Integrated Power and Attitude Control System (IPACS)
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Abstract. Recent advances in materials, circuit integration, and power switching have given the concept of dynamic energy and momentum storage important weight, size, and operational advantages over the conventional momentum wheel-battery configuration. Simultaneous momentum and energy storage for a three axes stabilized spacecraft can be accomplished with a topology of at least four wheels where energy (a scalar) is stored or retrieved in such a manner as to keep the momentum vector invariant. This study, instead, considers the case of two counter-rotating wheels in one axis to more effectively portray the principles involved. General scalable system design equations are derived which demonstrate the role of momentum storage when combined with energy storage.

Introduction
General scalable system design equations are derived that demonstrate the role of momentum storage when combined with energy storage. Scaling to "Smaller than Small"- the motif of this conference - may require innovative design and a search for suitable vendors.

Rotor and Enclosure Mass
Parameters are defined in the glossary and units are SI. Two areas of mass minimization are considered. One is the derivation of the expression for minimum mass sum of rotor and enclosure masses. It is assumed a pressure enclosure is required corresponding to an unmanned application. Additional mass for catastrophic containment is not considered. A second area is mass minimization of the motor-generator (M/G) and that solar array mass fraction required to support its copper loss. Mass comparisons are made between a conventional momentum wheel-battery system, battery replacement by dual wheels, and an IPACS system.

Practical implementation is discussed and IPACS advantages listed.

Configuration
A pair of counter-rotating wheels on three orthogonal axes is simplest to perceive. Another arrangement utilizes large angle control moment gyros with important power and weight advantages. A third geometry employs four wheels with momentum vectors perpendicular to the faces of a tetrahedron. This study is largely confined to the study of two counter-rotating wheels. Extension to other geometries is straightforward.

The rotor is assumed to be a hollow cylinder with outer radius \( r_1 \). The height and inner radius are defined in Table 1. With \( k_1 \) and \( k_2 \) fixed, rotor shape remains invariant as size changes. Rotor inertia and mass are given in Eqs. (1) and (2).

\[
J = \frac{M}{2} (1 + k_1^2) r_1^2 \tag{1}
\]

\[
M = \pi \rho k_2 (1 - k_1^2) r_1^3 \tag{2}
\]

The moment of inertia required by Eq. (4) is obtained with the aid of Fig. 1. and the fundamental Eq. (3).

\[
r_1 \omega_1 = \sqrt{\frac{\sigma}{\rho}} \tag{3}
\]

This gives the outer radius tangential velocity and for the representative values of \( \sigma \) and \( \rho \) given in the table yields 1643 meters/sec. or 1.643 Km/sec. (3700 miles/hr.). \( \sigma \) is the operating stress which may be smaller for a given application than the value given in Table 2. It should be as high as possible consistent with safety and operating life.

Fig. 1
Two Counter-rotating Wheels
\[
J = \frac{r_1 E + (1 + k)H}{\sqrt{\rho}} \frac{\sigma}{(1 - k^2) \frac{\sigma}{\rho}}
\]  
(4)

Eq. (4) is the inertia required to support H and E with angular velocity ranging between \( k \omega_s \) and \( \omega_s \). This is set equal to Eq. (1) after substitution of Eq. (2) into Eq. (1). A power equation in \( r_1 \) results:

\[
\pi k_1 \sigma \left(1 - k^4\right) - 2Er_1 - 2(1 + k)H \frac{\sigma}{\rho} = 0
\]

Eq. (5)

Given the two rotor shape parameters, \( k_1 \) and \( k_2 \), Eq. (4) uniquely determines the outer radius. There is no mass minimization. With the values in Table 1, \( r_1 = 0.0842 \text{ m.} \) and the rotor mass, from Eq. (2), is 2.52 Kg.

Another approach is to let \( k_2 \) be a free parameter and consider a housing weight constraint. Then a radius can be found that minimizes the sum of rotor and housing masses and also fixes \( k_2 \). Its value, however, cannot be too large, leading to a rod shape, difficult to dynamically stabilize. The governing equation is:

\[
M + M_m = \frac{2\left[ E + (1 + k)H \frac{\sigma}{\rho} \right]}{r_1} + k_3 r_1^3
\]

(6)

The first term is derived from Eqs. (1), (2), and (4) and the second term results from analysis of a working momentum wheel. Setting the derivative of Eq. (6) with respect to \( r_1 \) equal to zero, the value which produces minimum mass is:

\[
r_1 = \left[ \frac{2H}{3k_3 (1 - k)(1 + k_1^2) \frac{\sigma}{\rho}} \right]^{1/4}
\]

(7)

For the values given in Table 2, \( r_1 = 0.07749 \) and the rotor weight has decreased from 2.52 Kg to 1.91 Kg. But \( k_2 \) has increased from 2 to 3.97.

**Motor-Generator Size**

The motor-generator (MG) size is based upon its copper loss. The \( I^2 R \) dissipation is invariant with winding impedance for a given size but decreases as motor size (and mass) increase. This power loss must be provided by the solar array with its attendant mass. The goal is to find an analytical expression that minimizes the sum of these two masses.

The basic steady state DC motor equation is:

\[
V = IR + k_s \omega
\]

(8)

Multiplying by I;

\[
VI = P = I^2 R + k_s I \omega
\]

(9)

Substitute for I using \( T = kr_1 \) where \( k_r \) is the torque constant. In SI Units \( k_s = k_r = k_s \)

\[
P = T^2 \frac{R}{k_s} + T \omega
\]

(10)

This is generalized by invoking the motor constant which is commonly available from the vendor.

\[
K_m = \frac{k_s}{\sqrt{R}}
\]

(11)

\[
P = \frac{T^2}{K_m^2} + T \omega
\]

(12)

The first term is the copper loss, independent of winding impedance, and the second term is the mechanical power. It remains to find the relationship between \( K_m \) and motor mass, \( M_m \).

Let \( b = \frac{M_m}{K_m} \)

(13)

A survey of 29 Inland motors ranging in mass from 45 grams to 8.3 Kg gave a value for \( b \) of 5.23 with a standard deviation of 1.28 with rare earth magnet
motors having smaller values in this set. Finally the motor copper loss is:

$$P = \left( \frac{Tb}{M_m} \right)^2$$  \hspace{1cm} (14)

Eq. (14) is solved for $M_m$:

$$M_m = \frac{Tb}{\sqrt{P}}$$  \hspace{1cm} (15)

The solar array has a figure of merit, $c$ watts/Kg. Hence:

$$M_r = \frac{P}{c}$$  \hspace{1cm} (16)

This is the incremental array mass required to support the MG copper loss. The sum of Eqs. (15) and (16) is differentiated with respect to $P$, set to zero, and the resulting value of $P$ given by Eq. (17) is substituted into Eq. (15).

$$P = \left( \frac{Tbc}{2} \right)^{\frac{2}{3}}$$  \hspace{1cm} (17)

$$M_m = \left[ \frac{2b^2T^2}{c} \right]^\frac{1}{3}$$  \hspace{1cm} (18)

$\text{b and c are fixed so that the MG mass increases as the 2/3 power of torque. To put this in perspective, the benchmark often used here, is 300WHRs storage. The discharge power is then 900 watts assuming 30 minute eclipse and 90 minute orbit. If rotor speed is 4000 radians/sec. (half full speed), then the torque is 0.225 Nm and with b given above and c=12 watts/Kg the MG mass given by Eq. (18) is 0.61 Kg. From Eq. (17) the power is 3.68 watts.}$

It might seem that increasing the wheel depth of discharge would decrease rotor mass. However this would increase motor mass since, for a constant power withdrawal during eclipse, ($P=To$) the lower the speed the greater the torque. Another negative effect is the increased complexity of the power conditioning circuits required to utilize power over a greater voltage range as well as the array element complexity needed to efficiently charge at a lower voltage.

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**Comparisons**

Three cases are examined.

a) Energy storage only  
b) Conventional battery-wheel system  
c) IPACS

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**Energy Storage Only**

In Eq. (4) $H=0$ and then from Eqs. (1) and (4):

$$M = \frac{\rho}{\sigma} \frac{2E}{(1 + k^2_1)(1 - k^2)}$$  \hspace{1cm} (19)

This is independent of $r_1$ and $k_1$. In the limit as $k \rightarrow 0$ and $k_1 \rightarrow 1$

$$M = \frac{\rho}{\sigma} E$$  \hspace{1cm} (20)

Eq. (20) represents the ideal minimum mass thin shell rotor, useful for comparison.

Using the values in Table 2 Eq. (19) provides a mass of 0.682 Kg. for the mass of each of two counter-rotating wheels.

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**Table 1**

Wheel Parameters for 300 WHrs

<table>
<thead>
<tr>
<th>Item</th>
<th>Mass Kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotor</td>
<td>0.68</td>
</tr>
<tr>
<td>MG</td>
<td>0.61</td>
</tr>
<tr>
<td>bearing</td>
<td>0.55</td>
</tr>
<tr>
<td>housing</td>
<td>0.62</td>
</tr>
<tr>
<td>Total</td>
<td>2.46</td>
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</table>

For two wheels the mass is 4.92 Kg. Energy storage wheels are not temperature sensitive as batteries are, have no limit on depth of discharge as a function of number of cycles and no time dependent failure modes. It would be advantageous on GEO spacecraft since it could be completely de-energized except at equinoxes.
A state of the art nickel-hydrogen battery is used for comparison. It will be de-rated to 10 year LEO use where on each discharge cycle it is discharged to only 67% of its rated capacity. A nickel-hydrogen battery rated at 15 AH at 28 volts weighs 8.9 Kg. A 30 ah battery, scaled up to 32.1 AH and de-rated by 2/3 will provide 300 WHRs usable energy and weigh 19.0 Kg. The dynamic energy storage mass is therefore about 25% of battery mass. Array element switching and battery unique circuit masses are not considered; both are small and have similar values. The mass saving for direct battery replacement is 14.1 Kg.

**Conventional Battery-Wheel System**

A mass estimate for a conventional battery and wheel system is made to provide a comparison with an IPACS system. The battery mass will be the same as calculated in the previous section (19.0 Kg). A three axis stabilized system will require a 50Nms wheel on each axis with the following mass;

\[
M_{\text{mw}} = 2.8H^3 = 12.1 \text{ Kg}
\]  

(21)

The total mass for this system is then 55.3 Kg. (Other ACS masses common to all systems, such as momentum de-saturation torquer bars are not included.)

**IPACS**

Two wheels for each of three axes is base-lined for direct comparison with the conventional system. It is assumed that 50 Nms capability is required on each axis and that 300 WHr storage is divided between three axes for 100 WHr per axis. Note that E in the equations has the units of joules. Other parameters are listed in Table 2. Then from Eq. (6) the rotor radius is 0.07749 and with this value used in Eq. (7) the motor and housing mass equal 1.228. From the argument following Eq.(18), the torque is one third the former value. Then Eq. (16) gives an MG mass of 0.295. Including the bearing estimate from Table 1 the single wheel mass is 2.07 Kg, and for six wheels 12.4 Kg. This is 23% of the conventional system mass.

One reason for the advantage is poor battery utilization (only a third of its capacity can be used). Another is the efficient use of the high performance inertial rotors. The mass saving over a conventional system is 43 Kg.

**Spin Axis Orientation Scheme**

Magnetic bearings have the design latitude to provide a small degree of freedom, typically + 1 degree in spin axis orientation. An interesting but limited system may be designed on this basis. A tetrahedronal array of magnetically suspended wheels is considered where each has a small transverse angular motion capability, \( \alpha \). All four wheels operate at the same speed over the range from \( k_\omega \) to \( 2\omega \). With nominal orientation the net momentum is zero. To keep it simple and symmetrical, momentum is desired in the direction of one of the spin axes. This net momentum is obtained by symmetrically changing the spin axis orientation of the three remaining wheels. This is probably a worst case as well. Wheel speeds are assumed at the discharged value, which is worst case. Only the finally derived equation is presented; based upon some of the foregoing relationships.

\[
H = -k\sqrt{2\rho} \left( E' \right)^{1/2} \frac{E}{\alpha k_\omega (1-k_\omega^2)(1-k_\omega^3)\alpha^2} \left( 1 - \frac{\sqrt{2}}{8} \alpha \right) (22)
\]

H depends upon energy, carbon fiber properties, rotor form factors, and per unit speed at discharge. The curious 4/3 power relationship between H and E is due to the increase in moment of inertia and outer radius with E.. With values from Table 2, N is 3.6 Nms for E= 5E6 joules. It seems that bigger is better in this instance but a parametric analysis of disturbance momentum as energy (and presumably spacecraft size) increase, is needed.

**Rotor Shape**

There are differing opinions about rotor shape with advocates of rod shapes, disk shapes, and a compact shape with height equal to outer diameter. The foregoing equations show how these parameters influence the design on a theoretical basis. More important is choosing viable mechanical designs that produce a rotor that stays together with minimal modal problems. Then the system response to successful rotor geometries can be assessed via these equations. Design need not be confined to a cylindrical section. For those containing a central shaft the cross sectional shape approaches a truncated Gaussian curve for most efficient use of fiber.
Conclusions

Simple battery replacement by two counter-rotating energy wheels saves appreciable mass but the IPACS system provides the greatest mass saving, even with large momentum storage capability. Coupled with greater transient discharge capability and temperature insensitivity, the energy wheel will be a major future contender.

Another advantage of IPACS is the elimination of dynamic unbalance torque by virtue of the magnetic bearings which can be designed to have low gain at higher frequencies such that the rotor will rotate about its mass center rather than geometric center.

The intent of this paper is to provide a primer on design of an energy wheel system in general scalable terms suitable for large and small systems. As such, the numerical value of carbon filament modulus, for example, is not to be taken as authoritative. Numerical values are illustrative and in the ball park.

References


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>b</td>
<td>MG figure of merit</td>
<td>( \frac{Kg}{\sqrt{\text{watt}}} )</td>
<td>5.23</td>
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<tr>
<td>c</td>
<td>Solar array figure of merit</td>
<td>( \frac{\text{watts}}{Kg} )</td>
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<td>E</td>
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<td></td>
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<td>h</td>
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<td>H</td>
<td>momentum</td>
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<td>J</td>
<td>Single rotor moment of inertia</td>
<td>( \frac{Kg}{m^4} )</td>
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<td>k</td>
<td>Per unit lower speed limit</td>
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<td>( \sigma )</td>
<td>Carbon fiber maximum working modulus</td>
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