DICTIONARY APPROACHES TO IMAGE COMPRESSION AND RECONSTRUCTION

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Technical Areas: Image coding and compression, Medical Image Processing, Signal Reconstruction, Wavelets

ABSTRACT
This paper proposes using a collection of parameterized waveforms, known as a dictionary, for the purpose of medical image compression. These waveforms, denoted as $\phi_\gamma$, are discrete time signals, where $\gamma$ represents the dictionary index. A dictionary with a collection of these waveforms is typically complete or overcomplete. Given such a dictionary, the goal is to obtain a representation image based on the dictionary. We examine the effectiveness of applying Basis Pursuit (BP), Best Orthogonal Basis (BOB), Matching Pursuits (MP), and the Method of Frames (MOF) methods for the compression of digitized radiological images with a wavelet-packet dictionary. The performance of these algorithms is studied for medical images with and without additive noise.
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1 INTRODUCTION

We will discuss the advantages and disadvantages of using four methods of decomposition for image compression and restoration. The methods are Method of Frames (MOF) [1], Best Orthogonal Basis (BOB) [3], Matching Pursuit (MP) [4], and Basis Pursuit (BP) [2]. What these methods have in common is a requirement to use waveforms from a "dictionary" to represent an image. A dictionary, \( \Phi \), is simply a collection of parameterized waveforms, \( \phi_\gamma \), used as a basis for analysis. The parameter \( \gamma \) is dependent upon the dictionary type, e.g., if using a frequency dictionary, then \( \gamma \) is the indexing frequency. We are interested in these methods because they offer a flexible mechanism to customize a dictionary with known waveforms. This would allow higher compression of images using a customized dictionary.

2 METHOD OF FRAMES

Given a discrete dictionary of \( p \) waveforms (each of length \( n \)) that are collected as columns of an \( n \times p \) matrix, \( \Phi \), the decomposition problem is:

\[
\Phi \alpha = f. \tag{1}
\]

The Method of Frames (MOF) uses either a wavelet packet or cosine packet dictionary to pick out among all solutions of equation (1), the solution whose coefficients have the minimum \( L^2 \) norm:

\[
\min\|\alpha\|_2 \quad \text{subject to } \Phi \alpha = f \tag{2}
\]

The MOF solution is obtained by the use of a conjugate gradient method [5] to solve the equation.

There are two key limitations with the Method of Frames (MOF). The first is that MOF is not sparsity-preserving. MOF tends to use all the basis functions nonorthogonal to the signal yielding a very non-sparse representation. If the signal can be represented by a minimal set of the dictionary, then the coefficients found by MOF are likely to be more than this minimal set. The second limitation is that the MOF is resolution-limited. Specifically, no object can be reconstructed with features sharper than those allowed by the analysis and synthesis operators. This has been shown in [2].

3 BEST ORTHOGONAL BASIS

The Best Orthogonal Basis (BOB) method originated by Coifman and Wickerhauser [3,6] seeks to find a best
basis out of an orthogonal set of vectors relative to a
given signal. Thus, overall information cost is optimized.
This method uses a library of orthogonal waveforms that
has a natural dyadic tree structure. Utilizing this type of
structured dictionary makes it easy to construct orthogonal
bases by an O(N log N) search algorithm.

Given a library as a tree structure, the best basis of a
signal is found by traversing the tree and selecting nodes
that correspond to a minimization of the entropy function.
The union of these nodes correspond to the best basis [3].
Shannon’s entropy function is used as the selection
criteria

4 MATCHING PURSUIT

Mallat et. al. [4] has introduced an algorithm that can
provide a decomposition of signals that vary widely in
both time and frequency. It decomposes any signal into a
linear expansion of waveforms that are selected from a list
or dictionary of functions. It chooses a waveform that best
matches the signal structure of the signal at each iteration.
The remaining portion that is unmatched reprocessed at
the next iteration and matched to another signal in the
dictionary. This process continues until a specified error
tolerance is reached.

This algorithm can be expressed as a simple
decomposition by inner product of dictionary elements on
successive residuals.

\[ f = \sum_{n=0}^{n-1} \langle R^n, \phi_n \rangle \phi_n + R^n \]  
(3)

\[ R^n \] is the residue vector after approximating \( f \) at the \( n \)th
iteration. In this algorithm, one begins by computing the
inner products in a dictionary. The elements of the
dictionary are chosen in a way such that

\[ \langle R^n, \phi \rangle = \max_{\alpha} \langle R^n, \phi \rangle \]  
(4)
i.e. find the \( \phi \) that produces the maximum inner
product.

The Matching Pursuit algorithm is greedy. This
means that it must compute all the inner products within
the dictionary to compute its solution. As a result, this
method will take longer to compute in overcomplete
dictionaries because it must first make a calculation for an
atom that would be the best fit on the data. After this
initial guess, the residue function could turn out to be
more complex and the MP algorithm continues in a
fashion to correct the errors from initial guess. This will
result in sub-optimal fitting of the other terms in the
decomposition. It will however do well with orthogonal
dictionaries

5 BASIS PURSUIT

Basis Pursuit (BP) determines a signal representation
such that the coefficients selected have a minimal L^1 norm
[2]. BP differs from the Method of Frames only by the L^2
norm being replaced with the L^1 norm; however, this
changes the form of the solution considerably. In BP, one
solves the problem:

\[ \min \| \alpha \|_1, \ \text{subject to} \ \Phi \alpha = f \]  
(5)

where \( \Phi \) is an \( n \times p \) matrix of waveforms where \( p > n \)
(overcomplete dictionary) and \( \alpha \) is the vector of
coefficients. The MOF requires the solution of a quadratic
optimization problem, and so the minimization is found
in the first derivative where the minimum can be easily
found. In contrast, Basis Pursuit requires the solution of a
convex optimization problem with inequality constraints.
Here it is necessary to use a conjugate gradient method to
find the solution.

Because of the non-differentiability of the L^1 norm,
BP leads to decompositions that can have very different
properties from the Method of Frames. BP
decompositions can be much sparser. Because Basis
Pursuit always delivers a decomposition in an optimal
basis and not necessarily an orthogonal basis, it seems
better than the Best Orthogonal Basis method in resolving
nonorthogonal structures; however the cost to achieve this
is at the expense of greater computational complexity.

6 EXPERIMENTS

The ability of these methods for preserving the
resolution in the reconstructed images with the wavelet
packet dictionary for and MRI image, X-ray, and a
photograph are observed. Initial results show MP with a
compression ratio of 100:1 while the other methods show
ratios from 16:1 to 30:1. From the figures, the
reconstruction from MOF, BOB, and BP looks good to
the naked eye. MP did not do well in these examples. This
also shows that BP does not offer much of an
improvement over MOF even with the added algorithmic
complexity. MP does not perform well to reconstruct
images as the other methods, but does yield superior
compression ratios. The Peak-Signal-to-Noise Ratio
(PSNR) is used to give a qualitative analysis of the
images and their reconstruction [7]. The PSNR is given
by:

\[ PSNR = 10 \log_{10} \left( \frac{255^2}{D} \right) \]  
(6)

\( D \) is the Mean Square Error (MSE), \( E \{ (x - y)^2 \} \).
Investigation into applying dictionary methods to the problem of image compression has produced promising results. The characteristics of wavelets which include "compact support", overcomes some of the limitations of image compression seen in traditional approaches. In contrast, traditional methods such as Fourier based dictionaries provide an effective means for representing signals that are smooth in nature or do not contain abrupt changes or variations; however, these types of dictionaries are not sufficient for representing a signal that may have many irregularities, possess unique features, or exhibits transient behavior [4]. As a result, wavelet dictionaries have been shown to perform well as or better than standard approaches. Table 1 and Chart 1 show the Peak Signal-to-Noise Ratio of the images in tabular and graphical form respectively for each method. The methods showing the best results, Basis pursuit and Best Orthogonal Basis are very close. Since the complexity of MOF is $O(n \log(n))$, BP is $O(n \sqrt{n \log(n)})$, MP is quasi $O(n \log(n))$, and BOB is $O(n \log(n))$, the complexity may become a factor in the selection of the best method.

<table>
<thead>
<tr>
<th>Method of Frames</th>
<th>Basis Pursuit</th>
<th>Matching Pursuit</th>
<th>Best Orthogonal Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image A - mammogram</td>
<td>25.3324</td>
<td>49.8391</td>
<td>24.7532</td>
</tr>
<tr>
<td>Image B - chest x-ray</td>
<td>24.4679</td>
<td>49.4032</td>
<td>18.2503</td>
</tr>
<tr>
<td>Image C - neck x-ray</td>
<td>25.0605</td>
<td>49.3017</td>
<td>24.0115</td>
</tr>
<tr>
<td>Image D - nonmedical image</td>
<td>24.0712</td>
<td>49.1011</td>
<td>22.4513</td>
</tr>
</tbody>
</table>

Chart 1 - Peak Signal-to-Noise Ratios of four methods on four different images.

Table 1 - Data from Chart 1
8 REFERENCES


More information can be found at

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