Mars may have substantially changed its average axial tilt over geologic time due to the waxing and waning of water ice caps. Depending upon Mars' climate and internal structure, the average obliquity could have increased or decreased through climate friction by tens of degrees. A decrease could account for the apparent youthfulness of the polar layered terrain. Alternatively, Mars' average obliquity may have changed until it became "stuck" at its present value of 24.4°.
In previous papers (1), I speculated that ice ages on Mars, driven by the $10^5$ year obliquity oscillations (2), could have caused its average obliquity to increase, perhaps accounting for all its present value of $24.4^\circ$. The process was climate friction (also called obliquity-oblateness feedback in Bills’ (3) felicitous phrase). The idea was that the megaregolith contains vast reservoirs of adsorbed carbon dioxide, which escapes at low obliquity to form giant polar caps, sending Mars into an ice age (in this case dry ice); the CO$_2$ then goes back into the ground again at high obliquity (4-5). The deformation of the solid planet from the weight of the caps, plus the caps themselves, alter Mars’ flattening and thus its precession rate. Because of the effective viscosity of Mars’ mantle, the oscillation in the precession rate is out of phase with the obliquity oscillations, producing a slow secular increase in the average obliquity over geologic time. Climate friction may also operate on Earth (3,6-7), but here water ice caps instead of CO$_2$ come and go with the ice ages.

The present paper concerns climate friction due to water ice on Mars. Like the Earth, Mars has a substantial amount of water which may cycle in and out of polar caps. Thus I examine here possible secular obliquity changes due to large H$_2$O ice caps waxing and waning with the obliquity cycle on the Red Planet; see Fig. 1. In particular, I find that climate friction from H$_2$O caps may have decreased Mars’ tilt, instead of increasing it as found earlier. Hence Mars may have had a larger average axial tilt in the past. With stronger insolation at the poles and weaker ice ages, Mars may not have laid down appreciable layers in the polar regions with each Milankovitch cycle of obliquity. The present lower obliquity with its stronger ice age regime may have produced the observed layered terrain at the poles, thereby accounting for their apparent youthful age of a few hundred million years. In another scenario, Mars’ average tilt may have changed until it became “stuck” at its present value.
For a homogeneous viscous sphere the secular change in obliquity $\Delta \epsilon_{\text{sec}}$ due to climate friction from a simple sinusoidal obliquity oscillation $\Delta \epsilon_0 \sin \omega t$ is (6):

$$\Delta \epsilon_{\text{sec}} = \frac{\alpha M I \Delta \epsilon_0 \cos \epsilon}{2M J_2} \left[ \cos \xi \sin (\delta + \xi) - \sin \delta \right] t$$

where $\epsilon$ is the average obliquity, $M_I$ the mass of the water cap, $t$ is the length of time climate friction has been operating, $\Delta \epsilon_0 = 7.2^\circ$ is the amplitude of the principal obliquity oscillation (1), $M = 6.46 \times 10^{23}$ kg is the mass of Mars, and the usual spherical harmonic coefficient $J_2 = (C - A)/(M R^2) = 1.955 \times 10^{-3}$ is a measure of the flattening of the planet, caused mostly by rotation. $C$ and $A$ are the axial and equatorial moments of inertia respectively, while $R = 3390$ km is Mars' radius. Also in Eq. 1

$$\alpha = (3J_2 G M_s)/\left(2 \lambda \hat{\theta} a^3 \right) = 8.3 \text{ arc sec year}^{-1},$$

where $G = 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ is the universal gravitational constant, $M_s = 2 \times 10^{30}$ kg is the mass of the Sun, $a = 1.56$ AU is the semimajor axis of Mars' orbit, $\hat{\theta} = 7.09 \times 10^{-5}$ s$^{-1}$ is the angular rotation rate of Mars, and $\lambda = C/(M R^2) = 0.365$. Further, $\delta$ is the phase lag of the water cap with respect to the obliquity oscillation; the ice mass oscillates like $\sin (\omega t - \delta)$. Likewise $\xi$ is the phase lag of the solid planet with respect to the loading from the cap, so that the deformation behaves like $\sin (\omega t - \delta - \xi)$, where

$$\tan \xi = \frac{19 \eta |\omega|}{2 \rho g R}.$$
Here \( \eta \) is the effective viscosity of Mars' mantle, \( \rho = 3933 \text{ kg m}^3 \) is the average density of Mars, and \( g = 3.73 \text{ m s}^2 \) is the gravitational acceleration at the surface. In this equation

\[
\omega = -\alpha \cos \varepsilon - \dot{\Omega} \cos I
\]  

(3)

is the angular frequency of the principal obliquity oscillation, where \( I = 3.6^\circ \) is the inclination of Mars' orbit with respect to the invariable plane, and \( \dot{\Omega} = -17.8 \text{ arc sec year}^{-1} \) is the rate of change of the ascending node of the orbit on the invariable plane. Currently the obliquity oscillation has a 126,000 year period.

The Mars Global Surveyor spacecraft has revealed that at the present time the amount of water ice in the residual north polar cap of Mars is about \( 10^{18} \text{ kg} \) (8). The cap may simply be a permanent cap associated with the present average obliquity. On the other hand, given that very high obliquities may sublime the entire cap and the layered terrain as well (9), a similar amount of water may well be available for cycling during the obliquity oscillations. Taking \( M_I = 10^{18} \text{ kg} \) along with the other values given above and plugging them into Eq. 1 yield

\[
\Delta \varepsilon_{\text{sec}} = 521^\circ \left[ \cos \xi \sin (\delta + \xi) - \sin \delta \right]
\]

if climate friction has been operating over \( 4.55 \times 10^9 \text{ years} \), the age of the solar system.

Spada and Alfonsi have criticized the homogeneous model as being too simplistic (10); their model cuts the rate of climate friction about in half. Adopting their more conservative amplitude but retaining the homogeneous viscous part as a hybrid model gives

\[
\Delta \varepsilon_{\text{sec}} \equiv 260^\circ \left[ \cos \xi \sin (\delta + \xi) - \sin \delta \right]
\]

(4)
as the approximate change in axial tilt assuming climate friction operates at a constant rate over the age of the solar system. The $\sin \delta$ term in the square brackets in Eq. 4 gives the change in tilt due to the water ice cap itself. Water ice cycling at the poles by itself would cause its obliquity to decrease as time goes on. This is just the opposite from the Earth, where the obliquity would increase due to the cap alone as time marches forward. The reason for the opposite signs between the two planets is that $\omega$ is positive for Mars and negative for Earth in Eq. 3. The $\cos \xi \sin (\delta + \xi)$ term comes from the deformation of the solid planet; the weight of the ice at the poles squeezes out a larger equatorial bulge, increasing the oblateness. This is just the opposite of what the ice does; hence the deformation acts to increase Mars' obliquity as time progresses. On the Earth the deformation would cause a decrease.

Eq. 4 is plotted as a function of $\delta$ and $\xi$ in Fig. 2. It is clear from the figure that the secular obliquity change $\Delta e_{sec}$ is potentially enormous for the model adopted here, changing by perhaps tens of degrees over geologic time. Moreover, climate friction can either increase or decrease the obliquity, with the direction of obliquity-oblateness feedback depending upon whether the ice cap or the deformation dominates the process. Which one dominates depends upon the relative phase lags $\delta$ and $\xi$.

Previously I adopted $\delta \equiv 0^\circ$ for the proposed giant CO$_2$ caps, because this is what the models indicated (1,5), and because I argued that Mars was a volatile-poor planet where the all the available volatiles would quickly cycle into and out of the poles almost in phase with the obliquity oscillations (6). A cap in phase with the obliquity cycle produces no secular obliquity change, and the shift in the average tilt comes from the solid planet
deformation, causing the obliquity to increase. But with $10^{18}$ kg of H$_2$O at Mars’ north pole, the present water cap is almost as huge relative to the mass of Mars as the Laurentide ice sheet was relative to the Earth when that ice sheet was at its peak 20,000 years ago (11). Thus it is not clear that Mars can necessarily be considered water-poor. Further, the phase lag of the Earth’s ice sheets is about $\delta = 90^\circ$ (12). In the absence of geologic data and reliable climate models for Mars, we cannot say what $\delta$ is, but may potentially be as large as 90°. If so, an out-of-phase cap could produce a substantial decrease in obliquity if it is not offset by the associated deformation.

The angle $\xi$, which indicates how much the deformation lags the ice loading, depends on the structure of Mars’ interior. For a homogeneous planet $\xi$ depends on the effective average viscosity $\eta$ of Mars’ mantle, as given in Eq. 2. The value for $\eta$ is unknown. The apparent isostatic compensation of the water cap indicates a range from very low values up to Earth-like values of $10^{21}$ Pa s or $10^{22}$ Pa s, if the cap indeed waxes and wanes on the $10^5$ year obliquity timescale (8). This would give values for $\xi$ between about 0° and roughly 70°. Since these limits on $\delta$ and $\xi$ are not very restrictive, Mars could essentially be anywhere in Fig. 2. This leads to two interesting scenarios.

One is that obliquity-oblateness feedback may explain the relative youthfulness of the polar layered terrain, which is believed to be linked with Martian Milankovitch cycles (13). In this scenario Mars is nearly rigid on the obliquity timescale (large $\eta$), and that the water cap substantially lags the obliquity oscillation (large $\delta$ like on Earth). In this case the water cap dominates the deformation and Mars’ obliquity is decreasing. With a larger obliquity in the past, the average annual insolation at the poles would have been higher (3) with presumably correspondingly weaker ice ages. Thus Mars’ climate may have been
balmier in the past, producing little or no polar layered terrain. Weakly operating climate friction may have slowly increased the axial tilt, which in turn enhanced the severity of the ice ages and accelerated the rate of tilting in a positive feedback loop. Eventually Mars put down a discernible layer with each obliquity cycle, thus creating the layered terrain within the last few hundred million years (14).

The other scenario is that climate friction has changed the tilt of Mars until it became "stuck" at its present average obliquity of 24.4°. The idea is that, like in the previous scenario, the intensity of the ice ages depends on the average obliquity; but here obliquity-oblateness feedback changes the average until the climate becomes such that the size and phase lag of the caps just balance the associated deformation. In this case climate friction goes to zero, there is no further secular change in obliquity, and Mars maintains its present average tilt indefinitely.

Fig. 2 is based on a simple model but embodies all of the qualitative features that must hold for the real planet. For instance, the edge $\xi = 0°$ in Fig. 2 must be at zero regardless of Mars' rheology, since this edge represents an instantaneous isostatic adjustment to the ice cap load, resulting in no obliquity change. Likewise for the real Mars the edge $\delta = 0°$ must rise above zero as shown, since this corresponds to the ice cap being in phase with the obliquity oscillations; the cap in this case gives no secular drift, leaving only a secular increase coming from whatever deformation is induced by the cap (1). Similarly, climate friction must dip to a negative minimum at $\xi = 90°$ and $\delta = 90°$; this corner represents a perfectly rigid planet with no deformation, with a decrease in tilt coming only from an ice cap that is maximally out of phase with the oscillations. Hence the topology of the surface shown in Fig. 2 must be correct, with positive and negative changes in axial tilt separated by a curve of no change.
The exact shape and amplitude of the surface shown in Fig. 2 depend upon Mars’ internal structure and its climate dynamics. Much more understanding of the climate and internal structure of Mars will be needed to see if either of the two scenarios proposed above holds (15).

References and Notes


11. The Laurentide ice sheet had a mass of about $2 \times 10^{19}$ kg; J. X. Mitrovica and W. R. Peltier, *J. Geophys. Res.* **94**, 13,651 (1989). This would put the ratios of ice mass to planetary mass at $1.5 \times 10^{-6}$ for Mars and $3.3 \times 10^{-6}$ for Earth.


The layered terrain may also be young due to its removal at very high obliquity and re-deposition in recent geologic times; see (9).

15. I thank Bruce G. Bills, David E. Smith, and Maria T. Zuber for helpful discussions, Dave Rowlands and Frank Lemoine for making gravity anomaly calculations, and Susan Sakimoto for producing Fig. 2.
Figure Captions

Figure 1. The interplay between Mars' obliquity oscillations, ice cap, and deformation, all greatly exaggerated. The obliquity $\epsilon$ is lowest at (a) and greatest at (c). The ice cap is arbitrarily shown to lag the obliquity forcing by 90°, so that the cap has the greatest extent at (b) and smallest at (d). The ice load arbitrarily lags the cap by 45°. The actual lags for Mars are unknown. The permanent rotational bulge is omitted for clarity. The waxing and waning of the ice cap as the obliquity oscillates could cause a slow secular change in the average obliquity. Figure based on ref. 6.
Figure 2. The secular obliquity change $\Delta e_{sec}$ for the simple model given in Eq. 4, as a function of $\delta$ and $\xi$. Here $\delta$ is the phase lag of the ice cap with respect to the obliquity forcing, and $\xi$ is the phase lag of the solid planet with respect to the ice cap loading.

Climate friction is assumed to operate at a constant rate over the age of the solar system. The average obliquity can either increase or decrease, depending upon $\delta$ and $\xi$. The warm colors give increases and the cooler colors decreases. The decrease is greatest for $\xi = 90^\circ$ (a rigid planet) and $\delta = 90^\circ$ (caps maximally out of phase with obliquity). The diagonal line across the center separates increases (background) from decreases (foreground). Changes in obliquity of hundreds of degrees are not expected, but tens of degrees may be possible.
OBLIQUITY, ICE CAP, AND DEFORMATION