tag{A12}$$

with

$$r=1 \qquad (0<\tau<1) \tag{A13a}$$

$$\frac{\partial r}{\partial \xi}\to 0 \qquad (\xi\to-\infty,\ 0<\tau<1) \tag{A13b}$$

$$r=0 \qquad (\tau=0,\ \xi<0) \tag{A13c}$$

The solution to equation (A12) is obtained by conventional Laplace transform methods and is written

$$r=e^{\epsilon\xi}\left[1-\operatorname{erfc}\left(\frac{\epsilon}{2}\tau^{1/2}+\frac{\xi}{2\tau^{1/2}}\right)\right]+\frac{1}{2}\operatorname{erfc}\left(\frac{\epsilon}{2}\tau^{1/2}-\frac{\xi}{2\tau^{1/2}}\right) \tag{A14}$$

The dimensionless heat-transfer rate is obtained from equation (A14) as

$$\left(\frac{\partial r}{\partial \xi}\right)_{\xi=0}=\frac{1}{(\pi\tau)^{1/2}}e^{-\frac{\epsilon^2}{4}\tau}-\frac{\epsilon}{2}\operatorname{erfc}\frac{\epsilon}{2}\tau^{1/2}+\epsilon \tag{A15}$$

The third term ϵ represents the dimensionless rate at which heat is absorbed by unit mass of material during its temperature rise from $r=0$ to $r=1$ and does not contribute to the heat content of the remaining material. This heat content in dimensionless form is simply

$$\int_0^\tau\left[(\pi\tau)^{-1/2}e^{-\frac{\epsilon^2}{4}\tau}-\frac{\epsilon}{2}\operatorname{erfc}\frac{\epsilon}{2}\tau^{1/2}\right]d\tau=\left(\frac{\tau}{\pi}\right)^{1/2}e^{-\frac{\epsilon^2}{4}\tau}-\frac{\epsilon}{2}\tau\operatorname{erfc}\frac{\epsilon}{2}\tau^{1/2}+\frac{1}{\epsilon}\operatorname{erf}\frac{\epsilon}{2}\tau^{1/2} \tag{A16}$$

When $t=t_f$, $\tau=1$, and $\epsilon=\epsilon_f$, equation (A14) reduces to

$$r_f=e^{\epsilon_f\xi}\left(1-\frac{1}{2}\operatorname{erfc}\frac{\epsilon_f+\xi}{2}\right)+\frac{1}{2}\operatorname{erfc}\frac{\epsilon_f-\xi}{2} \tag{A17}$$

which, as $\epsilon_f\to 0$, has the limiting form

$$r_f=\operatorname{erfc}\left(-\frac{\xi}{2}\right) \tag{A18}$$

The asymptotic form for large ϵ_f is

$$r_f\to e^{\epsilon_f\xi} \tag{A19}$$

Equation (A16) for $\epsilon=\epsilon_f$ can be written

$$\frac{Q_f}{(\rho_b c_b k_b t_f)^{1/2}}=\frac{2}{\pi^{1/2}}\left(\frac{1}{2}e^{-\epsilon_f^2/4}-\frac{\pi^{1/2}}{4}\epsilon_f\operatorname{erfc}\frac{\epsilon_f}{2}+\frac{\pi^{1/2}}{2\epsilon_f}\operatorname{erf}\frac{\epsilon_f}{2}\right)=\frac{2}{\pi^{1/2}}\zeta \tag{A20}$$

where $\zeta\to 1$ as $\epsilon_f\to 0$ and $\zeta\to\frac{\pi^{1/2}}{2\epsilon_f}$ as $\epsilon_f\to\infty$.

The quantity $\epsilon_f=\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2}\left(\frac{d\tilde{m}}{dt}\right)_f$ is now replaced by

$$\epsilon_f=\left(\frac{c_b t_f}{\rho_b k_b}\right)^{1/2}\frac{m}{t_f}\approx 0.17k_{II}\frac{\left(\frac{M}{C_D A R}\right)^{1/2}}{\alpha\frac{\bar{c}_p}{c_{p,2}}}(1-\eta)\left(\frac{c_b}{\rho_b k_b}\right)^{1/2} \tag{A21}$$

by using the relations

$$m=1.18k_{II}\frac{\left(\frac{M}{C_D A R}\right)^{1/2}}{\alpha\frac{\bar{c}_p}{c_{p,2}}}\overline{m}$$

and

$$\overline{m}=0.84\left(\frac{t_f}{27}\right)^{1/2}(1-\eta)$$

derived in the analysis of sublimation.

The product of dimensionless heat-transfer rate at the surface and the dimensionless heat content of the material (the quantity A which appears in eq. (56)) is found from equations (A15) and (A16) and is

$$A=\left[(\pi\tau)^{-1/2}e^{-\frac{\epsilon^2}{4}\tau}-\frac{\epsilon}{2}\operatorname{erfc}\frac{\epsilon}{2}\tau^{1/2}+\epsilon\right]$$

$$\times\left[\left(\frac{\tau}{\pi}\right)^{1/2}e^{-\frac{\epsilon^2}{4}\tau}-\frac{\epsilon}{2}\tau\operatorname{erfc}\frac{\epsilon}{2}\tau^{1/2}+\frac{1}{\epsilon}\operatorname{erf}\frac{\epsilon}{2}\tau^{1/2}\right]$$

It may be verified that $A=\frac{2}{\pi}$ when $\tau=0$ for all values of ϵ and $A\rightarrow 1$ as $\tau\rightarrow\infty$ for all values of ϵ. Thus it may be expected that

$$\frac{2}{\pi}\leqq A\leqq 1$$

That is, A is a slowly varying function of τ.

REFERENCES

1. Allen, H. Julian, and Eggers, A. J., Jr.: A Study of the Motion and Aerodynamic Heating of Ballistic Missiles Entering the Earth's Atmosphere at High Supersonic Speeds. NACA Rep. 1381, 1958. (Supersedes NACA TN 4047.)
2. Chapman, Dean R.: An Approximate Analytical Method for Studying Entry Into Planetary Atmospheres. NACA TN 4276, 1958.
3. Sutton, George W.: The Hydrodynamics and Heat Conduction of a Melting Surface. Jour. Aero. Sci., vol. 25, no. 1, Jan. 1958, pp. 29–32, 36.
4. Roberts, Leonard: On the Melting of a Semi-Infinite Body of Ice Placed in a Hot Stream of Air. Jour. Fluid Mech., vol. 4, pt. 5, Sept. 1958, pp. 505–528.
5. Lees, Lester: Similarity Parameters for Surface Melting of a Blunt Nosed Body in a High Velocity Gas Stream. ARS Jour., vol. 29, no. 5, May 1959, pp 345–354.
6. Roberts, Leonard: A Theoretical Study of Stagnation-Point Ablation. NASA TR R–9, 1959. (Supersedes NACA TN 4392.)
7. Bethe, Hans A., and Adams, Mac C.: A Theory for the Ablation of Glassy Materials. Jour. Aero/Space Sci., vol. 26, no. 6, June 1959, pp. 321–328, 350.
8. Roberts, Leonard: Stagnation-Point Shielding by Melting and Vaporization. NASA TR R–10, 1959.
9. Lees, Lester: Convective Heat Transfer With Mass Addition and Chemical Reactions. Presented at Third Combustion and Propulsion Colloquium of AGARD (Palermo, Sicily), Mar. 17–21, 1958.
10. Landau, H. G.: Heat Conduction in a Melting Solid. Quarterly Appl. Math., vol. VIII, no. 1, Apr. 1950, pp. 81–94.

U.S. GOVERNMENT PRINTING OFFICE: 1961