Design and Development of Lateral Flight
Director

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May 1999
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<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>DME</td>
<td>distance measuring equipment</td>
</tr>
<tr>
<td>GS</td>
<td>glideslope</td>
</tr>
<tr>
<td>HSI</td>
<td>horizontal situation indicator</td>
</tr>
<tr>
<td>ILS</td>
<td>instrument landing system</td>
</tr>
<tr>
<td>LOC</td>
<td>Localizer</td>
</tr>
<tr>
<td>MM</td>
<td>middle marker</td>
</tr>
<tr>
<td>nm</td>
<td>nautical miles</td>
</tr>
<tr>
<td>OM</td>
<td>outer marker</td>
</tr>
<tr>
<td>( \dot{\phi}_{\text{cmd}} )</td>
<td>roll rate command, deg/sec</td>
</tr>
<tr>
<td>( \dot{\phi}_{\text{err}} )</td>
<td>roll rate error, deg/sec</td>
</tr>
<tr>
<td>( \tau )</td>
<td>pilot and aircraft lag in response to roll angle command from flight director, (2.0 sec)</td>
</tr>
<tr>
<td>( \ddot{\phi} )</td>
<td>roll acceleration, deg/sec/sec</td>
</tr>
<tr>
<td>( \dot{\phi} )</td>
<td>roll rate, deg/sec</td>
</tr>
<tr>
<td>( \phi )</td>
<td>roll angle, deg, and right wing down</td>
</tr>
<tr>
<td>( \psi )</td>
<td>turn (yaw) rate, deg/sec</td>
</tr>
<tr>
<td>( \psi )</td>
<td>heading (yaw) angle, deg</td>
</tr>
<tr>
<td>( x_w )</td>
<td>downtrack wind velocity, ft/sec</td>
</tr>
<tr>
<td>( \dot{x} )</td>
<td>downtrack velocity, ft/sec</td>
</tr>
<tr>
<td>( x )</td>
<td>downtrack location of aircraft, ft</td>
</tr>
<tr>
<td>( y_w )</td>
<td>crosstrack wind velocity, ft/sec</td>
</tr>
<tr>
<td>( y )</td>
<td>crosstrack velocity, ft/sec</td>
</tr>
<tr>
<td>( y )</td>
<td>crosstrack location of aircraft, ft</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>localizer deviation, deg (positive when aircraft left of centerline)</td>
</tr>
<tr>
<td>r2d</td>
<td>radians to degrees</td>
</tr>
<tr>
<td>d2r</td>
<td>degrees to radians</td>
</tr>
</tbody>
</table>

\[
K_r = \frac{g}{V} \quad K_x = \tan \left( \frac{y}{x_{\text{loc}} - x} \right)
\]

- \( g \) = 32.2 feet per second per second due to gravity
- \( V \) = 220 feet per second, velocity (130 knots)
- \( x_{\text{loc}} \) = 10,000 feet from runway threshold, location of localizer transmitter
\[ A = \frac{1}{\text{distance from aircraft to the localizer transmitter}}, \quad x_{\text{loc}} - x, \quad 1/46,000 \text{ at outer marker, } 1/13,000 \text{ at middle marker.} \]

- \( K_p \): proportional gain for localizer deviation feedback
- \( K_i \): integrator gain for localizer deviation feedback
- \( K_d \): derivative gain for localizer deviation feedback
- \( K_v \): proportional gain for heading, yaw feedback
- \( K_b \): proportional gain for bank, roll feedback
- \( \omega_n \): natural frequency of localizer deviation
- \( t_s \): settling time (to 5% error) of localizer deviation
- \( t_{s5} \): settling time (to 5% error) of localizer deviation
- \( \zeta \): damping ratio of localizer deviation
- \( \phi_{\text{err}} \): roll angle error, deg
- \( \psi_{\text{err}} \): heading error, deg
- \( \psi_{\text{ref}} \): runway heading in degrees with respect to North
- \( \psi_{\text{cmd}} \): heading command, deg
- \( \phi_{\text{cmd}} \): roll angle command, deg
- \( \lambda_{\text{cmd}} \): command, deg (usually zero)
- \( d \): distance in feet the center of the airplane is from the centerline
- \( \text{err} \): localizer error, deg
- \( \dot{\phi} \): roll rate, deg/sec
- \( \phi \): roll angle, deg
- \( \dot{\psi} \): yaw rate, or turn rate, deg/sec
- \( \psi \): yaw angle or heading angle, deg
- \( g \): 32.2 ft/sec, gravity
- \( V \): 220 ft/sec, velocity, (assumed constant)
SUMMARY

The current control law used for the flight director in the Boeing 737 simulator is inadequate with large localizer deviations near the middle marker. Eight different control laws are investigated. A heuristic method is used to design control laws that meet specific performance criteria. The design of each is described in detail. Several tests were performed and compared with the current control law for the flight director. The goal was to design a control law for the flight director that can be used with large localizer deviations near the middle marker, which could be caused by winds or wake turbulence, without increasing its level of complexity.

INTRODUCTION

There are several navigation systems to aid a pilot in landing an aircraft. Commercial aircraft normally use the instrument landing system (ILS) to make an approach and land. ILS is limited to a ceiling of 200 ft and visibility of half a mile. The ILS has been the international standard for the past 40 years. The pilot has access to several types of instruments to help him navigate to the runway. The instrument landing system (ILS) approach provides the pilot vertical guidance as well as horizontal guidance to the runway. The ILS approach will guide the pilot down to his landing site (runway).

The ILS is composed of position information, range information, and visual information (figure 1). Two highly directional transmitting systems provide the position information: the localizer (LOC) and the glideslope (GS). The LOC provides the pilot information relating the horizontal position to the runway. The signal is transmitted from an
antenna located on the runway centerline at the far end, typically 10,000 feet from the approach end of the landing runway. The signal is usable from a distance of at least 20 nautical miles (nm). Reliable indications of being off course to the left or right can be received 35° either side of the runway centerline, but the instrument only indicates ±2.5°. The GS provides the pilot vertical information position to the runway. The antenna site is located 750 to 1,250 feet from the approach end of the landing runway and is offset 250 to 650 feet from the runway centerline. The beam transmitted is 1.4° wide and is angled upward at approximately three degrees to intercept the middle marker at 200 feet and the outer marker at about 1,400 feet above the runway elevation. The GS signal is normally usable to a distance of at least 10 nm from the antenna site. Marker beacons can provide the range information for the ILS. Normally an ILS has two marker beacons: the outer marker (OM) and the middle marker (MM). The marker beacon is a signal transmitted from an antenna array, which produces an elliptical pattern 2,400 feet long by 4,200 feet wide at an altitude of 1,000 feet above the antenna site. The signal is 3 watts or less in power and transmitted on a frequency of 75 MHz. The OM is located 4 to 7 nm from the runway threshold. The MM is located 3,500 feet from the runway threshold where the GS is 200 feet above the touchdown zone elevation. The visual information for the ILS consists of approach lights, touchdown zone lights, runway centerline lights, and runway lights. The LOC and GS signals are received by the airplane instruments and used by the computer to calculate the output of the flight director.
The flight director in figure 2 is an example of a digital display in the simulator here at NASA Langley in the Simulation Systems Branch. On the right is the GS and it indicates the plane is below the glideslope and on the bottom is the LOC and it indicates the plane is right of centerline. However the flight director reveals the plane needs to bank right.

To prepare for landing the pilot must first tune all necessary navigation radios to receive the LOC and GS signals. The radar controller will guide the pilot by issuing vectors (heading changes) until the aircraft is on a heading to intercept the LOC at approximately a 45° angle before reaching the outer marker. Once the pilot is within the 2.5° angle the flight director will guide the pilot by indicating what angle to bank the plane to achieve the desired location, centerline. The flight director will adjust to compensate for the effect of wind. If the flight director moves left, the pilot will bank to the left. If the flight director moves to the right, the pilot will bank to the right. The flight director will guide the pilot so that the plane will arrive at a heading that negates the effect of wind and
the plane will fly on the centerline. Once the pilot crosses the middle marker, to continue the descent and land, the pilot must see the runway or lights associated with the approach. The plane usually will touchdown by 1200 feet from the threshold of the runway.

PROBLEM DESCRIPTION

Landing an aircraft in high winds, rough air or at night requires highly accurate information on the position and angle of the aircraft with respect to the runway. The flight director can provide the pilot information to guide the aircraft to safety. With a flight director, the pilot simply moves the control wheel left or right toward the flight director needle to keep it centered. The flight director makes flight in bad weather and winds easier and safer to fly and reduces complexity for the pilot, but requires a control law in a computer to calculate the proper roll commands.

The lateral flight director currently used for the NASA Langley Boeing 737 simulation uses proportional feedback from the localizer and heading, and a proportional, integral, derivative control law for the roll angle command and roll feedback. This design has an overdamped response, and as a result is very slow to capture the centerline. It does not provide adequate guidance if the aircraft is very far off the centerline at the middle marker. In an emergency situation, if a gust of wind were to knock the plane off course at the middle marker the current flight director could not provide efficient guidance to land the plane safely.

The objective of this study is to design a lateral flight director for a commercial Boeing 737 that will calculate roll angle commands to capture and track the runway centerline, using methods different from the control law now in use. The new flight director should be able to guide a plane to a safe landing in normal circumstances, as the current flight director does, and in the stress case, when the localizer is pegged at the middle marker. The different candidate control laws that are designed and tested are summarized in the following table:

<table>
<thead>
<tr>
<th>Control Law</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOC Deviation</td>
<td>Heading</td>
</tr>
<tr>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>PI</td>
</tr>
<tr>
<td>3</td>
<td>PD</td>
</tr>
<tr>
<td>4</td>
<td>PID</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>PI</td>
</tr>
<tr>
<td>7</td>
<td>PD</td>
</tr>
<tr>
<td>8</td>
<td>PID</td>
</tr>
<tr>
<td>Current Law</td>
<td>P</td>
</tr>
</tbody>
</table>

Key
- P – Proportional
- I – Integrator
- D – Derivative

Table 1 Control Laws
Since, there are certain criteria to meet in order to land a commercial airline, several experiments have been performed to test the most common and not so common landings. The test cases include:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>Standard</td>
<td>Standard</td>
<td>Standard</td>
<td>Procedure</td>
<td>Baseleg</td>
<td>Pegged</td>
</tr>
<tr>
<td>Conditions</td>
<td>Crosswind from left</td>
<td>Crosswind from right</td>
<td>Turn</td>
<td>No wind</td>
<td>Middle Marker</td>
<td></td>
</tr>
<tr>
<td>x - location</td>
<td>-40,000</td>
<td>-40,000</td>
<td>-40,000</td>
<td>-60,000</td>
<td>-100,000</td>
<td>-3020</td>
</tr>
<tr>
<td>y - location</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>20,000</td>
<td>60,000</td>
<td>550</td>
</tr>
<tr>
<td>heading</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>45</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>wind velocity, fps</td>
<td>0</td>
<td>30</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>wind direction</td>
<td>0</td>
<td>-90</td>
<td>90</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 Test Cases

Note: X and Y location are based on an end of runway coordinate system, Page 6. The y location is positive if approach is from the right and negative if approach is from left.

PROBLEM SETUP: REQUIREMENTS AND LIMITATIONS

The information available consists of the localizer, which indicates the angular deviation from the runway centerline. However, it is limited to +/-2.5°. Any localizer deviation error beyond this, the needle is limited to maximum of 2.5° within a reception range of 35° and approximately 20 nm. The system becomes more sensitive as the airplane nears the transmitter. The reason is that a given lateral distance off the beam centerline corresponds to a larger error angle as the transmitter is approached. Figure 3 depicts the general lateral guidance geometry.

For this study, the outer marker (OM) beacon is placed at 36,000 ft from the runway, about 6 nautical miles. The middle marker (MM) beacon is placed at 3,000 ft from the runway, about 0.5 nautical miles. These marker beacons send signals, flash lights and make audio tones for a few seconds, which the aircraft picks up as it flies over the markers.

The pilot knows the aircraft heading and bank angle from the basic flight instruments. Roll angle feedback is obtained from the attitude indicator. Heading is obtained from the compass on the HSI. The runway heading and localizer transmitter frequencies are known from approach charts. The localizer needle shows the angular deviation (not distance) from the runway centerline. The pilot controls the aircraft by moving a control wheel that determines the roll rate. The roll (bank) angle and the speed of the aircraft determine the rate of change of heading. The aircraft heading may or may not be in the direction that the plane is moving, depending on the effects of winds. For the analysis, it is assumed that the pilot will roll to follow the flight director. From the instruments in the plane the flight director can use the heading as input to the controller, if needed.

The flight director should not command large angles or rates. Typical limits are 25° of bank, and a roll rate of 10° per second. The ailerons, movable hinged sections on the wing of an airplane for controlling rolling
movements, have a maximum deflection of 20° and produce a roll rate of about 12° per second in the landing configuration.

The following assumptions and conditions will apply for this study. The aircraft will fly at a constant airspeed, velocity relative to the air, of 220 feet per second or 130 knots. This is normal approach speed for a Boeing 737. It is assumed that the aircraft makes coordinated turns. The aircraft performs a coordinated turn by banking (rolling the wings) and moves in the direction it is pointing relative to the airmass. Airmass is a large body of air having virtually uniform conditions of temperature and moisture in a horizontal cross section, but not the ground. Winds affect the motion of the aircraft relative to the ground.

Since this study is only concerned with lateral guidance, the aircraft is assumed to follow the glideslope. The glideslope is generated by a transmitter, which tells the pilot if the aircraft is higher or lower than it should be during the approach. Some ILS's have a distance measurement equipment (DME) transmitter collocated at the localizer transmitter which provides the distance of the aircraft from the localizer. For this study a DME will not be used.

![Figure 3 General Lateral Guidance Geometry](image)

**Figure 3 General Lateral Guidance Geometry**

The problem begins with the plane heading roughly toward the outer marker. In real life the air traffic controller would tell the pilot which direction to fly. The angle the airplane intercepts the localizer beam is between ±90°, the preferred intercept angle (ψ-ψref) ±45° off centerline. Figure 3 displays a -30° approach. If the aircraft is coming in at an angle of more than ±90° a procedure turn (figure 4) is done so that the pilot approaches the centerline at an angle of ±45°.

![Figure 4 Procedure Turn](image)

**Figure 4 Procedure Turn**
It is necessary to reach ('capture') the centerline near the outer marker in a reasonable time. This depends on the speed and initial heading of the aircraft. (The approach and landing are usually flown at a nearly constant speed, but that speed is different for different aircraft.) This problem would be exactly the same for other aircraft except for the speed during approach.

Once the plane reaches the centerline it should not cross it more than once. That is, an initial overshoot is acceptable, but not significant oscillations. Once the aircraft crosses the centerline the localizer deviation should not exceed about 0.5°. A 0.5° overshoot corresponds to a 20% overshoot or a damping ratio about 0.5. The aircraft should be flying along the runway centerline ('tracking') well before reaching the middle marker.

At touchdown the aircraft should be flying nearly wings level and close enough to the centerline that the wheels are on the runway. A typical runway is 150 feet wide, and typical airliner's wheels are about 50 feet apart. Thus the aircraft must be within 50 feet of the centerline.

Once the requirements and limitations for landing are set the next step is to model the runway and the airplane dynamics. Figure 5 displays the runway coordinate system and figure 6 displays the airplane dynamics including the assumed pilot reactions. To simplify the model for analysis purposes, latitude and longitude are not used in this problem, instead the runway x, y coordinate system is used. The centerline of runway is y = 0 and the beginning of the runway is x = 0. To the right of the runway is y > 0, and to the left is y < 0. The outer marker is assumed to be at x = -36,000, y = 0, and the localizer is at x = 10,000, y = 0, according to this coordinate system. The reference heading angle is normally North, for simplicity it is set to equal the runway, $\psi_{ref} = 0$. The plane is heading within ±180 degrees relative to the centerline. The pilot does not have a true awareness of position, just if he is right or left of centerline.

![Figure 5 Runway Coordinate System](image)
Using the plant dynamics in figure 6, and adding a feedback from the roll angle, \( \phi \), the inner loop can then be analyzed to find the proportional gain \( K_\phi \), figure 7. This gain will be used throughout the eight control laws. If the pilot lag is ignored for the analysis of the roll, then transfer function reduces to \( \frac{K_\phi}{s + K_\phi} \). If roll angle command, \( \phi_{cmd} \), is 25°, and the gain, \( K_\phi \), is 2, then the roll rate command, \( \dot{\phi}_{cmd} \), will be 50° per second. The roll rate is not realistic for a commercial aircraft, therefore the gain needs to be held to a smaller value. A more realistic roll rate command is 12.5° per second, which is achieved using the same roll angle command and cutting the gain to 0.5.

This new roll rate exhibits a time constant of 2 seconds and will reach steady state roll angle in, (three times the time constant), 6 seconds. When a roll angle command of 25° is given, the physical parameters for a commercial aircraft...
allow the roll angle to be achieved within 1.5 to 5 seconds. It should not take a plane, once a roll command is given more than 5 seconds to reach the roll angle; however, it should take at least 1.5 seconds. In order to simplify the analysis and meet realistic aircraft reactions, let $K_\alpha = 1$. This implies that the unlimited roll rate command will be when the roll command is $25^\circ$, and $1^\circ$ when the roll command is $1^\circ$. The time constant is 1 second, and should reach roll angle steady state at approximately 3 seconds for roll errors less than $10^\circ$. Note roll rate is limited to 10 degree per second, so the response is nonlinear for roll errors of more than $10^\circ$.

CONTROL WITHOUT FEEDBACK FROM HEADING

To simplify analysis, assume $\phi_{\text{cmd}} = \phi$, because the roll response is much quicker than the response of the aircraft to get to the centerline. The linearized model with feedback from the localizer deviation is represented in figure 8. Note in the model all three gains are represented.

\[
\begin{align*}
K_i & \frac{1}{s} \\
K_p & \frac{g}{V} \\
K_d & \frac{1}{s} \\
\phi & \frac{1}{s} \\
\psi & \frac{1}{s} \\
d2r & V \\
\dot{V} & \frac{1}{s} \\
y & \Lambda \\
r2d & \lambda \\
\end{align*}
\]

Figure 8 Flow Diagram for Analysis

From figure 8, using Mason’s gain formula the transfer function is:

\[
y = \frac{\frac{K_i}{s} \frac{1}{V} \frac{1}{s} \frac{1}{s} A(r2d) + \frac{K_p}{V} \frac{1}{s} \frac{1}{s} \frac{1}{s} A(r2d) + \frac{K_d}{V} \frac{1}{s} \frac{1}{s} \frac{1}{s} A(r2d)}{1 + \frac{K_i}{s} \frac{1}{V} \frac{1}{s} \frac{1}{s} A(r2d) + \frac{K_p}{V} \frac{1}{s} \frac{1}{s} \frac{1}{s} A(r2d) + \frac{K_d}{V} \frac{1}{s} \frac{1}{s} \frac{1}{s} A(r2d)}
\]

\[
= \frac{\frac{K_i}{s^3} A + \frac{K_p}{s^2} A + \frac{K_d}{s} A}{1 + \frac{K_i}{s^3} A + \frac{K_p}{s^2} A + \frac{K_d}{s} A}
\]

\[
= \frac{K_d A s^2 + K_p A s + K_i A}{s^3 + K_d A s^2 + K_p A s + K_i A}
\]

LAW 1: PROPORTIONAL CONTROL

The first test case is proportional control, by setting $K_i$ and $K_i = 0$, then the transfer function becomes

\[
y = \frac{K_p A}{s^2 + K_p A}
\]

This system, with proportional control, is not stable. It is a second order system, with the
damping ratio equal to zero. The roots fall on either the imaginary axis or the real axis with opposite roots. This will create an oscillatory behavior. The aircraft would roll back and forth across the centerline and never line up.

**LAW 2: PROPORTIONAL, INTEGRAL CONTROL**

The second test case is proportional, integral control, by setting $K_i = 0$, the transfer function becomes
\[
\frac{y}{u} = \frac{K_p g A s + K_i g A}{s^3 + K_p g A s + K_i g A}.
\]
Using a root locus to display this third order system roots shows in figure 9 the system is unstable.

![Figure 9 Root Locus, with Positive K left, and Negative K right](image)

**LAW 3: PROPORTIONAL, DERIVATIVE CONTROL**

The third test case is proportional, derivative control, by setting $K_i = 0$, the transfer function becomes
\[
\frac{y}{u} = \frac{K_d g A s + K_p g A}{s^2 + K_d g A s + K_p g A}.
\]
This is a second order equation of the form $\frac{K s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ with a characteristic equation $s^2 + 2\zeta \omega_n s + \omega_n^2$. Let $\zeta = 0.707$, because this has an overshoot of less than 5%, the quickest settling time, and the smallest integral error for a linear system with a step input. $\omega_n$ determines how long the aircraft takes to line up with the centerline. A higher value of $\omega_n$ means a shorter oscillation period and lines up with the centerline more quickly. At the outer marker, flying at a speed of 220 ft/sec, the aircraft is about 37,000 feet or 168 seconds from touchdown. At the middle marker the aircraft is about 4,000 feet or 18 seconds from touchdown. So the most critical case for the gain determination is at the middle marker. This 18 seconds corresponds to the maximum allowable ‘settling time’ $t_s$ of the system, the time it takes to correct 95% of an initial error. Settling time is related to $\omega_n$ and $\zeta$ from the relationship $t_s = \frac{3}{\omega_n \zeta}$.

Solving for $\omega_n$ yields $\omega_n = \frac{3}{t_s \zeta}$. With $\zeta = 0.707$, and $t_s = 18$ yields $\omega_n = 0.24$. This is the maximum value of $\omega_n$ since the aircraft would have more time to respond, and a
smaller $\omega_n$, for any other case. Using $\omega_n = .24$, the characteristic equation becomes $\Rightarrow s^2 + .336s + .0576$. This results in $K_p = \frac{.0576}{gA}$, 82 at the outer marker, and 23 at the middle marker, $K_d = \frac{.336}{gA}$, 480 at the outer marker, and 136 at the middle marker. The simplest control law would be one with constant gains that work throughout the approach from the outer marker to touchdown. Since the middle marker is most critical, let $K_p = 23$ and $K_d = 136$. The next step is to verify the response using the ‘standard’ test case to ensure the gains are in the right ballpark. Figures 10 and 11 illustrate the standard approach test case, with no winds, heading is parallel to the runway, and velocity of plane is 220 feet per second. The plane starts past the outer marker.
The initial localizer error is about 1.15 degrees, and the response overshoots to 0.45 degrees. These gains produce an overshoot of about $0.45/1.15 = 39\%$. The overshoot and oscillation is produced by the lag, of the pilot and aircraft response, but also by the roll and roll rate limits and because distance to localizer changes making the system more sensitive. This is too much oscillation so it might be better to increase the damping ratio to 1.0. Then
\( \omega_n = 0.17 \) and the characteristic equation is \( s^2 + 3.4s + 0.0289 \), which implies the gains at the middle marker are

\[
K_p = \frac{0.0289}{gA} = 11, \quad K_d = \frac{34}{gA} = 137.
\]

Looking at the standard test case shows this reduces the oscillation.

LAW 4: PROPORTIONAL, INTEGRAL, DERIVATIVE CONTROL

The fourth test case is proportional, integral, derivative control, the transfer function is as before,

\[
y = \frac{K_d gA s^2 + K_p gA s + K_i gA}{s^3 + K_d gA s^2 + K_p gA s + K_i gA}.
\]

If we let \( K_d = 137 \) and \( K_p = 11 \), from the previous analysis, and let \( K_i = 1 \), the pole-zero plot, figure 12, shows where the poles and zeros lie in the \( s \)-plane.

![Figure 12 Left, Pole Zero Plot with gains, \( K_p = 11, K_i = 1, K_d = 136, K_q = \text{none} \), Right, Root Locus, with \( K = 0.1, 0.2, 0.3, 0.4, 0.5 \) and gains \( K_p = 11, K_i = 1, K_d = 136, K_q = \text{none} \)].

This indicates that a stable solution maybe found for the system. Using a root locus plot and letting \( K \) vary 0.1, 0.2, 0.3, 0.4, and 0.5, figure 12 also shows that as \( K \) gets bigger the pole on the \( x \)-axis goes further out. If \( K = 0.1 \) the oscillations are reasonable in preliminary tests.
CONTROL WITH FEEDBACK FROM HEADING

For this part of the problem there is an additional feedback from heading, $\psi$. The next step is to analyze the two inner feedback loops, roll and heading. The linearized model is represented in figure 13. Note in the control used for heading feedback is proportional.

Using Mason’s gain analysis techniques produces the following transfer function:

$$\frac{K_\psi K_\phi K_v}{s^2 + \frac{K_\phi + K_\psi K_\phi K_v}{s^2}} \Rightarrow \frac{K_\psi K_\phi K_v}{s^2 + K_\phi s + K_\psi K_\phi K_v}$$

This is a second order equation of the form $\frac{\omega^2}{s^2 + 2\zeta \omega s + \omega^2}$. From the previous analysis $K_\phi = 1$.

Given $K_v = \frac{g}{V} = 0.146$

Let $\zeta = 1$, then

$K_\phi = 1 = 1 * 2 * \omega \Rightarrow \omega = 0.5$

$\omega^2 = 0.25 = K_\phi K_v K_\psi \Rightarrow K_\psi = 1.71$

Settling time is $t_s = 3/(1*0.5) = 6$

Output to input becomes $\frac{\psi}{\psi_{cmd}} = \frac{0.25}{s^2 + s + 0.25}$. This system has a one second lag, and no overshoot.

Placing this transfer function in the overall system for the final analysis results in the linearized model in figure 14.
The next step is to analyze the entire system. To simplify analysis, assume $\Psi_{\text{end}} = \Psi_{\text{d}}$, because the heading response is much quicker than the response of the aircraft to get to the centerline. The linearized model with feedback from the localizer deviation is represented in figure 14. Note in the model all three gains are represented. From figure 14, using Mason’s gain formula the transfer function is:

$$
\frac{y}{u} = \frac{K_i (d2r)V = A(r2d) + K_p (d2r)V = A(r2d) + K_d s(d2r)V = A(r2d)}{1 + \frac{K_i (d2r)V = A(r2d) + K_p (d2r)V = A(r2d) + K_d s(d2r)V = A(r2d)}}
$$

$$
= \frac{K_i VA}{s^2} + \frac{K_p VA}{s} + \frac{K_d VA}{1} = \frac{K_d VAs^2 + K_p VAs + K_i VA}{(1 + K_d VA)s^2 + K_p VAs + K_i VA}
$$

LAW 5: PROPORTIONAL CONTROL

The fifth test case is proportional control, by setting $K_d$ and $K_i = 0$, the transfer function becomes

$$
\frac{y}{u} = \frac{K_p VA}{s + K_p VA}.
$$

Proportional control is a first order system with a lag. If I use the previous criteria letting the time constant be 6 due to the requirement to reach centerline in 18 seconds, then it follows $K_p VA = 0.167$, and at the middle marker, $K_p = 9.85$. Now to do a preliminary test for the gains, $K_p = 9.85$ and $K_i = 1.7$. The results are displayed in figures 15 and 16.
Figure 15 Position of Aircraft, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$

Figure 16 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$
LAW 6: PROPORTIONAL, INTEGRAL CONTROL

The sixth test case is proportional, integral control, by setting $K_d = 0$, the transfer function becomes

$$\frac{y}{u} = \frac{K_p V A s + K_i V A}{s^2 + K_p V A s + K_i V A}.$$  

This is a second order equation of the form $\frac{K_s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ with a characteristic equation $s^2 + 2\zeta \omega_n s + \omega_n^2$. From the previous problem we have $K_p = 9.85, K_i V A = 0.1667 = 2\zeta \omega_n$. Solving for $\omega_n$ with $\zeta = 0.707$, yields $\omega_n = 0.1179$ and with $\zeta = 1$, yields $\omega_n = 0.0833$. Using $\omega_n = 0.1179$, the characteristic equation is $s^2 + 0.1667s + 0.0139$. This results in $K_i = \frac{0.0139}{V A}$, 0.8212 at the middle marker. Using the gains for preliminary testing, let $K_p = 9.85, K_i = 0.8, K_d = 0$, and $K_o = 1.7$, the results are displayed in figure 17.

![Figure 17 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.8, K_d=0, K_o=1.7$](image)

The integrator certainly introduces some oscillation. This is too much oscillation so it might be better to increase the damping ratio to 1.0. Then $\omega_n = 0.0833$ and the characteristic equation is $s^2 + 0.1667s + 0.0069$. This results in $K_i = \frac{0.0069}{V A}$, 0.4105 at the middle marker. This produces less oscillation, within acceptable limits.
LAW 7: PROPORTIONAL, DERIVATIVE CONTROL

The seventh test case is proportional, derivative control, by setting \( K_i = 0 \), the transfer function becomes

\[
\frac{y}{u} = \frac{K_d VA s + K_p VA}{(1 + K_s VA)s + K_p VA}.
\]

This is a first order system with a lag. In the previous analysis 6 was used, this time let \( \tau = 6.5 \), then

\[
\frac{1 + K_s VA}{K_p VA} = 6.5.
\]

This value is chosen to prevent cancellation with the previous used value of 6. \( 0.08355 = K_s VA \Rightarrow K_s = 4.94 \), Testing the gains, \( K_p = 9.85, K_d = 5, K_v = 1.7 \), with the standard approach test case produces the graph in figure 19, which appears to be acceptable.

![Position of Aircraft](image)

*Figure 18 Position of Aircraft with gains, \( K_p=9.85, K_i=0, K_d=5, K_v=1.7 \)*

LAW 8: PROPORTIONAL, INTEGRAL, DERIVATIVE CONTROL

The eighth test case is proportional, integral, derivative control, the transfer function is as before

\[
\frac{y}{u} = \frac{K_d VAs^2 + K_p VAs + K_v VAs}{(1 + K_s VA)s^2 + K_p VAs + K_v VA}.
\]

Using all the gains from the previous analysis, \( K_d = 5, K_i = 0.4, K_p = 9.85, \) and \( K_v = 1.7 \), is sufficient to test without any further analysis. The plots of the results are in the next section, Observations.
OBSERVATIONS

From the previous analysis of the possible control laws, 1 and 2 were unstable, the remaining, which are to be tested thoroughly include:

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<th>$K_d$</th>
<th>$K_{yp}$</th>
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<tr>
<td>8</td>
<td>9.85</td>
<td>0.4</td>
<td>5</td>
<td>1.7</td>
</tr>
</tbody>
</table>

*Table 3 Gains that Passed Initial Testing*

Test Cases:

1. **Standard approach** – This is the standard approach made most of the time. The aircraft has a heading parallel to the runway and is past the outer marker.

2. **Standard with crosswind from left** – This approach is the same as the previous but wind is added from the left. This may test if the wind is blowing the aircraft towards the centerline or away from the centerline. Thirty knots of crosswind is the maximum wind a plane is expected to land in.

3. **Standard with crosswind from right** – This approach is the same as the previous but wind is added from the right. This may test if the wind is blowing the aircraft towards the centerline or away from the centerline. Thirty knots of crosswind is the maximum wind a plane is expected to land in.

4. **Procedure Turn** – This approach is when the airplane approaches the runway from an angle of more than 90 degrees or less than 90 degrees.

5. **Baseleg approach** – The aircraft approaches from a 90-degree angle.

6. **Pegged at middle marker** – The aircraft is at the middle marker and about 2.5 degrees off the localizer centerline.
CONTROL WITHOUT FEEDBACK FROM HEADING

LAW 3: PROPORTIONAL, DERIVATIVE CONTROL

Standard approach, starting near the outer marker.

Figure 19 Position of Aircraft with gains, $K_p=11, K_i=0, K_d=137, K_q=None$, and Test Case Standard, No Wind

Figure 20 Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0, K_d=137, K_q=None$, and Test Case Standard, No Wind
**Law 3: Proportional, Derivative Control**

Standard approach starting near the outer marker. The maximum crosswind speed a plane is expected to land in is about 30 feet per second. All crosswind tests use this speed.

*Figure 21 Position of Aircraft with gains, $K_P=11, K_I=0, K_D=137, K_W=\text{None}$, and Test Case Standard Approach from Right, with Crosswind from Right*

*Figure 22 Localizer, Roll, and Roll Rate with gains, $K_P=11, K_I=0, K_D=137, K_W=\text{None}$, and Test Case Standard Approach from Right, with Crosswind from Right*
Law 3: Proportional, Derivative Control

Standard approach starting near the outer marker. Ground track is nearly the same as before but heading is different.

![Position of Aircraft graph]

Figure 23 Position of Aircraft with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Standard Approach from Right, with Crosswind from Left

![Localizer Deviation, Roll, Roll Rate graphs]

Figure 24 Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Standard Approach from Right, with Crosswind from Left
Law 3: Proportional, Derivative Control

Localizer capture after a procedure turn, the normal flight procedure.

Figure 25 Position of Aircraft with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Procedure Turn from Right, No Wind, Heading $45^\circ$

Figure 26 Localizer Deviation, Roll, and Roll Rate with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Procedure Turn from Right, No Wind, Heading $45^\circ$
Law 3: Proportional, Derivative Control

Baseleg approach an extreme case not normally used.

Figure 27 Position of Aircraft with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Baseleg from Right, No Wind, Heading $90^\circ$

Figure 28 Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Baseleg from Right, No Wind, Heading $90^\circ$
Law 3: Proportional, Derivative Control

This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

![Position of Aircraft](image)

*Figure 29 Position of Aircraft with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$*

![Localizer Deviation and Roll Rate](image)

*Figure 30 Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0, K_d=137, K_w=None$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$*
LAW 4: PROPORTIONAL, INTEGRAL, DERIVATIVE CONTROL

Standard approach starting near the outer marker.

Figure 31 Position of Aircraft with gains, $K_p=11, K_i=0.1, K_d=137$, $K_w=None$, and Test Case Standard, No Wind

Figure 32 Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0.1, K_d=137$, $K_w=None$, and Test Case Standard, No Wind
Law 4: Proportional, Integral, Derivative Control

Standard approach starting near the outer marker from the right with crosswind from the right.

Figure 33 Position of Aircraft with gains, $K_p=11, K_i=0.1, K_d=137, K_w=None$, and Test Case Standard Approach from Right, with Crosswind from Right

Figure 34 Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0.1, K_d=137, K_w=None$, and Test Case Standard Approach from Right, with Crosswind from Right
Law 4: Proportional, Integral, Derivative Control

Standard approach starting near the outer marker from the right with crosswind from the left. Ground track is nearly the same as before but heading is different.

**Figure 35** Position of Aircraft with gains, $K_p=11, K_i=0.1, K_d=137$, $K_w=None$, and Test Case Standard Approach from Right, with Crosswind from Left

**Figure 36** Localizer, Roll, and Roll Rate with gains, $K_p=11, K_i=0.1, K_d=137$, $K_w=None$, and Test Case Standard Approach from Right, with Crosswind from Left
Law 4: Proportional, Integral, Derivative Control

Localizer capture after a procedure turn, the normal flight procedure.

Figure 37 Position of Aircraft with gains, $K_p=11$, $K_i=0.1$, $K_d=137$, $K_w=None$, and Test Case Procedure Turn from Right, No Wind, Heading $45^\circ$

Figure 38 Localizer, Roll, and Roll Rate with gains, $K_p=11$, $K_i=0.1$, $K_d=137$, $K_w=None$, and Test Case Procedure Turn from Right, No Wind, Heading $45^\circ$
Law 4: Proportional, Integral, Derivative Control

Baseleg approach an extreme case not normally used.

Figure 39 Position of Aircraft with gains, $K_p=11$, $K_i=0.1$, $K_d=137$, $K_w=None$, and Test Case Baseleg from Right, No Wind, Heading 90°

Figure 40 Localizer, Roll, and Roll Rate with gains, $K_p=11$, $K_i=0.1$, $K_d=137$, $K_w=None$, and Test Case Baseleg from Right, No Wind, Heading 90°
Law 4: Proportional, Integral, Derivative Control

This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

Figure 41 Position of Aircraft with gains, $K_p=11$, $K_i=0.1$, $K_d=137$, $K_w=None$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading 10°

Figure 42 Localizer, Roll, and Roll Rate with gains, $K_p=11$, $K_i=0.1$, $K_d=137$, $K_w=None$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading 10°
LAW 5: PROPORTIONAL CONTROL

Standard approach starting near the outer marker.

*Figure 43 Position of Aircraft with gains, $K_p=9.85, K_v=0, K_w=0, K_w=1.7$, and Test Case Standard, No Wind*

*Figure 44 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_v=0, K_w=0, K_w=1.7$, and Test Case Standard, No Wind*
Law 5: Proportional Control

Standard approach starting near the outer marker from the left with crosswind from the right.

Figure 45 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Right

Figure 46 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Right
**Law 5: Proportional Control**

Standard approach starting near the outer marker from the left with crosswind from the left.

---

**Figure 47** Position of Aircraft with gains, $K_p = 9.85, K_i = 0, K_v = 0, K_w = 1.7$, and Test Case Standard Approach from Left, with Crosswind from Left

---

**Figure 48** Localizer, Roll, and Roll Rate with gains, $K_p = 9.85, K_i = 0, K_v = 0, K_w = 1.7$, and Test Case Standard Approach from Left, with Crosswind from Left
Law 5: Proportional Control

Localizer capture after a procedure turn, the normal flight procedure.

![Position of Aircraft](image)

Figure 49 Position of Aircraft with gains, $K_p = 9.85, K_i = 0, K_d = 0$, and Test Case Procedure Turn from left, No Wind, Heading 45°

![Localizer Deviation](image)

![Roll](image)

![Roll Rate](image)

Figure 50 Localizer, Roll, and Roll Rate with gains, $K_p = 9.85, K_i = 0, K_d = 0$, and Test Case Procedure Turn from Left, No Wind, Heading 45°
Law 5: Proportional Control
Baseleg approach an extreme case not normally used.

![Graph of Aircraft Position](image)

*Figure 51 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$, and Test Case Baseleg from Left, No Wind, Heading 90°*

![Graphs of Localizer Deviation, Roll, and Roll Rate](image)

*Figure 52 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$, and Test Case Baseleg from left, No Wind, Heading 90°*
Law 5: Proportional Control

This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

![Graph of Position of Aircraft](image)

*Figure 53 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$*

![Graphs of Localizer Deviation, Roll, and Roll Rate](image)

*Figure 54 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=0, K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$*
LAW 6: PROPORTIONAL, INTEGRAL CONTROL

Standard approach starting near the outer marker.

Figure 55 Position of Aircraft with gains, \( K_p = 9.85, K_i = 0.4, K_d = 0, K_w = 1.7 \), and Test Case Standard, No Wind

Figure 56 Localizer, Roll, and Roll Rate with gains, \( K_p = 9.85, K_i = 0.4, K_d = 0, K_w = 1.7 \), and Test Case Standard, No Wind
Law 6: Proportional, Integral Control

Standard approach starting near the outer marker from the left with crosswind from the right

![Position of Aircraft](image)

*Figure 57 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Right*

![Localizer Deviation, Roll, and Roll Rate](image)

*Figure 58 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Right*
Law 6: Proportional, Integral Control
Standard approach starting near the outer marker from the left with crosswind from the left

![Position of Aircraft](image)

Figure 59 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_v=1.7$, and Test Case Standard Approach from Left, with Crosswind from Left

![Localizer Deviation](image)

Figure 60 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_v=1.7$, and Test Case Standard Approach from Left, with Crosswind from Left
**Law 6: Proportional, Integral Control**

Localizer capture after a procedure turn, the normal flight procedure.

![Position of Aircraft](image1)

*Figure 61 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_z=0, K_w=1.7$, and Test Case Procedure Turn from Left, No Wind, Heading 45°*

![Localizer Deviation](image2)

![Roll and Roll Rate](image3)

*Figure 62 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_z=0, K_w=1.7$, and Test Case Procedure Turn from Left, No Wind, Heading 45°*
Law 6: Proportional, Integral Control
Baseleg approach an extreme case not normally used.

![Figure 63 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_w=1.7$, and Test Case Baseleg from Left, No Wind, Heading 90°](image)

![Figure 64 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_w=1.7$, and Test Case Baseleg from left, No Wind, Heading 90°](image)
Law 6: Proportional, Integral Control

This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

Figure 65 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading 10°

Figure 66 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=0$, $K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading 10°
LAW 7: PROPORTIONAL, DERIVATIVE

Standard approach starting near the outer marker.

Figure 67 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Standard, No Wind

Figure 68 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Standard, No Wind
Law 7: Proportional, Derivative Control

Standard approach starting near the outer marker from the left with crosswind from the right.

Figure 69 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Right.

Figure 70 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Right.
Law 7: Proportional, Derivative Control

Standard approach starting near the outer marker from the left with crosswind from the left.

**Figure 71** Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Left.

**Figure 72** Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Standard Approach from Left, with Crosswind from Left.
Law 7: Proportional, Derivative Control

Localizer capture after a procedure turn, the normal flight procedure.

Figure 73 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Procedure Turn from Left, No Wind, Heading $45^\circ$

Figure 74 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Procedure Turn from Left, No Wind, Heading $45^\circ$
Law 7: Proportional, Derivative Control
Baseleg approach an extreme case not normally used.

Figure 75 Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Baseleg from Left, No Wind, Heading $90^\circ$

Figure 76 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Baseleg from Left, No Wind, Heading $90^\circ$
Law 7: Proportional, Derivative Control

This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

![Position of Aircraft](image)

*Figure 77* Position of Aircraft with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$

![Localizer Deviation](image)

*Figure 78* Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0, K_d=5, K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$
LAW 8: PROPORTIONAL, INTEGRAL, DERIVATIVE

Standard approach starting near the outer marker.

Figure 79 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=5, K_w=1.7$, and Test Case Standard, No Wind

Figure 80 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=5, K_w=1.7$, and Test Case Standard, No Wind
Law 8: Proportional, Integral, Derivative Control

Standard approach starting near the outer marker from the right with crosswind from the right

![Position of Aircraft Diagram](image)

**Figure 81** Position of Aircraft with gains, $K_p=9.85,K_i=0.4,K_d=5$, $K_w=1.7$, and Test Case Standard Approach from Right, with Crosswind from Right

![Localizer Deviation Diagram](image)

**Figure 82** Localizer, Roll, and Roll Rate with gains, $K_p=9.85,K_i=0.4,K_d=5$, $K_w=1.7$, and Test Case Standard Approach from Right, with Crosswind from Right
**Law 8: Proportional, Integral, Derivative Control**

Standard approach starting near the outer marker from the right with crosswind from the left.

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**Figure 83** Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_c=5, K_v=1.7$, and Test Case Standard Approach from Right, with Crosswind from Left

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**Figure 84** Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_c=5, K_v=1.7$, and Test Case Standard Approach from Right, with Crosswind from Left
Law 8: Proportional, Integral, Derivative Control

Localizer capture after a procedure turn, the normal flight procedure.

Figure 85 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=5, K_w=1.7$, and Test Case Procedure Turn from Right, No Wind, Heading $45^\circ$

Figure 86 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=5, K_w=1.7$, and Test Case Procedure Turn from Right, No Wind, Heading $45^\circ$
Law 8: Proportional, Integral, Derivative Control
Baseleg approach an extreme case not normally used.

Figure 87 Position of Aircraft with gains, $K_p = 9.85, K_i = 0.4, K_d = 5, K_w = 1.7$, and Test Case Baseleg from Right, No Wind, Heading 90°

Figure 88 Localizer, Roll, and Roll Rate with gains, $K_p = 9.85, K_i = 0.4, K_d = 5, K_w = 1.7$, and Test Case Baseleg from Right, No Wind, Heading 90°
**Law 8: Proportional, Integral, Derivative Control**

This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

![Position of Aircraft](image)

*Figure 89 Position of Aircraft with gains, $K_p=9.85, K_i=0.4, K_d=5, K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$*

![Figure 90](image)

*Figure 90 Localizer, Roll, and Roll Rate with gains, $K_p=9.85, K_i=0.4, K_d=5, K_w=1.7$, and Test Case Pegged at Middle Marker, Stress Case, No Wind, Heading $10^\circ$*
One of the main objectives was to meet the criteria of developing a lateral flight director that may aid while landing in a stress case, pegged at the middle marker, while also meeting the minimum criteria. The criteria includes:

- Does plane make it to centerline in time to land, within the 50-foot range?
- Is the overshoot less than 20%?
- Is the system too oscillatory?
- Is the system stable?

The following is a table of all the control laws that are compared for easy reference:

<table>
<thead>
<tr>
<th>Control Law</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOC Deviation</td>
<td>Heading</td>
</tr>
<tr>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>PI</td>
</tr>
<tr>
<td>3</td>
<td>PD</td>
</tr>
<tr>
<td>4</td>
<td>PID</td>
</tr>
<tr>
<td>5</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>PI</td>
</tr>
<tr>
<td>7</td>
<td>PD</td>
</tr>
<tr>
<td>8</td>
<td>PID</td>
</tr>
<tr>
<td>Current Law</td>
<td>P</td>
</tr>
</tbody>
</table>

*Table 4 Control Laws*

Summary of results are in the following table:

<table>
<thead>
<tr>
<th>Control Law</th>
<th>Standard</th>
<th>Standard Crosswind from left</th>
<th>Standard Crosswind from right</th>
<th>Procedure Turn</th>
<th>Baseleg No wind</th>
<th>Pegged at Middle Marker</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Unstable</td>
</tr>
<tr>
<td>2</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Failed</td>
<td>Unstable</td>
</tr>
<tr>
<td>3</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Failed, 32%</td>
<td>Failed ose.</td>
<td>Passed</td>
<td>Overshoot</td>
</tr>
<tr>
<td>4</td>
<td>Failed, 33%</td>
<td>Failed, 33%</td>
<td>Failed, 33%</td>
<td>Failed, 40%</td>
<td>Failed ose</td>
<td>Failed</td>
<td>Overshoot</td>
</tr>
<tr>
<td>5</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
</tr>
<tr>
<td>6</td>
<td>Failed 43%</td>
<td>Failed 43%</td>
<td>Failed 43%</td>
<td>Failed 34%</td>
<td>Failed 32%</td>
<td>Failed</td>
<td>Overshoot</td>
</tr>
<tr>
<td>7</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
</tr>
<tr>
<td>8</td>
<td>Failed 53%</td>
<td>Failed 53%</td>
<td>Failed 53%</td>
<td>Failed 40%</td>
<td>Failed 40%</td>
<td>Failed</td>
<td>Overshoot</td>
</tr>
<tr>
<td>current law</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Passed</td>
<td>Failed</td>
<td>Landing</td>
</tr>
</tbody>
</table>

*Table 5 Test Cases Results*
Roll angle feedback is required for all control laws. The roll angle control law designed, which is used for all tested control laws, is proportional. This is derived from the assumption that the pilot closes the inner loop. Supposing the pilot flies by referencing the flight director instrument, then the roll command will be proportional to roll error. The amount of roll rate activity on the plots indicates the pilot's workload in following the flight director.

Control laws 1, 2, 3, 4, 6, and 8 were rejected for several reasons. Control law 1 and 2 failed the initial design, because of the instability in the systems neither one could be fully designed. Control law 1 is neutrally stable with a damping ratio of zero and thus, the system oscillates continuously regardless of the gains used. Control law 2 is unstable for any gains as indicated in the root locus plots. Control law 3 initially passed the standard approach, but failed the procedure turn with an overshoot of 32%. Figure 95 is a magnified version of the localizer deviation. The overshoot is $0.8/2.5=32\%$. Adding an integrator on control laws 4, 6, and 8 added unnecessary overshoot. Control laws 4, 6, and 8 failed to meet the criteria for the stress case at the middle marker.

Based on the gains derived, only control law 5 and control law 7 pass all tests, while the current law fails. The test results of the current control law are listed in appendix A. The current law is smooth but very sluggish, and cannot recover if pegged at the middle marker. It failed the stressed case because at touchdown its heading is not parallel with the runway, which indicates that the plane is not line up with centerline and would land off the runway at an angle to the runway. The plane would be approximately 59 feet off centerline at touchdown, which exceeds the safe landing criteria. Control law 5 and 7 are almost identical in every way except for the pegged at the middle marker. They provide a smooth transition upon approach to the centerline. At the centerline, control law 5 doesn't reach steady state before touchdown, as opposed to control law 7, which gets there much quicker and stabilizes before touchdown. For safety reasons control law 7 should be chosen. If the pilot preferred not to do too much work then the current control law can meet his needs but if wind blows him off course then the pilot cannot recover the centerline alignment of his aircraft. Control law 7 would meet this need.
CONCLUSIONS

From the eight control laws tested, only control law 7 with feedback from roll, heading and localizer deviation met all criteria successfully. The goal of this study was to determine a system that is less complicated than the current system and successfully performs the middle marker stress test. Given this goal, gains were chosen to be static rather than dynamic which results in a simpler controller. In addition, using a heuristic method for designing the control laws, the gains were chosen based on analysis, but were not changed to be able to test for optimal control. The research was conducted primarily to test different control laws as opposed to finding the optimal one. Control law 7 is less complicated than the current law and this implies less hardware complexity. Even though control law 7 passed all tests presented in this paper, further tests and investigation should be done before installing the control law on a Boeing 757 aircraft. Other parameters that may be included in the future tests include noise in the localizer signal, turbulence, and variable winds.
APPENDIX A

CURRENT LAW

Standard approach starting near the outer marker.

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Figure 92 Current Law, Standard Approach from Right, and No Wind

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Figure 93 Current Law, Standard Approach from Right, and No Wind
Standard approach starting near the outer marker from the left with crosswind from the right.

Figure 94 Current Law, Standard Approach from Left, with Crosswind from Right

Figure 95 Current Law, Standard Approach from Left, with Crosswind from Right
Standard approach starting near the outer marker from the left with crosswind from the left.

Figure 96 Current Law, Standard Approach from Left, with Crosswind from Left

Figure 97 Current Law, Standard Approach from Left, with Crosswind from Left
Localizer capture after a procedure turn, the normal flight procedure.

Figure 98 Current Law, Procedure Turn from the Right, No Wind, Heading 45°

Figure 99 Current Law, Procedure Turn from the Right, No Wind, Heading 45°
Baseleg approach an extreme case not normally used.

Figure 100 Current Law, Baseleg from Right, No Wind, and Heading 90°

Figure 101 Current Law, Baseleg from Right, No Wind, and Heading 90°
This is the most severe case an aircraft would encounter, without having to abort the landing, localizer at limits near middle marker.

Figure 102 Current Law, Pegged at Middle Marker, Stress Case

Figure 103 Current Law, Pegged at Middle Marker, Stress Case
REFERENCES


**Design and Development of Lateral Flight Director**

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**NASA Langley Research Center**
Hampton, VA 23681-2199

**National Aeronautics and Space Administration**
Washington, DC 20546-0001

Part of the information presented herein was included in a project entitled "Design and Development of a Lateral Flight Director (For a Boeing-737 Commercial Aircraft)" submitted in fulfillment of the requirements for the degree of Master of Engineering, Old Dominion University, Norfolk, Virginia, July 1997.

The current control law used for the flight director in the Boeing 737 simulator is inadequate with large localizer deviations near the middle marker. Eight different control laws are investigated. A heuristic method is used to design control laws that meet specific performance criteria. The design of each is described in detail. Several tests were performed and compared with the current control law for the flight director. The goal was to design a control law for the flight director that can be used with large localizer deviations near the middle marker, which could be caused by winds or wake turbulence, without increasing its level of complexity.