
**A preliminary model study of the large-scale seasonal cycle in bottom pressure over the global ocean**

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**Abstract**

Output from the primitive equation model of Semtner and Chervin is used to examine the seasonal cycle in bottom pressure ($p_b$) over the global ocean. Effects of the volume-conserving formulation of the model on the calculation of $p_b$ are considered. The estimated seasonal, large-scale $p_b$ signals have amplitudes ranging from less than 1 cm over most of the deep ocean to several centimeters over shallow, boundary regions. Variability generally increases toward the western sides of the basins, and is also larger in some Southern Ocean regions. An oscillation between subtropical and higher latitudes in the North Pacific is clear. Comparison with barotropic simulations indicates that, on basin scales, seasonal $p_b$ variability is related to barotropic dynamics and the seasonal cycle in Ekman pumping, and results from a small, net residual in mass divergence from the balance between Ekman and Sverdrup flows.
1. Introduction

Sea level pressure has played a historical and essential role in atmospheric science, from the study of storm systems and numerical weather prediction to the characterization of climate events such as the El Niño/Southern Oscillation and the North Atlantic Oscillation. Much of this prominence has been fostered by the wide coverage and availability of barometric pressure observations. Drawing a parallel with the equivalent variable for the oceans, bottom pressure ($P_b$), reveals a drastically different situation. There have been some focused programs that included pressure measurements at the deep sea floor, like MODE [Brown et al., 1975] and more recently BEMPX [Luther et al., 1990], and a few deep observations in the Southern Ocean, mainly connected with studies of the Antarctic Circumpolar Current [e.g., Hughes and Smithson, 1996; Woodworth et al., 1996]. Most other available observations are from relatively shallow depths. Overall, though, the data coverage is insufficient to get even a crude estimate of the seasonal cycle over the large scale.

In their work on the seasonal variability of the ocean, Gill and Niiler [1973; hereafter referred to as GN] assumed an ocean governed by geostrophic and Sverdrup dynamics in the interior, and Ekman dynamics near the surface, to derive a relatively simple equation for $P_b$ of the form

$$\frac{\partial (H \csc \phi, P_b)}{\partial (\phi, \lambda)} = \csc \phi \cot \phi \frac{\partial P}{\partial \lambda} - 2 \Omega^2 \rho_o \cos \Phi \Phi_E,$$

where $H$ is the ocean depth, $\phi$ and $\lambda$ are latitude and longitude, $\alpha$ and $\Omega$ are the Earth's radius and rotation rate, $\rho_o$ is a mean ocean density, $\Phi_E$ is the Ekman pumping velocity, and $P$ is the potential energy of the water column per unit area, defined as

$$P = \int_{-H}^0 g \rho dz,$$

with $g$ and $\rho$ being the acceleration of gravity and ocean density, respectively. Variables in (1-2) represent deviations from a time mean. The $P_b$ equation should be valid for seasonal and long spatial scales (>1000 km or longer), away from western boundary regions. The two terms on the right represent, respectively, baroclinic processes associated with density changes over the water column, and forcing by the wind stress curl. For given $P$ and $\Phi_E$ fields, integration along $f/H$ contours ($f = 2\Omega \sin \phi$) from the eastern boundary yields $P_b$; the method fails in regions of closed $f/H$ contours. GN found that, over the tropics, the potential energy term is important and baroclinic effects play more of a role, but poleward of $30^\circ$ latitude, the $\Phi_E$ term dominates. Thus, the barotropic response to Ekman pumping at mid and high latitudes should play a leading role in the dynamics of seasonal fluctuations in $P_b$. Their theory remains essentially untested to date due to lack of appropriate data.

The ability to characterize fluctuations in $P_b$ and understand the underlying dynamics has become increasingly important in recent years. The advent of global altimetry has highlighted the complementary nature of sea level ($\zeta$) and $P_b$ observations made clear by GN. Knowing both $\zeta$ and $P_b$ could lead to more accurate estimates of density (or conversely steric height) fluctuations over the water column, and e.g. the possibility for differentiating between eustatic and steric contributions to global mean sea level trends. In some cases, improved estimates of surface heat fluxes would be possible [e.g., Wang and Koblinsky, 1997]. It has also become clear that the interaction of $P_b$ signals with topography leads to important transfers of momentum between the ocean and solid Earth, on both short and long spatial scales [e.g. Holloway, 1992; Ponte and Rosen, 1994]. Knowledge of $P_b$ is of equal importance for many other current geophysical problems, be it the determination of loading effects on the solid Earth [van Dam and Herring, 1994] or the interpretation of satellite geodesy measurements in terms of the Earth’s variable gravity field [Dong et al., 1996].

Given its broad geophysical relevance, there is a clear need for better understanding the variability in $P_b$. The subject gains increased significance in light of the planned launching, in a few years, of the Gravity Recovery and Climate Experiment (GRACE) satellite mission, dedicated to the study of the Earth's gravity field. Knowledge of $P_b$ field may be improved through indirect estimation using simple theories such as that of GN, or more complex modeling efforts, but such studies have been rare. In this paper, we attempt a characterization of the large-scale, seasonal cycle in $P_b$, as simulated by the model of Semtner and Chervin [1992]. Some subtle issues in diagnosing $P_b$ from moored sea level and density fields are discussed first. The seasonal cycle in $P_b$ is then analyzed and the underlying dynamics are examined, in comparison with simulations from a simpler, barotropic model, and in relation to wind stress forcing.
2. Calculating Bottom Pressure

In most primitive equation ocean models, \( p_b \) is not carried as a prognostic variable and must instead be diagnosed from the sea level and density fields. From integration of the hydrostatic equation over the whole water column, \( p_b \) is given by

\[
p_b = g \int_{-H}^{0} \rho dz + p_a \cong g \rho_0 \zeta + g \int_{-H}^{0} \rho dz + p_a \tag{3}
\]

where \( p_a \) is the surface atmospheric pressure. Given the general tendency for an inverted barometer (IB) sea level response to \( p_a \) [Ponte, 1993; Gaspar and Ponte, 1997], we assume without loss of generality that \( \zeta = \zeta^b + \zeta' \), i.e., IB term plus dynamic signals \( \zeta' \) related to pressure, wind, or any other forcing, where as usual

\[
\zeta^b = \frac{1}{g \rho_0} (\overline{p_a} - p_a) \tag{4}
\]

and \( \overline{p_a} \) is the averaged pressure over the global oceans (overbar will denote spatially averaged quantities throughout the paper). Then (3) reduces to

\[
p_b = g \rho_0 \zeta' + g \int_{-H}^{0} \rho dz + \overline{p_a}. \tag{5}
\]

For later use, it is convenient to normalize (5) by \( g \rho_0 \) to obtain

\[
\frac{p_b}{g \rho_0} = \zeta' - \zeta^b + \overline{\zeta_a}. \tag{6}
\]

All terms have units of length, with \( \zeta_a \) being the steric height, and \( \overline{\zeta_a} \) being the signal in centimeters of water equivalent to \( \overline{p_a} \). Throughout the paper, bottom pressure will refer to the normalized pressure in (6) in centimeters of water.

The diagnoses of \( p_b \) from (3) or (6) must be consistent with the model dynamics and thermodynamics, and the treatment of the boundary forcing. In this regard, two issues need to be taken into account. First, under the IB assumption, \( p_a \) forcing is neglected in most modeling efforts. Thus, for all purposes, \( p_a \) is set to zero and its effects on \( p_b \), represented by \( \overline{\zeta_a} \), are not considered, as they are not important dynamically. For applications requiring "true" \( p_b \) (as measured by a pressure gauge), estimates of \( \overline{\zeta_a} \) would have to be obtained from available atmospheric data.

A second, more important issue in calculating \( p_b \) has to do with the fact that most current ocean models use a continuity equation that implies volume conservation, i.e., \( \zeta^b = 0 \) [Greatbatch, 1994]. However, in reality ocean volume (or for that matter ocean mass) is not strictly conserved. One way for volume changes to happen is through the expansion or contraction of water parcels associated with any density changes. In volume-conserving models, changes in density are not accompanied by corresponding volume changes. Thus, for example, a uniform warming of the global ocean, with no changes in mass involved, would still show up as a signal in \( p_b \), because the changes in \( \zeta_a \) are not compensated by changes in \( \zeta' \).

To correct for the missing physics, one needs to consider how the ocean reacts to the unmodeled volume changes. A simplified approach to the problem is discussed in detail by Greatbatch [1994] and Meller and Ezer [1995]. Volume changes act essentially as a barotropic load on the ocean. Under the assumption of an equilibrium adjustment and decoupling between barotropic and baroclinic processes, the unmodeled volume changes translate into a spatially homogeneous sea level signal given by

\[
\delta \overline{\zeta_a} = -\frac{1}{A} \int_V \frac{\delta p}{\rho_0} dV \tag{7}
\]

\( (A \text{ and } V \text{ are the surface area and volume of the global ocean}) \), and involving the averaged density or steric height changes over the global ocean. Here, we shall take this correction to be valid for the seasonal time scale. At these periods, the equilibrium hypothesis likely holds [see Ponte, 1993]. We note, however, that the assumption of barotropic and baroclinic decoupling needs more careful future investigation.

Another way for effecting oceanic volume changes is through freshwater fluxes at the atmosphere and land interfaces. These effects are, however, not represented in volume-conserving models. In fact, the freshwater flux condition is usually implemented as a restoring to observed surface salinity fields, involving a virtual salt flux instead of a real freshwater flux (see Huang [1993] for a detailed treatment of these issues). Again under the same assumptions used to arrive at (7), the missing effects of real freshwater fluxes on sea level can be approximated by

\[
\overline{\zeta_f} = -\frac{1}{A} \int_A \int Q dt dA, \tag{8}
\]

which involves the time-integrated, spatial average of the freshwater flux \( Q \) (evaporation minus precipitation and runoff). Any net \( Q \) is basically spread uniformly over the global oceans, with corresponding effects on \( p_b \).

The sea level correction factor \( \delta \overline{\zeta_a} \), defined by (7), is easily calculated from respective model density fields. On the other hand, calculation of \( \overline{\zeta_f} \) and
\( \zeta \) requires averaged \( Q \) and \( p_a \) data, either from climatologies or atmospheric operational products, that are likely to have large uncertainties. Their treatment will not be attempted here, but there are a couple of points worth discussing for future reference. First, one should note that \( \zeta_q \) and \( \zeta_a \) are likely related, to the extent that changes in the mean atmospheric pressure over the oceans may involve variability in the atmospheric water content, through evaporation and precipitation processes, besides mass shifts from land- to ocean-covered regions and vice versa. Therefore, \( \zeta_q \) and \( \zeta_a \) effects should be considered together and estimated in a consistent manner. In particular, \( \zeta_a \) should be calculated based on \( p_a \) values that fully represent effects of atmospheric water content, which is not likely the case with at least some of the current operational products [van den Dool and Saha, 1993]. Second, effects of \( \zeta_q \) and \( \zeta_a \) on \( p_a \) may not be negligible. Operationally derived \( p_a \) values point to a seasonal cycle in \( \zeta_a \) with peak-to-peak amplitudes of 0.5–1 cm [Ponte, 1993; Wunsch and Stammer, 1997]; similar seasonal amplitudes for \( \zeta_a \) are obtained from available \( Q \) climatologies. These values may seem small, but as discussed next, are of the same order of magnitude of dynamically relevant seasonal signals in \( p_a \).

3. Model Estimates of the Seasonal Cycle in \( p_a \)

In the absence of appropriate data sets to characterize in detail the large-scale, seasonal cycle in \( p_a \), we examine such fields as determined from output of the Parallel Ocean Climate Model (POCM) of Semtner and Chervin [1992]. We use monthly averaged output from the so-called POCM.4B run, which is described in detail and analyzed in comparison with observations by Stammer et al. [1996]. Forcing for this run consisted of 3-day averaged wind stresses calculated from European Centre for Medium-Range Weather Forecasts (ECMWF) twice-daily wind fields, ECMWF-based monthly mean heat fluxes from Barnier et al. [1995], and restoring conditions to Levitus et al. [1994] for the surface temperature and salinity fields. The POCM output thus represents signals in \( \zeta' \) and \( \zeta_a \) related to both mechanical wind stress forcing and thermodynamical forcing, but not pressure forcing. Given the model’s volume-conserving formulation, all the points discussed in the previous section are relevant in interpreting the calculated \( p_a \) fields.

The model was run initially for eight years (1987–1994). Monthly averaged temperature, salinity, and sea level fields for this period were used in (6) to calculate time series of \( p_a \) at each model grid point (spacing of 0.4° in longitude and 0.4° cos \( \phi \) in latitude). Given the focus on large scale patterns, averages over 4°x4° boxes were constructed and used in the analyses. In addition, to minimize any effects of transient adjustment of the model fields to the new boundary forcing over the initial period, the analysis is restricted to the last six years of output (1989–1994).

Before analyzing the spatial character of the seasonal variability, the influence of volume effects associated with \( \delta \zeta_a \) are examined. Figure 1 shows the time series of \( \delta \zeta_a \), calculated by integrating the density field over the volume of the model as in (7), with \( \rho_o = 1.03 \text{ g/cm}^3 \) and \( A = 3.47 \times 10^{18} \text{ cm}^2 \), the surface area of the model ocean. There is a long term trend of -2 cm over the six year period (approximately 3 mm/year), which is fairly linear over the first four years and tends to flatten out in the last two years. Models are known to have a tendency to drift with time, due to slow diffusive adjustment, incompatibilities in boundary forcing and model physics, etc. (see Stammer et al. [1996] for a brief discussion of drifts in POCM). The implicit decrease in global mean sea level in Fig. 1 (model ocean is getting denser) seems too large to be realistic. Examination of these trends should be an important consideration for studies of interannual and lower frequency variability in \( p_a \), but need not concern us in the study of the seasonal cycle.

Superposed on the long term trend in Fig. 1, there is a clear seasonal cycle in \( \delta \zeta_a \) with peak-to-peak amplitudes of approximately 0.3–0.4 cm, and minimum occurring around August. This signal is a measure of hemispheric asymmetries in the ocean-land distribution and the seasonal cycle in steric height. Melor and Ezer [1995] have calculated \( \delta \zeta_a \) from available hydrographic data and estimate a seasonal cycle with a peak-to-peak amplitude of approximately 0.8–1 cm, and minimum in July (see their Fig. 11). The tendency of POCM to underestimate the seasonal cycle in steric height, apparently due to lack of proper heat uptake by the mixed layer, results in a weaker seasonal signal in \( \delta \zeta_a \), and the phase shift by approximately one month with respect to the observations may be partly due to phase errors of the forcing ECMWF heat fluxes [Stammer et al., 1996]. The difference \( \delta \zeta_a \) may of course be also due to the climatological, sparse hydrographic records used in Melor and Ezer’s calculation, as opposed to the 6-year, dense sampling of the model.
The \( p_b \) fields analyzed here include the model calculated \( \delta \zeta_p \) term shown in Fig. 1. After removing means and trends from each \( p_b \) time series, values were averaged for each season (December-February, March-May, June-August, and September-November). The corresponding mean seasonal variability in \( p_b \) over the 6-year period of model output is shown in Fig. 2. Most of the deep ocean, including tropical latitudes, exhibit \( p_b \) fluctuations on the order of 1 cm or smaller. (Note that these amplitudes are of the same order of magnitude of the seasonal cycle in \( \zeta_p \); the effects of \( \delta \zeta_p \) should thus be taken into account in the study of \( p_b \), in contrast with their less important role for the variability in \( \zeta_p \).) Larger variability is seen over the Southern Ocean, the western North Pacific, and near boundary regions, particularly over shallow depths (e.g., North Sea, Bering Sea, Sea of Okhotsk, South China Sea), where amplitudes of a few centimeters are attained.

The large, basin scale character of seasonal \( p_b \) signals is striking. The Atlantic, Indian, and to some extent the Southern Ocean tend to show single sign patterns. In northern winter, most of the Atlantic and Southern Oceans show a positive mass anomaly, with the Pacific and Indian Oceans having a negative anomaly. The pattern is reversed in northern summer. Besides the apparent seasonal interbasin mass exchange, there is a clear oscillation in the North Pacific, with a node at approximately 30°N, which suggests seasonal strengthening of the depth-averaged gyre circulation in northern winter and spring, and mass exchanges between subtropical and higher latitudes.

To assess the seasonal variability in Fig. 2 as a function of frequency, Fig. 3 shows the power spectrum of \( p_b \), calculated by averaging the periodogram over all grid boxes in the model. Each time series was detrended and tapered using a cosine bell function over 10% of the points on each end, prior to Fourier transformation. Detrending does decrease the power slightly at the first two harmonics (6 and 3 years), but has no effect on higher harmonics. The \( p_b \) spectrum shows a large dominant peak at the annual period, and a weaker peak at the semiannual period. Besides these peaks, the spectrum exhibits a dependence on frequency close to \( \omega^{-0.5} \). The important contribution of the annual time scale to the seasonal variability in \( p_b \), seen in Fig. 2, is clear.

Given the focus of many studies on the annual period, we show in Plate 1 the amplitude and phase of the annual cycle in \( p_b \), calculated by simply Fourier transforming each time series after detrending. Most of the features discussed in relation to Fig. 2 are present in Plate 1. The annual cycle has amplitudes smaller than 1 cm over most of the global oceans, with higher values (1-3 cm) over the Southern Ocean and the western North Pacific, and over most of the shallow coastal regions in the model. The phase exhibits, for the most part, a large scale pattern: the lack of a distinct phase opposition between the Northern and Southern Hemispheres, observed in the case of \( \zeta_p \), is almost striking. Maximum \( p_b \) values in the Atlantic and Pacific Oceans occur mostly in the northern winter (Dec–Feb) and summer (Jun–Aug) seasons, respectively. Thus, the Atlantic and the Pacific appear to fluctuate approximately out of phase, as noted by Stammer et al. [1996], based on the averaged \( p_b \) field over the Atlantic and Pacific Oceans (see their Fig. 15).

Before attempting to interpret the seasonal \( p_b \) signals, it is also useful to examine the relation between \( p_b \) and \( \zeta_p \) by calculating

\[
A = \frac{(\rho_0)^{-1}F(p_b)}{F(\zeta_p)},
\]

which is essentially the admittance between \( p_b \) and \( \zeta_p \), with \( F \) denoting Fourier transform. In physical terms, \( A \) quantifies how much of a bottom pressure signal results from a sea level load at the surface. From (6), one expects \(|A| < 1 \) if \( \zeta_p \) and \( \zeta_p \) are correlated. Likewise, phases near zero indicate a simple in-phase relationship among \( p_b \), \( \zeta_p \), and \( \zeta_p \). The amplitude and phase of \( A \) for the annual period are shown in Plate 2. Variability in \( p_b \) is weak relative to \( \zeta_p \), with \(|A| < 0.4 \) over most of the oceans, away from shallow regions and the Southern Ocean. The small \( p_b \) amplitudes are expected as a result of the well-known correlation between \( \zeta_p \) and \( \zeta_p \). There is, however, extensive regions, particularly in the Southern Ocean where \(|A| > 0.4 \). Over most shallow coastal regions, strong correspondence between \( p_b \) and \( \zeta_p \) is indicated by amplitudes near unity and phases near zero. These results most likely indicate the essentially barotropic nature of variability in these regions.

A tendency for nearly in-phase behavior is clear in the Southern Ocean and the North Pacific (north of ~30°N), but phases span the whole range. In particular, the approximately out-of-phase behavior over equatorial Pacific regions is worth noting, as indicative of the presence of first (or more precisely odd) baroclinic mode variability [e.g., Brown et al., 1975]. The regional dependence of the annual admittance in
Plate 2 suggests a complex relation between $p_b$ and $\zeta'$ and the difficulty in extracting information about $p_b$ from the sea level fields alone.

4. Dynamics and Forcing of Seasonal Variability in $p_b$

As discussed in the Introduction, the importance of barotropic dynamics and wind stress curl forcing for large-scale, seasonal variability in $p_b$ at mid and high latitudes was pointed out by GN. Model results in Fig. 2 are qualitatively similar to the North Atlantic and North Pacific estimates of GN (see their Fig. 3), but more general evidence for the importance of barotropic dynamics to the seasonal $p_b$ variability can be obtained by comparing the POCM results to those from a recent run of the near-global, shallow-water model of Ponte [1993]. For details about model formulation, resolution, friction, etc., the reader should consult Ponte [1993] and Gaspar and Ponte [1997]. The available model output is from a run originally performed for other purposes and forced by four-times daily reanalysis wind stress and pressure fields from the National Centers for Environmental Prediction (NCEP) for the years 1993–1995, with a spinup period taken to be the last two months in 1992. Thus, $\zeta'$ represents the effects of both pressure and wind stress, but the latter dominates at the seasonal time scale [Ponte, 1994]. For the barotropic model, there is of course no role for thermodynamics, $\zeta_p$ is assumed constant, and calculation of $p_b$ is equivalent to that of $\zeta'$.

From the four-times daily model output, seasonal means of $\zeta'$ for the months Dec-Feb, Mar-May, Jun-Aug, and Sept-Nov were calculated. The resulting seasonal variability, based on years 1993–94, is shown in Fig. 4 for comparison with POCM results derived for the same 1993–94 period and shown in Fig. 5. The comparison is qualitative, as different operational products were used for the forcing fields in the two models, in addition to differences in resolution, topography, coastal geometry, etc. Nevertheless, besides a noticeable tendency for stronger signals in the barotropic model, the similarities between the POCM and barotropic results are clear. Correspondence between patterns of positive and negative $p_b$ anomalies, and regions of larger and smaller amplitudes, is qualitatively good, especially for the winter and summer seasons. The western intensified, North Pacific oscillation between subtropical and higher latitudes is particularly well reproduced in the barotropic results.

Some of the characteristics of the $p_b$ seasonal variability in Figs. 4 and 5 can be traced to the nature of the wind stress curl and the corresponding Ekman pumping fields in (1). Figure 6 shows the seasonal variability of the forcing $w_E$ term in (1), calculated from monthly averages of the NCEP wind stress fields for the years 1993-94 that were used to force the barotropic model. Derivatives of the wind stress involved in the calculation were evaluated at every grid point using centered differences; the calculation was confined to latitudes poleward of 5°, as Ekman theory formally breaks down at the equator. Without attempting a full integration along $f/H$ contours as done by GN, we note that the seasonal forcing signals are generally stronger and more zonally homogeneous in the North Pacific than in the North Atlantic. These two factors provide for the stronger $p_b$ variability and the clear dipole pattern in the North Pacific, as compared to the North Atlantic (the longer extent of the Pacific may also lead to stronger variability toward the western boundary). More generally, there is qualitative correspondence between the positive and negative large scale patterns of the $w_E$ and $p_b$ fields, respectively, and vice versa, which is consistent with (1). The correspondence is particularly good for seasons and basins for which the $w_E$ fields are more spatially homogeneous.

The relation between $p_b$ and $w_E$ is also apparent when examining their frequency spectra. It can be easily shown from (1) that, if the spectrum of $w_E$ is separable in frequency and wavenumber, i.e.,

$$\phi(K, \omega) \sim \phi_1(K)\phi_2(\omega)$$

then the frequency dependency of the $p_b$ spectrum is simply given by $\phi_2(\omega)$. Given that the spectrum of $w_E$ is essentially that of the wind curl, the approximation in (10) is a reasonable one at low frequencies [e.g., Willebrand et al., 1980]. We have calculated the periodogram of the $w_E$ term in (1) at every grid point (again excluding latitudes within 5° of the equator) and averaged them together to obtain the frequency spectrum shown in Fig. 7. The frequency dependency of that spectrum is remarkably similar to the spectrum of $p_b$ shown in Fig. 3, both in its general background shape (close to $\omega^{-0.5}$) and in the relative height of the peaks at annual and semiannual periods.

Seasonal fluctuations in $p_b$ field, highlighted in Fig. 2 and Plate 1, represent large-scale divergence or convergence of mass brought about by the seasonally varying flow field. Horizontal oceanic motions are largely nondivergent, however, and only a small
divergence is involved in effecting \( p_b \) changes. For example, to accumulate 1 cm of water over a \( 10^4 \times 10^4 \) kilometer area in six months, one needs only a net transport of \( \sim 6 \times 10^4 \) m\(^3\)s\(^{-1}\), much smaller than typical horizontal transports in the ocean. To explore the dynamics behind the divergent mass fluxes leading to the horizontal transports in the ocean. The derivation is given in Appendix A using Cartesian coordinates and neglecting nonlinear and dissipation effects for simplicity. The equation for \( p_b \) is

\[
\frac{p_{bt}}{g} + J \left( p_b, \frac{H}{f} \right) - \frac{f_x}{f^2} P_x = \left( \frac{\tau_y}{f} \right)_z - \left( \frac{\tau_x}{f} \right)_y - \rho_o Q + O \left( \frac{\omega}{f} \right) \text{ terms.} \tag{11}
\]

As pointed out in Appendix A, (11) associates local mass divergence, represented by \( p_b \), with four different processes: geostrophic flow divergence due to variable \( f \) and \( H \), which includes the baroclinic term \( P_x \); Ekman flow divergence forced by the wind stress; mass fluxes at the surface due to \( Q \); and geostrophic effects of \( O(\omega/f) \) relative to the geostrophic term.

Introducing a spatial scale of motion \( L \), \( p_b \) term is \( O(\omega/f)(L/L_R)^2 \) relative to geostrophic term, where \( L_R = \sqrt{gH/f} \) is the external Rossby radius (\( \sim 2000 \) km). Seasonal \( Q \) forcing is generally negligible compared to the Ekman terms. For seasonal time scale and away from the equator (\( \omega/f \ll 1 \)), and for typical \( L \sim 1000 \) km, one has the traditional quasi-steady Sverdrup balance, as in (1), with wind-driven Ekman divergence being for the most part balanced by geostrophic flow divergence. The net residual divergence is associated with small geostrophic flow correction to the Sverdrup solution, at scales larger than \( L_R \). Basin-scale signals discussed in Fig. 2 are likely dominated by these processes. At scales shorter than \( L_R \), divergence from \( O(\omega/f) \) ageostrophic effects should also become important, as well as dissipation and nonlinear effects not treated in (11).

5. Final remarks

It is difficult to assess the realism of the model \( p_b \) fields in Fig. 2 in the absence of global pressure records. From the available point measurements over the deep ocean, only a few are longer than one year [Woodworth et al., 1997; Mofjeld et al., 1996]. The observations just to the east of the Drake Passage, analyzed by Woodworth et al. [1996], show fluctuations over a variety of time scales; amplitudes of the seasonal cycle are not provided, but visual inspection of their Figs. 5a, 6, 7, 8a is not inconsistent with amplitudes at the centimeter level. The records of Mofjeld et al. [1996], taken near 46°N, 130°W in the North Pacific, show substantial contamination from instrument drift after 30-day running means are applied to them (Mofjeld, personal communication, 1997), but all indications point to seasonal amplitudes with order of magnitude similar to the records from Woodworth et al. Not much can be said quantitatively about the agreement or disagreement between model \( p_b \) and these records, except to note that observed values seem larger than (but of the same order of magnitude of) the POCM results. Point measurements may of course relate to local processes not relevant to large-scale variability examined in the model. Direct, more quantitative evaluation of model estimates of \( p_b \) against observations must await the launching of the gravity missions currently being planned. Conversely, the seasonal fluctuations in the mass field over the oceans estimated here provide a crude measure of the oceanic signals that will need to be resolved by the planned gravity missions.

Besides comparisons with point measurements, there are other more indirect methods by which the model \( p_b \) fields can be evaluated. Given the importance of \( p_b \) variability in the excitation of Earth rotation signals, particularly seasonal polar motion [Ponte et al., 1998], and also its effects on the variable gravity field, as observed by satellite tracking methods [Dong et al., 1996], estimation of these oceanic effects based on the model \( p_b \) fields and comparison with other relevant geophysical observations on Earth orientation and gravity field provide an independent consistency check on the \( p_b \) fields, as shown in the case of a different ocean model by Ponte et al. [1998]. Analysis of the \( p_b \) fields from the POCM model in this broader geophysical context is currently being pursued by this and other investigators.

In all these comparisons, one must keep in mind that signals related to \( \zeta_a \) and \( \zeta_q \) may contribute to the local \( p_b \) measurements, and may also have a significant projection onto the integrated measurements represented by Earth rotation and large-scale gravity parameters. Given the crude estimates of \( \zeta_a \) and \( \zeta_q \) available, and depending on their relative phase, these effects may impact the estimates of the seasonal cycle in regions with \( p_b \) signals at millimeter amplitudes. Ideally, one would like to have pressure forcing and real freshwater fluxes implemented in circulation...
models, which would permit verification of the equilibrium response assumption at seasonal time scales and examination of higher frequency variability where that assumption becomes less useful [Ponte, 1993].

Finally, regarding the inference of surface heat fluxes or heat content in the water column from altimetry sea level records, as attempted recently by Wang and Koblinsky [1997], Chambers et al. [1997], and also White and Tai [1995], Plate 2 provides a crude estimate of the error induced by neglecting contributions to $\zeta$ which are truly related to $p_b$ and not $\zeta_\rho$, for the annual cycle. Depending on the region, estimates may easily be in error by 20 to 40%. However large these values may seem, the sea level-based estimates are still meaningful given the large uncertainties in the surface heat fluxes and the lack of sufficient hydrographic data sampling to estimate heat content. The possibility of improving those estimates by using model-derived $p_b$ fields is, nevertheless, noted.

**Appendix A: Derivation of General Equation for $p_b$**

Starting from the linearized horizontal momentum and continuity equations, vertical integration over the full depth $H$ leads to

\[
\begin{align*}
\frac{\partial u}{\partial t} - f v &= -\frac{1}{\rho_o H} \int_{-H}^{0} \rho p_x dz + \frac{\tau^x}{H \rho_o} \\
\frac{\partial u}{\partial t} + f u &= -\frac{1}{\rho_o H} \int_{-H}^{0} \rho p_y dz + \frac{\tau^y}{H \rho_o} \\
\zeta_t + (H u)_x + (H v)_y &= \zeta_{\rho t} - Q
\end{align*}
\]  
(A1)

Here, $u$ and $v$ are depth-averaged flows, $\zeta$ is sea level, $\zeta_\rho$ is steric height, and $Q$ denotes freshwater flux (evaporation minus precipitation and runoff). We work with Cartesian coordinates and neglect bottom and lateral dissipation effects for simplicity. The ocean is acted upon by surface wind stresses $\tau^{x,y}$.

To write (A1) in terms of $p_b$, one needs to work on the pressure integrals. Moving the derivative outside the integral, and using the fact that $F_p(z) = \frac{p_b - g \rho_0 z}{\rho_0}$, the integrals become

\[
\int_{-H}^{0} p_{x,y} dz = \left( H \rho_b - g \int_{-H}^{0} dz \int_{-H}^{0} \rho_0 dz' \right)_{x,y} - p_b H_{x,y}
\]

The double integral can be integrated by parts and after carrying out the derivatives, we have simply

\[
\int_{-H}^{0} p_{x,y} dz = H \rho_{b,x,y} + P_{x,y}
\]  
(A2)

with $P$ as defined in (2).

With the use of (A2) and the hydrostatic relation

\[
\frac{p_b}{g \rho_0} = \zeta - \zeta_\rho
\]  
(A3)

(effects of $\rho_0$ are neglected here), we can write (A1) as

\[
\begin{align*}
\frac{\partial u}{\partial t} - f v &= -\frac{p_{b,x}}{\rho_0} - \frac{P_x}{H \rho_o} + \frac{\tau^x}{H \rho_o} \\
\frac{\partial u}{\partial t} + f u &= -\frac{p_{b,y}}{\rho_0} - \frac{P_y}{H \rho_o} + \frac{\tau^y}{H \rho_o} \\
\frac{\partial p_b}{\partial t} + (H u)_x + (H v)_y &= -Q
\end{align*}
\]  
(A4)
These equations are very similar to the linear shallow-water equations forced by wind stress and freshwater fluxes, with \( p_b \) replacing \( \zeta \) and extra forcing terms \( P_{z,y} \) associated with baroclinic effects. With time dependence of the form \( e^{i\omega t} \), it is straightforward to solve the momentum equations for \( u, v \) and substitute in the continuity equation to get a very general, but complicated, equation for \( p_b \). For seasonal time scales and away from the equator, i.e., \( \omega \ll f \), and without explicitly treating \( O[\omega/f] \) terms arising from \( u_t, v_t \) terms in (A4), one arrives at a simpler equation:

\[
\frac{p_b}{g} + J\left(p_b, \frac{H}{f}\right) - \frac{f v}{f^2} P_z = \left(\frac{\tau_y}{f}\right)_z - \left(\frac{\tau_x}{f}\right)_y - \rho_0 Q + O\left[\frac{\omega}{f}\right] \quad \text{terms (A5)}
\]

If one partitions the flow field in the usual geostrophic and Ekman components in (A4), it is easy to see that the wind stress terms in (A5) represent the divergence of the Ekman transports, whereas the Jacobian and \( P \) terms represent the divergence of geostrophic transports. Ageostrophic effects due to \( u_t, v_t \) terms in momentum equation are represented in \( O[\omega/f] \) terms. Any imbalance of mass fluxes due to these Ekman, geostrophic, and ageostrophic flows, and to surface mass inputs (\( Q \) term), lead to variability in \( p_b \).

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Figure 1. Time series of $\bar{\zeta}_p$ (cm) for the period January 1989–December 1994 calculated from POCM output as described in the text. Origin of the time series is arbitrarily set to zero.

Figure 2. Anomalies in $p_b$ averaged over the months of December-February, March-May, June-August, and September-November. Contour interval is 0.5 cm; negative values are lightly shaded.

Figure 3. Power spectrum obtained by averaging individual periodograms of $p_b$ (no area weighting) calculated at all model points. No averaging in frequency is applied.

Figure 4. Anomalies in $p_b$ as in Fig. 2, but calculated from the barotropic model based on two years of output (1993–1994). Contour intervals are 1 cm; negative values are lightly shaded. Dark shading represents land points in the model.

Figure 5. Anomalies in $p_b$ as in Fig. 2, but calculated from the POCM model based on only two years (1993–1994). Contour intervals are 1 cm; negative values are lightly shaded.

Figure 6. Seasonal anomalies in the Ekman pumping forcing term in (1), based on wind values for 1993–94. Values within 5° of the equator were not calculated. Contour interval is $10^{-4}$ g s$^{-2}$; negative values are lightly shaded. The land mask is that of the barotropic model in Fig. 4.

Figure 7. Power spectrum of the Ekman pumping term in (1), calculated by averaging periodograms at all points for which data is shown in Fig. 6. No averaging in frequency is applied.

Plate 1. Amplitude (in cm) and phase of the annual cycle in $p_b$. Note the changing contour intervals for the amplitude. Phases are plotted every other point and indicated by the direction of the line segments (origin denoted by the dots); 0° corresponds to eastward direction, and phase increases anticlockwise. Values between $-45^\circ$ and $45^\circ$ correspond approximately to a maximum in northern winter season (Dec–Feb), between $45^\circ$ and $135^\circ$ to a maximum in northern spring, and so on.

Plate 2. Amplitude and phase of the admittance between $p_b$ and $\zeta'$ for the annual harmonic. Phase is plotted as in Plate 1. Line segments pointing eastward (westward) imply in-phase (out-of-phase) relation between $p_b$ and $\zeta'$. 
Months since January 1989

\( \zeta \) (cm)
The graph shows the variation in $g_r^2/s^4$ (cycles/month) over a range of cycles/month from 0.01 to 1.00, with a peak near 0.05 cycles/month. The x-axis represents months, ranging from 36 to 3 months. The y-axis represents $10^{-5}$ to $10^{-7}$ cycles/month.