Design and Implementation of Satellite Formations and Constellations

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The direction to develop small low cost spacecraft has led many scientists to recognize the advantage of flying spacecraft in constellations and formations to achieve the correlated instrument measurements formerly possible only by flying many instruments on a single large platform. Yet, constellations and formation flying impose additional complications on orbit selection and orbit maintenance, especially when each spacecraft has its own orbit or science requirements.

The purpose of this paper is to develop an operational control method for maintenance of these missions. Examples will be taken from the Earth Observing-1 (EO-1) spacecraft that is part of the New Millennium Program (NMP) and from proposed Earth System Science Program Office (ESSPO) constellations. Results can be used to determine the appropriateness of constellations and formation flying for a particular case as well as the operational impacts. Applications to the ESSPO and NMP are highly considered in analysis and applications.

After constellation and formation analysis is completed, implementation of a maneuver maintenance strategy becomes the driver. Advances in technology and automation by GSFC's Guidance, Navigation, and Control Center allow more of the burden of the orbit selection and maneuver maintenance to be automated and ultimately placed onboard the spacecraft, mitigating most of the associated operational concerns. This paper presents the GSFC closed-loop control method to fly in either constellations or formations through the use of an autonomous closed loop three-axis navigation control and innovative orbit maintenance support. Simulation results using AutoCon™ and FreeFlyer™ with various fidelity levels of modeling and algorithms are presented.

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INTRODUCTION

Missions such as those of the Earth System Science Program Office (ESSPO) and New Millennium Program (NMP) emphasize the use of multiple spacecraft to collect Earth-imaging scientific data. These programs consist of multiple spacecraft in various orbits which include the Earth Observing System's EOS AM-1, EOS PM, EOS CHEM, and the EOS Laser Altimetry (ICESATs) missions. Other related spacecraft such as the next generation of Landsats are also considered part of this initiative. The EO-1 spacecraft of the NMP also is using the ESSPO requirements to promote technologies and correlated measurements. The orbit characteristics of several of these missions presented in the following table seem very similar in orbital mechanics terms, however the science goals are varied to achieve a wide range of Earth observations in the areas of ground imaging, atmospheric research, and ice sciences. These various spacecraft form a constellation of related spacecraft, potentially taking coincident or sequential measurements of the same location on the Earth's surface, or correlating measurements of related atmospheric phenomena. The reasons for these temporal measurements range from cross-calibration of the instruments as follow-on spacecraft are launched into the same orbit to sequential measurements made by instruments on spacecraft in different polar orbits.

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<td>Delta</td>
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As these programs mature, the maintenance of a constellation or formations of spacecraft drives the need for further analysis regarding the design of the spacecraft orbits. Analysis regarding the impacts of a design on subsequent missions and their requirements becomes more important and has highlighted challenges in determining the feasibility of proposed solutions to scientific questions, in accounting for monetary constraints, and in accommodating new technologies which have also posed challenges in the areas of orbit control and temporal observations. Extended analysis has also been driven by the imposition of constellation requirements on future low Earth orbiting spacecraft.

FORMATION AND CONSTELLATION DESIGN DRIVERS

Design drivers for formations and constellations come from both scientific and technological disciplines, and include:

- Small spacecraft flown as virtual platforms or ESSPO mission segments to meet instrument or scientific requirements.
- Navigation and communications requirements.
- Spacecraft and instrument operational considerations.
Constellations and formations offer the advantages of reduced launch risk per instrument, the separation of instrument and spacecraft bus schedules, and the implementation of new technology. However, the use of several spacecraft instead of one large spacecraft bus also has some disadvantages when coincident or sequential observations or calibration of instruments are required. The use of one instrument’s imaging data by another for planning, near-real time operations, or ground data processing can become a significant driver.

The proposed use of ground stations instead of the space network for communication support is another consideration in constellation and formation design. ESSPO spacecraft are considering the use of X-band direct downlink for scientific data return. In order to assure that direct downlinking of data from numerous spacecraft will be possible without overlap in viewing from the ground station, an analysis was performed of the separation in a constellation which would minimize science data collection concerns. Therefore, in considering the maintenance of a constellation or formation, fuel budgets must be analyzed. The goal is to minimize the required fuel for constellation maintenance by combining this maneuver with other maneuvers already planned to meet other mission requirements such as ground track control.

Navigation system selection also will impact the choice and design of constellations and formations not to mention the impact to the available onboard computer hardware and Attitude Control Systems (ACS). Recently, GPS has come to the forefront for real-time onboard navigation, but other technologies exist which may compliment the spacecraft hardware and provide a robust real-time navigation system. The technology of cross-links between spacecraft for both data communication and relative navigation has yet to be fully explored, but for a true closed-loop design, a real-time cross link must be available.

Orbit mechanics and the need to meet all mission orbit requirements place a great burden on the selection of the constellation and its maintenance. For example, most EOS missions have both ground track and mean local time (MLT) of node crossing control requirements. These orbital requirements must be met in order to successfully collect scientific data. Also, physical impossibilities will inhibit wishful thinking in the selection of some constellations or the achievement of the formations directly from the launch vehicle. Some constellations may take a long duration to establish and can impose increased constraints on the launch vehicle to meet injection targets. The operations associated with these maneuvers may also become a driver if the instruments are required to physically change their modes, such as covering up optics during maneuvers to protect against contamination or sun impingement.

**Formation And Constellation Definitions**

While often used together, achieving and maintaining a constellation are independent concepts from that of formation flying. A constellation is defined as two or more spacecraft in similar orbits that perform separate control of their orbits. They may provide global or localized science data, but mostly in a post-processing sense. They do not provide real-time communications between spacecraft. In general, a constellation could contain spacecraft that have no hard requirement concerning maintenance of a relative position. For a large difference in orbital anomalistic angles, relative cross track separations vary over the orbit since the spacecraft are really in different orbit planes. This orbit plane difference in nodal crossing is used as an advantage for constellation maintenance to meet sequential observations by accounting for the Earth rotation. The concern is that the result of relative drift in the along-track direction between two spacecraft yields a different sub-satellite point, thereby impeding the coincident observation requirement on every orbit. However, for the NMP problem, in order to achieve a higher percentage of coincident observations, the spacecraft have the additional requirement to maintain a formation within the constellation.

Formation flying is an orbital operations concept design in which a spacecraft maintains a predetermined trajectory relative to a reference position without making a physical attachment. This reference position may be occupied by another spacecraft if desired. Consider two spacecraft placed in the same orbital plane and at the same altitude, with an initial anomaly separation angle small enough that atmospheric density and gravitational perturbations can be considered constant. These spacecraft will be similarly affected by atmospheric drag and by the gravitational potential field of the Earth provided that they have identical ballistic properties. Ballistic properties are defined here as the ratio of mass to the product of frontal area and coefficient of drag. If the spacecraft are separated in the radial direction, and the
respective ballistic properties are different, their orbit velocities are also different, and one spacecraft (the formation flyer) will appear to drift relative to the other (the reference flyer). The drift is most apparent in the along-track (orbital velocity) direction. The approach for determining the formation flying maintenance was formulated using basic orbital mechanics and formation flying concepts which are derived from Hill's or Clohessy-Wiltshire Equations of motion. 8

DESIGN METHODOLOGY

To consider the methodologies of maintaining constellations and formations, an example from each is discussed in detail. The first methodology discussed is the constellation.

Constellation Design and Maintenance

The mean anomaly separation between spacecraft is used as the basis for our analysis. While some separations may seem exceedingly large, it is determined by the science temporal requirement for coincident/sequential observations and by communication requirements. Also, for spacecraft to observe the same location, their orbit planes must be oriented to account for the rotation rate of the Earth during the time lag between one spacecraft seeing the location and the other spacecraft passing over the same location. To characterize the definition of location, it is assumed that the sequential instrument fields of view are large enough to have an imaging expectancy of at least 80%. 9 A first order approximation to analyze the constellation was completed based on orbital mechanics found in any textbook. While high order Geopotential and third body effects can be ignored in the analytical results, they should be considered when verifying results. These values were verified in high order simulations using AI Solutions' AutoCon™, or FreeFlyer™. 10 The analysis of constellations was based on information in Table-1 and on the following assumptions and requirements:

- The spacecraft must maintain a minimum true anomaly separation.
- All spacecraft must meet their groundtrack requirements, therefore, maneuvers must be performed at intervals defined by the atmospheric conditions and not the constellation maintenance.
- The range of spacecraft ballistic coefficient differences are no larger than 15% with a baseline of 50 kg/m².
- Atmospheric conditions are considered to be relatively uniform over the separation in the orbit planes and between spacecraft.
- The maximum separation in radial altitude to meet the maximum ground track requirement is 2 km (+/- 1 km about a reference altitude).

Other mission orbit requirements place additional constraints on the constellation maintenance. These are ground track control, frozen orbit control, inclination control, mean local time control, and repeating orbits. The principal driver of these is ground track maintenance, which has the most stringent orbit requirements. To meet science requirements for Earth observing instruments, the repeating groundtrack of the sub-satellite must be controlled. Ground track maintenance is performed by varying elements of the orbit to ensure that the orbit repeat cycle is met and reference points at the equator are overflown each orbit. The ground track accuracy is maintained by changing the orbital nodal period with respect to the fixed Earth rotation rate. The nodal period is adjusted by changes to the semi-major axis. The number and times of the maneuvers to accomplish this are determined by atmospheric conditions. For ESSPO spacecraft, this maneuver frequency varies between one month and six months. Frozen orbit control can be accomplished through strategic placement of the ground maintenance maneuvers at no additional fuel cost. The other orbit parameters are rarely adjusted and are not considered here.

Constellation Targets

To maintain the constellation, maneuvers must be performed to control the drifting between spacecraft due to the differential decay rates. The targets used for constellation maintenance are dependent upon the individual requirements of the science goals, operations, and constraints. An example of the targets used most often for polar orbiting ESSPO type missions are semi-major axis (sma) and eccentricity. One can maintain an ESSPO constellation by adjusting these parameters to control the individual orbit or to
maintain the constellation separation. A change to the sma will adjust the drifting in an orbit period between the spacecraft while the eccentricity can adjust the orientation of the relative orbit elements such as argument of periapsis. The sma can be targeted to meet the ground track requirements and to maintain the constellation.

**Maintenance Using Ground Track Control**

If one follows the ground track control theme then constellation maintenance is reduced to meeting the mission requirements. The ground track control is realized by a change to the sma and the adjustments made to this parameter will result in a differential drift in the relative mean anomaly. There is no control of the magnitude of the drift between the spacecraft as the drift distance is dependent upon when the maneuvers are performed for the ground track control. The targeted sma is the required sma to maintain the mission ground track which can be computed via differential correction methods in FreeFlyer™.

**Maintenance Using Mean Anomaly Control**

If one follows the differential mean anomaly rate theme, one can adjust the time it will take to transverse a delta mean anomaly between the spacecraft. The selection of the sma of the maneuvering spacecraft can be used as a target to bring about a controlled drift over a given delta anomaly in a given time. The derivation of this sma target is simply an algebraic expansion of the mean anomaly rates as shown below.

The mean anomaly difference over time can be computed as,

\[ \Delta \theta = (n_1 - n_2)t \]

where

\[ n_1 = \frac{\mu}{a_1^3} \]
\[ n_2 = \frac{\mu}{a_2^3} \]

with \( a_n = \text{mean sma} \), \( a_{0_0} = \text{initial mean sma} \), \( a_{id} = \text{sma decay rate} \), \( \mu = \text{gravitational constant} \), \( n = \text{mean motion} \), and \( t = \text{time} \).

Using a desired angular difference and time, this can be expanded to,

\[ \Delta \theta \frac{t}{\mu} = \sqrt{\frac{\mu}{(a_{0_0} \pm (a_{id} \cdot \frac{t}{2})^3) - \frac{\mu}{\sqrt{(a_{0_0} \pm (a_{id} \cdot \frac{t}{2})^3)}}} \]

Solving for the target semi-major axis, \( a_{2_0} \), and using an assumption that the decay rates are subject only to the differential ballistic coefficients yields,

\[ a_{2_0} = \left[ \frac{\mu}{\sqrt{\mu}} \left( \frac{\mu}{(a_0 \pm (a_{id} \cdot \frac{t}{2})^3) - \frac{\mu}{\sqrt{(a_0 \pm (a_{id} \cdot \frac{t}{2})^3)}}} \right)^{1/3} + \left( (a_{id} \cdot \frac{t}{2}) \cdot (B_{C_1}/B_{C_2}) \right) \right]^{1/3} \]

where,

\[ a_{2_{id}} = a_{id} \cdot (B_{C_1}/B_{C_2}) \]
Ground Track Control Results

To consider a sample scenario, this analysis assumes that ground track maintenance starts with one spacecraft half-way through the ground track maintenance cycle to account for the maximum radial separation (and therefore maximum in-track velocity difference) over time. Orbital decay rates were calculated at the solar flux maximum, based on +2 sigma predictions. If the ballistic coefficient \( (B_c) \) is 50 kg/m\(^2\), the decay rate at 705 km at the beginning of the mission (June 1998) is approximately 0.0028 km/day. If the \( B_c \) equals 40 kg/m\(^2\), the decay rate is approximately 0.0034 km/day. Decay rates for the -2 sigma solar flux values can be orders of magnitude less (e.g. \( B_c=50\text{kg/m}^2 \), decay rate ~ 0.0003 km/day four years later) and could give significantly different results. The ground track maneuvers periodically change the relative semi-major axes of the spacecraft which results in a switching of the sign of the delta mean motion.

The maintenance of the ground track results in a repeating and somewhat uniform increasing and decreasing of the mean anomaly (along-track distance) between the spacecraft as maneuvers change the direction of the differential mean motion. The observed difference in the mean anomaly of each spacecraft varied by approximately +/- 15° over a several months. This difference suggest that ESSPO type separation angle requirements of 40° can easily be met. Furthermore, results suggest that multiple spacecraft can be initially 'stationed' at intervals of 60° to allow for drift. These spacecraft do not need to be in co-planar orbits, since the above sequential observations and station coverage must be met. More importantly, the ground track control results of this analysis suggest that no additional propellant is required to maintain a constellation separation if the coincident observations can be reduced to occurring at smaller time intervals. In Figure 1, a mean anomaly separation angle is shown for spacecraft with the same \( B_c \) but with different ground track requirements of +/- 20 km and +/- 5 km. Figure 2 presents the separation angle for spacecraft that have the same ground track requirements, but the \( B_c \) of the formation flyer is 15% that of the reference spacecraft (40 kg/m\(^2\) vs 50 kg/m\(^2\)).

Mean Anomaly Control Results

The results of using the equations derived above for the sma targets are shown in Figure 3 and 4. Figure 3 presents the required initial sma to drift a desired distance in a fixed time and the sma to drift a fixed distance in a desired time. Two examples in the figure show the effects of changing the fixed parameter. The results of this spreadsheet were numerically verified using the FreeFlyer™ system and the verified points are noted by the circles and squares. The initial reference sma was 7077 km, which represents a typical mean element of the sma of ESSPO orbits.
The mean anomaly control results of this analysis, while similar to the ground track results, suggest that any given constellation separation magnitude can be controlled. The separations and time can also be used as an input into the ground track control to minimize the separation drift distances and thereby increase the number of sequential instrument observations. The results suggest that ESSPO type separation angle requirements of 40° can easily be met. Furthermore, results suggest that multiple spacecraft can be initially 'stationed' at smaller separation angles. As with the ground track results, these spacecraft do not need to be in co-planar orbits, since the above sequential observations and station coverage must be met.

Figure 3 - Constellation Drift

Analytical Drift Equations

Figure 4 presents a general analytical method to compute the initial radial separation for maintaining a constellation given a desire to control the along track separation. The equations for this plot are analytical and only a meant to given a representative case. The point is that a controlled drift in the along track direction, both away from and back toward a reference spacecraft cannot be achieved by using the generic drift equations previously described. The radial separation required for a controlled drift is an order of magnitude smaller than that for general drift over a given time period. Since the orbit decay is inversely proportional to the ballistic coefficient, the chase spacecraft will decay at an average decay rate similar to that of the reference and is given by,

$$\dot{r}_2 = \dot{r}_1 \left( \frac{Bc_1}{Bc_2} \right)$$

and the differential orbital decay rate will be

$$\Delta \dot{r} = \dot{r}_1 - \dot{r}_2$$

the initial radial separation can then be given by

$$\Delta r = 0.5 \cdot \Delta \dot{r} \cdot t$$

The maximum downrange drift rate can then be given by substitution into the differential angular rates

$$\dot{D}_{max} = 1.5 \cdot \sqrt{\mu / r^3} \cdot \Delta r$$

and the maximum drift is then

$$D_{max} = \dot{D}_{max} \cdot 0.5 \cdot t$$

The equations are presented here as a general guideline and do not hold up under a high fidelity modeling which includes higher order Geopotential terms and differential orbital perturbations due to large angular separations. Figure 4 presents the drift and initial radial separation only for the decay rates used in the constellation analysis and need to be modified for each individual case.
MECHANICS AND DEFINITIONS OF FORMATION FLYING

In order to meet the coincident observation requirement without a large variation in the anomaly as previously presented, a formation strategy must be developed and followed. Assuming that the fields of view of the instruments are circular (on the order of one kilometer in diameter) and nadir pointed, a control box can be determined to ensure that the FOVs will overlap to a given percentage. It is assumed here that this control box is 50 kilometers in the along track direction, given the assumption that the ground error is equal to the along-track error for a sma of 7077 km. Therefore, to meet this two kilometer requirement, an initial altitude displacement for the formation flying spacecraft with respect to the reference is required to affect the formation flying theory.

Formation flying involves position maintenance of multiple spacecraft relative to measured separation errors. It involves the use of an active control scheme to maintain the relative positions of the spacecraft. Optimally, this process will be performed autonomously onboard the spacecraft and is called Enhanced Formation Flying, such as that which will be implemented by GSFC for the New Millennium EO-1 mission. A complete description of the fundamental of formation flying was previously published\textsuperscript{11}. An example of the orbit dynamics of formation flying is shown in Figure 5.

![Formation Flying Example](image)

FIGURE 5. Formation Flying Example

Formation flying techniques can be used to meet a variety of mission separation requirements. When the mission requirements call for a tightly controlled separation (kilometer range), whether the overall separation is small or large, frequent control becomes necessary. Formations of spacecraft are identified using tight or loose control methods. While some separations may seem exceedingly large, they are determined by the science requirement to view coincident sites or a communication requirement of a ground station to view only one spacecraft at a time. For large separations, one must consider the rotation of the Earth if the formation is used to meet concurrent or sequential imaging of the same locations on the ground. Therefore, relative crosstrack separations are used to follow the reference ground track for any temporal requirement. A patent rights application was submitted to the GSFC patent counsel by two of the authors for the application of Autonomous Closed Loop 3-Axis Navigation Control Of Spacecraft.\textsuperscript{12}

Formation Flying and Targeting Algorithm Description

The algorithm enables the spacecraft to execute complex 3-axis orbital maneuvers autonomously. Figure 6 illustrates the basic sets of information required for formation targeting as it is incorporated into AutoCon\textsuperscript{TM}. The algorithm is suited for multiple burn scenarios but is explained here in a two-burn approach for clarity. The simplest formation flying problem involves two spacecraft orbiting the Earth. One spacecraft, referred to as the control spacecraft, orbits without performing any formation flying maneuvers. The second spacecraft is the chase spacecraft. It monitors the control spacecraft, and performs maneuvers to maintain the desired formation phasing. The goal of the formation flying algorithm is to perform...
maneuvers to move the chase spacecraft along a specified trajectory, called the transfer orbit, from its initial state \( S_0 = (r_0, v_0) \) at a given time \( t_0 \) to a target state \( S_t = (r_t, v_t) \) at a later time \( t_t \).

- Determine \((r_t, v_t)\) at \( t_0 \) (where you are at time \( t_0 \)).
- Determine \((R_t, V_t)\) at \( t_t \) (where you want to be at time \( t_t \)).
- Project \((R_t, V_t)\) through \(-\Delta t\) to determine \((r_t, v_t)\) (where you should be at time \( t_0 \)).
- Compute \((\delta r, \delta v)\) (difference between where you are and where you want to be at \( t_0 \)).

**FIGURE 6. Formation Orbital Parameters**

This goal is accomplished by finding the state the spacecraft would have at the current time in order to achieve the target state at the target epoch without maneuvering. This new state is called the desired state \( S_d = (r_d, v_d) \); it is the target state propagated backwards in time from the target epoch to the epoch of the initial state. The difference between the initial state and the desired state is:

\[
\delta S = \begin{pmatrix}
\delta r \\
\delta v
\end{pmatrix} = \begin{pmatrix}
r_0 - r_d \\
v_0 - v_d
\end{pmatrix}
\]

Then, following the derivation of the state transition matrix given in Battin \(^{13}\), the relevant state transition matrix submatrices are:

\[
R(t_s) = \left[ \frac{r_d}{\mu} \right] (I - F) \left[ r_t - r_d \right] v_d^T - \left[ v_t - v_d \right] x_d^T + \frac{C}{\mu} \left[ v_d v_d^T \right] + G \left[ I \right]
\]

\[
\tilde{R}(t_s) = \left[ \frac{r_t}{\mu} \right] \left[ v_t - v_d \right] \left[ v_t - v_d \right]^T + \frac{1}{\left[ r_t \right]^2} \left[ r_t \right] (I - F) \left[ r_t x_d^T + C v_d x_d^T \right] + F \left[ I \right]
\]

The expressions for \( F, G, \) and \( C \) are derived from the universal variable. From these submatrices, the \( C^* \) matrix is computed as follows:

\[
R^*(t_0) = -R^T(t_s)
\]

\[
V^*(t_0) = \tilde{R}^T(t_s)
\]

\[
C^*(t_0) = V^*(t_0) \left[ R^*(t_0) \right]^T
\]

The expression for the impulsive maneuver follows immediately:

\[
\Delta v = C^*(t_0) \delta r - \delta v
\]

**Keplerian and Non-Keplerian Transfer Orbits**

The transfer trajectory for constellations and formations does not need to be of a Hohmann type. Having established both actual and desired states of a spacecraft's location using any navigation filter, all that is needed is a means of autonomously zeroing the difference between the two states. Given two Keplerian trajectories and a chronologically defined maneuver window, a reference non-Keplerian trajectory may be determined which will smoothly transport the spacecraft from its position on the first Keplerian path at the beginning of the maneuver window to a desired position on the second Keplerian path.
at the conclusion of the maneuver window. Control points on the reference trajectory in Figure 7 are calculated at regular time intervals consistent with the ability of the spacecraft to receive and process position data, fire its thrusters, and account for the effects of each firing. At each step in the process, the next control point on the reference path is examined and back-computed along a Keplerian path to determine small differences between spacecraft position and velocity on the reference path and determine which Keplerian path would intersect the reference path at the next control point. These differences are then fed into a system of linearized state transition matrices to determine the incremental \( \Delta V \) required to get the spacecraft to the next control position on the reference trajectory. At the conclusion of the maneuver window, a final burn is required to match the velocity required to maintain the new Keplerian trajectory. One can use single or multiple maneuvers to achieve the target condition.

![Figure 7. Non-Keplerian Reference Trajectory During Maneuver](image)

**Algorithm Targets**

For the formation, the orbit target is described as a location relative to the reference so that the drifting due to ballistic coefficient difference can be utilized. For example, the EO-1 relative position has a three dimensional target that is 450 km behind the reference spacecraft in the along-track direction, a sub-kilometer altitude above the reference, and also a cross track differential to account for the rotation of the Earth to meet the observation requirements. This target can easily be misinterpreted as a simple rotation in true anomaly and altitude for the along-track direction and altitude and a node displacement for the cross track requirement. If a true anomaly is used to compute the along-track difference, a completely different orbit will be designed as the change on keplerian elements doesn’t take into consideration the true orbit with the perturbations included. It can be shown that propagating an orbital element set with a delta true anomaly either before or after the change in altitude will not give the desired results. Therefore, if one wants the orbit or the formation flying spacecraft to ‘fly’ a predetermined trajectory, the following method can be used. This method will account for the cross track component as well.

- Initially use the reference orbit cartesian state
- Offset the altitude of the formation flyer by the desired amount
- Propagate (numerical methods suggested) the initial state backward/forward by the required time delta, e.g. plus or minus one minute.
- Change the Epoch of the final propagated state of the formation flyer to the original time to effect a change in the cross track to meet coincident observations
- Change the coordinate system into ECI from ECEF

**Formation Flying Results**

The following results are taken directly from the AutoCon\textsuperscript{TM} ground system which utilizes the GSFC algorithm. The results are divided into two formation flying scenarios of two spacecraft which maintain either a close or a dynamic formation.\textsuperscript{7} The initial conditions were derived from the orbit elements.
for the Landsat-7 mission which has a sun-synchronous orbit with a descending node MLT of 10:00 a.m. and a ground track repeat of 233 orbits in 16 days. The results show formation flying evolution and the effect on the mission groundtrack requirements. Evolution Figures are presented in a control spacecraft rotating coordinate system with the radial direction being the difference in radius magnitude and the alongtrack direction being the arc between the position vectors.

Close Formations

The first two figures present the maintenance of a formation that has a 10 meters radial separation only. Figure 8 presents the formation evolution in radial and separation distances for a period of 90 days. To re-initialize this orbit, two maneuvers are used in a Hohmann-like transfer. The first DV to re-establish the 10 m radial position separation by using the algorithm targeting method with a ½ orbit period and the second DV by using the same method with a .01 orbit period to adjust the velocity components. Figure 9 presents the ground track of these orbits. The initial orbital condition placed the ground track at the "0" error location for convenience.

Figures 10 and 11 present results of starting with an initial along track separation of 0 m and an initial radial separation of 20 m and then targeting to a 10 m radial and 0 along track separation whenever
either spacecraft performs a maneuver. Therefore, the first maneuver of the formation flyer is to adjust to both the groundtrack of the control spacecraft after its maneuver and to re-establish the initial formation parameters. Figure 10 presents the results when ground track maneuvers have occurred for the control spacecraft. As seen in Figure 11, a ground track maneuver takes place slightly before the time when the along track separation is near zero. The smaller parabola represent the maintenance of the formation to the 10 m radial separation. The formation evolution in radial and separation distances is presented for a period of 90 days.

Dynamic Formations

The next simulation consists of maintaining a dynamic formation where the formation flying spacecraft was in a different orbit plane with an along track separation on the order of 450 km. To simulate this, the initial state of the control spacecraft was propagated backward for 1 minute (450 km at 7.5 km/s) and to maintain the ground track requirement the right ascension of ascending node was adjusted to account for a one minute Earth rotation. Figures 12 and 13 present the formation evolution in the radial versus along track and cross track versus along track separation for several days. The effect of the perturbations on the orbit elements has an immediate effect in the osculating orbital elements. This results in a very large radial separation approaching +/- 1km. A cross track of +/- 30km was anticipated since that is the effect of the node difference. As the formation evolved, a maneuver was required to re-established the formation at the initial separation of 0 m alongtrack and 30 km cross track at a radial separation of 10 m. Figure 12 presents the trajectory of the formation flyer. The figure shows the radial separation change from approximately 500 m to +10 m and an along track separation from 450 km to 0 km. After this state was targeted, a maneuver was performed to maintain a formation similar to the close formation. Figure 13 presents formation evolution after the maneuver.

EFF TECHNOLOGY DESCRIPTION

The control of the constellations and formations mentioned above use an algorithm that is part of a new technology called AutoCon™, which features flight software that is capable of autonomously planning, executing, and calibrating routine spacecraft orbital maneuvers. The autonomous EO-1 formation flying control software AutoCon™ builds on this existing capability for the maneuver planning, calibration, and evaluation tasks. A fuzzy control engine is ideal for this application because it can easily handle conflicting constraints between spacecraft subsystems.

The AutoCon™ flight control system will need data from additional sensors and spacecraft subsystems such as propulsion, groundtrack, navigation, and attitude data. It will then be possible to autonomously generate, analyze, and execute the maneuvers required to initialize and maintain the formation between Landsat-7 and EO-1. Figure 14 shows a functional diagram of the AutoCon™ system. Because these calculations and decisions are performed onboard the spacecraft, the lengthy period of ground-based planning currently required prior to maneuver execution will be eliminated. The system is general and modular so that it can be easily extended to future missions. Furthermore, the AutoCon™ flight
control system is designed to be compatible with various onboard navigation systems (i.e. GPS, or an uploaded ground-based ephemeris). This formation flying technology will demonstrate the capability of EO-1 to fly over the same groundtrack as Landsat-7 within +/-3 kilometers at the equator while autonomously maintaining the formation for extended periods to enable paired scene comparisons between the two satellites.

**Autonomous Control Architecture Design**

The Enhanced Formation Flying (EFF) system for the EO-1 application is designed by GSFC, AI Solutions, Inc., and the Hammers Company, who has responsibility for the EO-1 attitude control system (ACS). The flight software, AutoCon-Flight™, will serve as the overall architecture and execute the Goddard developed control algorithm for maneuver decision, design, and execution. This control algorithm will provide a delta-velocity magnitude, burn epoch, and duration to the ACS for execution. Maneuver implementation is the responsibility of the ACS. Maneuver calibration will be performed autonomously within AutoCon™. Integration testing and system verification will be performed with the ACS flight software prior to the mission to demonstrate technology readiness. Ground simulation equipment will be used for system integration, testing, and performance evaluation. Verification of the flight system performance during the operational phase will be conducted according to a validation plan. This validation will occur in several incremental steps starting with ground verification of the maneuver parameters that have been computed onboard, and will culminate with full onboard autonomous maneuver prediction, planning, and closed-loop onboard maneuver execution. A subsystem interface is shown in Figure 15.

![Figure 14 - AutoCon Functional Diagram](image)

![Figure 15 - AutoCon Sub-System Diagram](image)

**CONCLUSIONS**

In considering the use of constellations to meet scientific objectives, one must take into account the physical limitations and restrictions imposed. Our results suggest that no additional propellant is required to maintain a large constellation separation if the coincident observations can be reduced to occurring at small time intervals when the orbit mechanics would naturally provide this event. The maintenance of a constellation for constant coincident viewing can be quite complicated but is feasible if one manages and plans for this endeavor. This planning should assess emerging technology and the system engineering aspects of the spacecraft development. It should assess the spacecraft ballistic coefficient in particular as well as the amount of fuel required. The amount of coincident observations that are required to meet mission objectives versus the amount desired should be addressed. The formation can be established to provide coincident observations on a timed schedule, but may miss targets of opportunity for calibration or extra coverage.

This paper shows that the formation flying algorithm presented is a feasible technology that can be used in a closed-loop design to meet science and mission requirements of Low Earth Orbiting missions in the NMP and ESSPO. The algorithm is very robust in that it supports not only benign ground track control, but demanding 3-D control for inclination and non-Keplerian transfers. To best meet the NMP EO-1
requirements, this innovative technology will be flown onboard the spacecraft which launches in May 1999. The algorithms are being integrated into AutoCon™ for both ground support validation and closed-loop onboard autonomy. This system will be implemented as a close-loop flight code onboard the NMP Earth Orbiter-1 (EO-1) spacecraft thereby yielding the name of Enhanced Formation Flying while an open-loop system will be implemented on the ground for verification. The application of this algorithm and AutoCon™ system to other NMP or ESSPO programs is unlimited and can be used to fully explore the NASA mandate of faster, better, cheaper spacecraft.

REFERENCES

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