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Parameterizations of Pion Energy Spectrum in Nucleon-Nucleon Collisions

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Abstract

The effects of pion ($\pi$) production are expected to play an important role in radiation exposures in the upper atmosphere or on the Martian surface. Nuclear databases for describing pion production are developed for radiation transport codes to support these studies. We analyze the secondary energy spectrum of pions produced in nucleon-nucleon (NN) collisions in the relativistic one-pion exchange model. Parametric formulas of the isospin cross sections for one-pion production channels are discussed and are used to renormalize the model spectrum. Energy spectra for the deuteron related channels ($NN \rightarrow d\pi$) are also described.

1. Introduction

A convenient representation of the differential cross section in energy of particles created in the interactions of space and atmospheric radiation with materials is required for radiation transport com-
reactions, an abundance of mesons is produced through the nuclear force. High-energy reactions increase in relative importance in the upper atmosphere due to the Earth's magnetic field, reducing lower energy ion components which have insufficient energy to overcome the threshold for meson production. The one-pion production channels dominate the meson production for energies up to about 1.5 GeV/amu and represent an important contribution at higher energies. The energy region below 1.5 GeV/amu extends above the galactic cosmic ray (GCR) peak at 0.2 to 0.6 GeV/amu. Also, most of the energy range of particles seen in solar particle events (SPE) are dominated by the one-pion exchange interactions. The scattering of high-energy protons or neutrons on hydrogen is important because protons and neutrons represent over 90 percent of the particle flux in materials and also because hydrogenous materials in tissue and material structures are important. The description of the one-pion production mechanism in nucleon-nucleon (NN) collisions is also needed for modeling the pion production mechanism in inclusive proton and neutron collisions on target atoms in applying reaction theory or in Monte Carlo simulations (refs. 4 and 5).

In this report, we describe the calculation of the secondary energy spectrum in proton-proton (pp) and neutron-proton (np) reactions. The one-pion production channels are modeled by using the relativis-
tic one-pion exchange (OPE) model. Parametric models for the isospin components in NN reactions and $\pi$N reactions are discussed. The one-pion channels discussed here can be appended with parameteriza-
tions of high-energy models (refs. 6 through 8) to provide the energy spectrum for inclusive pion pro-
duction in NN collisions at overall energies of interest for space radiation studies.

2. One-Pion Production Cross Sections

The cross sections for production of a single pion in nucleon-nucleon (NN) collisions may be written in terms of four independent cross sections by applying isospin conservation. For a transition from an initial isospin state $I_i$ to a final isospin state $I_f$ the cross section is denoted $\sigma_{I_i I_f}$. For formation of a deuteron $d$ in the final state, a superscript $d$ is used. In table 1 we list the isospin and masses of the

the various reaction channels in NN collisions. Various authors have considered parameterizations of these cross sections: $\sigma_{10}$, $\sigma_{01}$, $\sigma_{11}$, and $\sigma_{10}^d$. The fits of Wilson and Chun (1988) are useful because they extend over all energies. The work of VerWest and Arndt (ref. 9) is significant because they have
reconsidered discrepancies in older data sets, resulting in a large reduction in the isospin zero cross section \( \sigma_{01} \) over previous estimates and are in good agreement with the more recent measurements (ref. 9). The formula of VerWest and Arndt (ref. 9) for \( \sigma_{10}^d \) is (in units where \( \hbar = c = 1 \))

\[
\sigma_{10}^d(s) = \frac{\pi}{2p^2} \alpha \left( \frac{p_r}{p_o} \right)^\beta \frac{m_o^2 \Gamma^2}{(s_{\pi N} - m_o^2)^2 + m_o^2 \Gamma^2} 
\]

(2.1)

where

\[
p_r^2 = s/4 - m_N^2 \\
s = 4m_N^2 + 2m_NT_L \\
s_{\pi N} = (s - m_N)^2 \\
p_r^2(s) = \frac{[s - (m_d - m_\pi)^2][s - (m_d + m_\pi)^2]}{4s} \\
p_o^2(s) = s/4 - m_N^2 \\
s_o = (m_N + m_o)^2 
\]

with \( m_N \) the nucleon mass, \( m_N = 0.939 \text{ GeV} \), \( m_\pi \) the pion mass, and \( m_\pi = 0.138 \text{ GeV} \). The formula used in reference 9 for \( \sigma_{11f} \) is

\[
\sigma_{\pi}(s) = \frac{\pi}{2p^2} \alpha \left( \frac{p_r}{p_o} \right)^\beta \frac{m_o^2 \Gamma^2 (q^4 q_o^3)}{(s^* - m_o^2)^2 + m_o^2 \Gamma^2} 
\]

(2.2)

where \( s^* = \langle M \rangle^2 \),

\[
p_r^2(s) = \frac{[s - (m_N - \langle M \rangle)^2][s - (m_N + \langle M \rangle)^2]}{4s} \\
q_r^2(s^*) = \frac{[s^* - (m_N - m_\pi)^2][s^* - (m_N + m_\pi)^2]}{4s} \\
q_o = q \left( m_o^2 \right) \\
\langle M(s) \rangle = M_o + (\arctan Z_+ - \arctan Z_-)^{-1} \left( \frac{1 + Z_+^2}{1 + Z_-^2} \right) 
\]
where

\[ Z_+ = (\sqrt{s} - m_N - M_o)(2/\Gamma_o) \]
\[ Z_- = (m_N + m_\pi - M_o)(2/\Gamma_o) \]

with \( M_o = 1.22 \text{ GeV} \) and \( \Gamma_o = 0.12 \text{ GeV} \). The parameters of the model are listed in table 3. The formulas of equations (2.1) and (2.2) are valid only for \( T_L < 1.5 \text{ GeV} \).

Wilson and Chun have considered the following parameterizations of the isospin cross sections for all energies:

\[
\sigma_{10}(d) = \begin{cases} 
2.563 e^{-0.435T_L} - 17.47 e^{-6.044T_L} & (T_L \leq 0.6 \text{ GeV}) \\
7.531 e^{2.82T_L} + 44.8 e^{-5.69T_L} & (0.6 < T_L \leq 1.3 \text{ GeV}) \\
0.22 e^{0.0885T_L} - 1.96 e^{-1.754T_L} & (1.3 \leq T_L) 
\end{cases} 
\tag{2.3}
\]

\[
\sigma_{10}(np) = \frac{36}{T_L^{1.2}} \left[ 1 - e^{-0.7(T_L - T_{th})^{1.6}} \right] 
\tag{2.4}
\]

\[
\sigma_{01} = \frac{7.2}{T_L^{1.1}} \left[ 1 - e^{-1.1(T_L - T_{th})^{1.4}} \right] \left[ 1 - e^{-2(T_L - T_{th})^{2}} \right] 
\tag{2.5}
\]

\[
\sigma_{11} = \frac{5}{T_L^{0.522}} \left[ 1 - e^{-3.75(T_L - T_{th})^{2}} \right] 
\tag{2.6}
\]

where \( T_{th} \) is the single-pion production threshold and \( T_L \) is the laboratory nucleon energy.

Comparisons of the fits described above are shown in figure 1. Large discrepancies exist, especially for \( \sigma_{01} \) and \( \sigma_{10} \). Because the model of VerWest and Arndt (ref. 9) is more accurate at lower energies, we will use this model below 1.3 GeV. The resulting fit is shown in figure 2, and comparisons to data (ref. 10) for \( \pi^+ \) and \( \pi^0 \) production in proton-proton (pp) collisions are shown in figure 3. The result of joining equations (2.1) and (2.2) with equations (2.3) to (2.6) will represent the one-pion production data quite accurately over the energy range of interest for radiation transport codes and will be used to renormalize the model spectra described in the next section.

### 3. Pion Energy Spectrum

The deuteron production channels are two-body final states and are therefore much easier to parametrize than the other pion production channels. By isospin conservation (neglecting Coulomb effects), we have the relationship

\[ \sigma(pn \rightarrow \pi^0 d) = \frac{1}{2} \sigma(pp \rightarrow \pi^+ d) = \frac{1}{2} \sigma_{10}^{d} \]  
\tag{3.1}
The angular distribution is parameterized as

\[
\frac{d\sigma_{10}^d}{d\Omega} = \sigma_{10}^d N_{10}^d \left( A_{10}^d + \cos^2 \theta - B_{10}^d \cos^4 \theta \right)
\]  

(3.2)

where \( N_{10}^d \) is a normalization constant and \( \theta \) is the cm scattering angle. The energy dependent parameters \( A_{10}^d \) and \( B_{10}^d \) are

\[
A_{10}^d = 0.27 \left[ 1 + 0.13 \cos \left( \frac{T_L}{0.182} \right) \right]
\]  

(3.3)

and

\[
B_{10}^d = \begin{cases} 
0 & (T_L < 0.4 \text{ GeV}) \\
0.6 \left[ 1 - \exp \left( \frac{-\left( T_L + 0.4 \right)}{0.3} \right) \right] & (T_L \geq 0.4 \text{ GeV})
\end{cases}
\]  

(3.4)

Comparisons of equations (3.1) to (3.4) to experimental data (refs. 11 and 12) are shown in figure 4(a) for the \( \pi^+ d \to pp \) reaction and in figure 4(b) for the \( pp \to \pi^+ d \) reaction.

The energy distribution in the laboratory frame of the final deuteron is related to the center of mass (c.m.) angular distribution by

\[
\frac{d\sigma}{dT_d} = \frac{2\pi s}{m_N^2 \lambda(s, m_N^2, m_N^2) \lambda(s, m_d^2, m^2_\pi)} \frac{d\sigma_{10}^d}{d\Omega}
\]  

where the function \( \lambda \) is defined

\[
\lambda(s, A, B) = (s - A - B)^2 - 4AB
\]  

(3.6)

and \( s \) is the Mandelstam variable given by \( s = \left( p_{N_1} + p_{N_2} \right)^2 \).

In applying equation (3.5), we use

\[
\cos \theta = \frac{m_d^2 - m_N^2 - 2m_Nm_d + 2E_\pi E_p - 2m_N T_d}{2p_p p_\pi}
\]  

(3.7)
where barred quantities are the c.m. values given by

\[ E_\pi = \frac{s + m_d^2 - m_{\pi}^2}{2/s} \]  

\[ \bar{E}_d = \frac{s + m_{\pi}^2 - m_d^2}{2/s} \]  

\[ E_p = \frac{s}{2} \]

The kinematical limits on the kinetic energy of the deuteron \( T_d \) are found from equation (3.7) by observing that \(|\cos \theta| \leq 1\).

The energy distribution in the laboratory reference frame for the pion is

\[ \frac{d\sigma}{dT_\pi} = \frac{2\pi s}{m_N \lambda(s, m_{\pi}^2, m_N^2) \lambda(s, m_d^2, m_{\pi}^2)} \frac{d\sigma_{10}^d}{d\Omega} \]

with

\[ \cos \theta = \frac{m_{\pi}^2 - m_d^2 - 2m_Nm_{\pi} + 2E_dE_\pi - 2m_NT_\pi}{2p_p p_d} \]

and a similar expression is found for the deuteron spectrum. Calculations of energy spectra of secondary pions and deuterons for several beam energies are shown in figures 5 and 6, respectively.

The NN \( \rightarrow \) NN\(\pi\) reactions are assumed to proceed through the formation and decay of the \( \Delta \) resonance. The mechanism is described by figure 7(a). The \( \Delta \) forms an isospin quartet \((I = 3/2)\), and it is useful to consider the coupling of the \( \pi N \) system wave functions in isospin space to understand the \( \Delta \) decay properties. Denoting the nucleon isospin by \( I_N \) and the pions by \( I_\pi \), we have

\[ |N\pi\rangle = \sum_{I, I_\pi} |II_\pi\rangle \langle II_\pi | I_N I_{N'\pi} I_\pi I_{\pi'\pi} \rangle \]

Equation (3.13) is used to obtain the components of the \( \Delta \) wave function with the result

\[ |\Delta^{++}\rangle = |\pi^+ p\rangle \]

\[ |\Delta^+\rangle = \frac{1}{\sqrt{3}} [ |\pi^+ n\rangle + \sqrt{2} |\pi^0 p\rangle ] \]
Branching ratios for the formation of various pion species in NN collisions are obtained by using equations (3.14).

The invariant differential cross section for NN → NΔ has been evaluated in the relativistic one-pion exchange (OPE) model. We use this model and the assumption of isotropic decay of the Δ in its rest frame to obtain the momentum distribution and energy spectrum of pions and nucleons in the NN → NNπ channels. The Mandelstam variables for the reaction are defined in terms of the four momentum vectors of the various particles defined in figure 7 and given by

\[ s = (K_1 + K_2)^2 \]  
\[ t = (K_1 - K_3)^2 \]  
\[ u = (K_2 - K_3)^2 \]

The cross-section distribution in \( t \) is written in terms of the matrix element \( M \) as

\[ \frac{d\sigma}{dt} = \frac{1}{64\pi} |M|^2 |t|^\frac{1}{2} \]

where

\[ I_F = \sqrt{(K_1K_2)^2 - m_N^2} \]

The matrix element \( M \) is decomposed as

\[ |M|^2 = |M_{\text{dir}}|^2 + |M_{\text{ex}}|^2 + |M_{\text{INT}}|^2 \]

The direct amplitude is given by (ref. 13)

\[ |M_{\text{dir}}|^2 = \left( \frac{f_\pi f_\pi^*}{m_\pi} \right)^2 F^4(t) \frac{t[t-(m_\Delta-m_N)^2][(m_\Delta+m_N)^2-t]^2}{(t-m_\pi^2)^2 3m_\Delta^2} \]

where \( f_\pi \) and \( f_\pi^* \) are the coupling constants with values 1.008 and 2.202, respectively, and \( m_\pi \) and \( m_\Delta \) are the pion-fixed Δ mass with values 0.139 GeV and 1.232 GeV, respectively. In equation (3.21), \( F(t) \) is the form factor for the off-shell meson which is parameterized as
\[ F(t) = \frac{\Lambda^2 - m^2}{\Lambda^2 - t} \] (3.22)

where the value of the parameter \(\Lambda\) will be discussed below. The exchange term in equation (3.20) is equivalent in form to equation (3.21) with the replacement \((u \leftrightarrow t)\).

The interference term is given by

\[
\frac{1}{4} |M_{\text{INT}}|^2 = \left( \frac{f_\pi f'_\pi}{m_\pi} \right) \left( \frac{F^2(t)F^2(u)}{(t-m_\pi^2)(u-m_\pi^2)} \right) \frac{1}{6m_\Delta^2} \\
\times \left\{ tu + \left( m^2_\Delta - m^2_N \right)(t + u) - \left( m^4_\Delta + m^4_N \right) \right\} \left\{ tu + m_\Delta(m_\Delta + m_N)\left( m^2_\Delta - m^2_N \right) \right\} \\
+ \left\{ tu - \left( m^2_\Delta - m^2_N \right)(t + u) + (m_N + m_\Delta)^4 \right\} \left\{ tu - m_\Delta(m_\Delta - m_N)\left( m^2_\Delta - m^2_N \right) \right\} \\
\] (3.23)

In order to account for the finite width of the \(\Delta\) resonance, a mass distribution \(\rho(\mu^2)\) is introduced through

\[
\frac{d\sigma}{dt \ d\mu^2} = A(s, t, u)\rho(\mu^2) \tag{3.24}
\]

where \(\mu^2 = (K_4 + K_\pi^2)^2\) and \(A(s, t, u)\) is the invariant cross section of equation (3.18) with the fixed \(\Delta\) mass \(m_\Delta\) replaced by \(\mu\). The mass distribution is parameterized in terms of the elastic pion-nucleon cross section and the \(\Delta\) width as (ref. 13)

\[
\rho(\mu^2) = \frac{K_\Delta^2 \sigma_{\pi N}}{8\pi^2 m_\Delta \Gamma} \tag{3.25}
\]

where \(K_\Delta\) is given by

\[
K_\Delta(\mu^2, m_\pi^2) = \sqrt{\frac{\mu^2 + m_N^2 - m_\pi^2}{4\mu^2 - m_N^2}} \quad (3.26)
\]

The width is parameterized as

\[
\Gamma = \Gamma_o \left| \frac{K_\Delta(\mu^2, m_\pi^2)}{K_\Delta(m_\Delta^2, m_\pi^2)} \right|^3 Z(\mu^2, m_\pi^2) \tag{3.27}
\]
with the form factor \( Z \) given by

\[
Z(\mu^2, m^2_\pi) = \frac{K^2_\Delta(m^2_\Delta, m^2_\pi) + \kappa^2}{K^2_\Delta(\mu^2, m^2_\pi) + \kappa^2}
\] (3.28)

and parameter values \( \Gamma_o = 0.12 \text{ GeV} \), and \( \kappa = 0.2 \text{ GeV} \). By neglecting Coulomb effects, the shape of the secondary energy spectrum for the components of the isospin triplet of pions is largely determined by the \( \pi N \) cross section in equation (3.25), with the remaining kinematic factors in equations (3.18) to (3.28) being identical for each pion species. The description of the \( \pi N \) cross sections is given in the appendix.

The decay of the \( \Delta \) (\( \Delta \to N\pi \)) is assumed to be isotropic in the \( \Delta \) rest frame such that the pion distribution from the decay is

\[
\frac{d\sigma}{d\mu^2 d\Omega^*_\pi} = \frac{1}{4\pi} A(s, t, u)\rho(\mu^2)
\] (3.29)

and the nucleon distribution from the decay is

\[
\frac{d\sigma}{d\mu^2 d\Omega_N^*} = \frac{1}{4\pi} A(s, t, u)\rho(\mu^2)
\] (3.30)

where starred quantities are in the \( \Delta \) rest frame. A second contribution to the nucleon spectrum comes from the nondecaying nucleon line in figure 7, as discussed below. The \( \Delta \) mass is found as

\[
\mu^2 = \left( E^*_4 + E^*_\pi \right)^2
\] (3.31)

and using

\[
d\mu^2 = \frac{\partial\mu^2}{\partial E^*_\pi} dE^*_\pi
\] (3.32)

allows the invariant momentum distribution of the pion to be written

\[
E^*_\pi \frac{d\sigma}{dp^*_\pi} = \frac{1}{4\pi^2} \int d\Omega_3 \left( \frac{p_1 p_3}{p^*_\pi} \right) \frac{\partial\mu^2}{\partial E^*_\pi} A(s, t, u)\rho(\mu^2)
\] (3.33)
with a similar expression for the decay nucleon spectrum. The energy spectrum of the pion in the laboratory system is then

\[
\frac{d\sigma}{dE_\pi} = \frac{1}{4\pi^2} \int d\Omega_3 d\Omega_\pi p_\pi \left( \frac{p_1 p_3}{p_\pi^2} \right) \frac{\partial \mu^2}{\partial E_\pi} A(s, t, u) \rho(\mu^2)
\]  

(3.34)

The laboratory energy spectrum of the decay nucleon is

\[
\frac{d\sigma^D}{dE_N} = \frac{1}{4\pi^2} \int d\Omega_3 d\Omega_N p_N \left( \frac{p_1 p_3}{p_N^2} \right) \frac{\partial \mu^2}{\partial E_N} A(s, t, u) \rho(\mu^2)
\]  

(3.35)

The energy spectrum of the recoil nucleon is given by (ref. 14)

\[
\frac{d\sigma^R}{dE_N} = \int_{0}^{\theta_{\text{max}}} d(\cos \theta_N) \frac{m_N^2 \mu^2}{\pi m_\Delta^2} \frac{p_N}{p_o} A(s, t, u) \rho(\mu^2)
\]  

(3.36)

where \( p_o \) is the laboratory momentum of the incident nucleon.

A comparison of equation (3.24) to experimental data for the \( pp \rightarrow n\Delta^+ \) reaction (refs. 10 and 15) is shown in figure 8, with good agreement found. The parameter \( \Lambda \) in the form factor of equation (3.22) has a strong effect on the calculations. As noted by Jain and Santra (ref. 14), the inclusion of distorted waves effects, the value of \( \Lambda \) chosen, and a value of \(-1\) GeV provides the best fit to data. Calculations of energy spectrum for \( \pi^+ \) and \( \pi^0 \) production in pp collisions and \( \pi^+, \pi^0, \pi^- \) production in neutron-proton (np) collisions are shown in figures 9 and 10, respectively. The variations in the \( \Delta \rightarrow \pi N \) decay vertices due to the isospin dependence provide only a modest change in shape between the various production channels. Coulomb effects which are not treated herein will provide further dependence on the pion charge.

4. Inclusive Pion Production Spectrum

As the kinetic energy of nucleons increases, the threshold for \( 2, 3 \ldots \) pion production is reached and so is production of heavier mesons such as the kaon. The threshold energies for several production processes are listed in table 4. The production threshold is also dependent on charge conservation when individual species of mesons in the reaction are produced. Inclusive meson production data have been parameterized in convenient forms by several authors (refs. 6 through 8). The spectrum in one-pion, two-pion, and other channels will be somewhat distinct due to the mechanisms involved. The two-pion production channels have been considered by Sternheimer and Lihdenbaum (ref. 16) using a purely kinematic form of the isobar model. There are two distinct mechanisms for two-pion production which are through the excitation of two \( \Delta \)'s:

\[
N + N \rightarrow \Delta_1 + \Delta_2 \rightarrow N + N + \pi + \pi
\]  

(4.1)

or through the excitation of higher mass nucleon resonances:

\[
N + N \rightarrow N + N^* \rightarrow N + N + \pi + \pi
\]  

(4.2)

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where $N^*$ is a nucleon resonance of higher mass than the $\Delta$, which is assumed to decay through the emission of two pions. The mechanisms of equations (4.1) and (4.2) are illustrated in figures 7(b) and 7(c), respectively. The mechanism of equation (4.2) will have a cross-section structure similar to the one-pion model, $NN \rightarrow NN\pi$, described above. However, with $\Delta \rightarrow N\pi$, the vertex is replaced by the $N^* \rightarrow N\pi\pi$ vertex. The mechanism of equation (4.3) will contain the mass distribution of two $\Delta$'s with a structure such as

$$
\frac{d\sigma}{dt\, d\mu_1^2\, d\mu_2^2} = A(s, t, u)\rho_1\left[\frac{\mu_1^2}{\rho_2}\right]\rho_2\left[\frac{\mu_2^2}{\rho_2}\right]
$$

(4.3)

thus requiring one additional numerical integral to obtain the pion energy spectrum.

The inclusive pion spectrum can be represented as a sum over the spectrum for each multiplicity at pion as

$$
\frac{d\sigma}{dE_\pi} = \frac{d\sigma}{dE_{\pi_1}} + \frac{d\sigma}{dE_{\pi_2}} + \ldots
$$

(4.4)

where $\frac{d\sigma}{dE_{\pi_1}}$ represents the spectrum described by equation (3.34). The inclusive spectrum can be represented by the one-pion channel plus the high-energy model of Schneider, Norbury, and Cucinotta (ref. 7) renormalized to exclude the one-pion contributions, or alternatively, by including the two-pion production channels separately, as described previously.

5. Concluding Remarks

One-pion production channels will dominate pion production for a significant fraction of galactic cosmic ray (GCR) and solar particle event (SPE) exposures in free space, in the upper atmosphere, or on the Martian surface. In this report, the one-pion production cross sections were discussed, and a convenient formula for their numerical representation was found. These models will be appended with high-energy models to span all energies of importance in GCR studies. The formula described here also can be used to model pion and nucleon production spectra in nucleon-nucleus and nucleus-nucleus reactions.

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Appendix

Pion-Nucleon Scattering

The $\pi N$ scattering amplitude is written to represent the pion isospin $I_\pi$ and nucleon isospin $\frac{1}{2}\tau_N$ as

$$f_{\pi N} = \frac{1}{3}(f_{1/2} + 2f_{3/2}) + \frac{1}{3}(f_{3/2} - f_{1/2})I_\pi \cdot \tau_N \quad (A1)$$

where $f_{1/2}$ and $f_{3/2}$ are amplitudes for total isospin 1/2 or 3/2, respectively. Introducing the total isospin of the $\pi N$ system,

$$T = I_\pi + \frac{1}{2}\tau_N \quad (A2)$$

and $\pi N$ wave functions

$$|TT_Z\rangle = \sum_{I_Z} \langle I_I^I_I^F_z N^Z \tau_z | TT_Z\rangle | I_I^I_I^F_z N^Z \tau_z \rangle \quad (A3)$$

and noting

$$I_\pi \cdot \tau_N = \left( T^2 - \frac{11}{4} \right) \quad (A4)$$

leads to the following relations for elastic scattering:

$$\langle \pi^+ p \mid f_{\pi N} \mid \pi^+ p \rangle = f_{3/2} \quad (A5)$$

$$\langle \pi^0 p \mid f_{\pi N} \mid \pi^0 p \rangle = \frac{1}{3}(f_{1/2} + 2f_{3/2}) \quad (A6)$$

$$\langle \pi^- p \mid f_{\pi N} \mid \pi^- p \rangle = \frac{1}{3}(f_{3/2} + 2f_{1/2}) \quad (A7)$$

$$\langle \pi^+ n \mid f_{\pi N} \mid \pi^+ n \rangle = \frac{1}{3}(f_{3/2} + 2f_{1/2}) \quad (A8)$$

$$\langle \pi^0 n \mid f_{\pi N} \mid \pi^0 n \rangle = \frac{1}{3}(f_{1/2} + 2f_{3/2}) \quad (A9)$$

$$\langle \pi^- n \mid f_{\pi N} \mid \pi^- n \rangle = f_{3/2} \quad (A10)$$

with similar relations found for charge-exchange matrix elements.
The total cross sections can be found from the optical theorem. Numerous measurements exist for the total $\pi^+ p$ and $\pi^- p$ reactions. From equations (A5) to (A10), we can find solutions for the other $\pi N$ collision pairs (neglecting Coulomb effects) by using equations (A11) through (A14).

\[
\sigma_{\pi^+ p}^T = \frac{1}{2} \sigma_{\pi_n}^T + \sigma_{\pi_n}^T - \sigma_{\pi_n}^{EX}(\pi^- p) \tag{A11}
\]

\[
\sigma_{\pi_n}^T = \sigma_{\pi_n}^T \tag{A12}
\]

\[
\sigma_{\pi_n}^T = \sigma_{\pi_n}^T \tag{A13}
\]

\[
\sigma_{\pi_n}^T = \sigma_{\pi_n}^T \tag{A14}
\]

12
References


Table 1. Isospins and Masses of Particles

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<thead>
<tr>
<th>Particle</th>
<th>Isospin</th>
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<tr>
<td>$p$</td>
<td>1/2</td>
<td>1/2</td>
<td>0.9383</td>
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<tr>
<td>$n$</td>
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<td>-1/2</td>
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<td>$\pi^+$</td>
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<td>1.1396</td>
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<td>$\pi^0$</td>
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<td>0</td>
<td>1.1350</td>
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<tr>
<td>$\pi^-$</td>
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<td>-1</td>
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<td>$\Delta^+$</td>
<td>3/2</td>
<td>3/2</td>
<td>1.232</td>
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<td>1/2</td>
<td>1.232</td>
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<td>1.232</td>
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<tr>
<td>$\Delta^+$</td>
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<td>$d$</td>
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Table 2. Isospin Components of One-Pion Production Channels in Nucleon-Nucleon (NN) Collisions

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<tr>
<th>Channel</th>
<th>Isospin cross section $\sigma_{1/2,\ell}$</th>
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<tr>
<td>$pp \rightarrow \pi^+ d$</td>
<td>$\sigma^d_{10}$</td>
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<tr>
<td>$pp \rightarrow pp\pi^0$</td>
<td>$\sigma_{11}$</td>
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<tr>
<td>$pp \rightarrow pn\pi^+$</td>
<td>$\sigma_{11} + \sigma_{10}$</td>
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<tr>
<td>$np \rightarrow \pi^0 d$</td>
<td>$\frac{1}{2}\sigma^d_{10}$</td>
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<tr>
<td>$np \rightarrow np\pi^0$</td>
<td>$\frac{1}{2}(\sigma_{10} + \sigma_{01})$</td>
</tr>
<tr>
<td>$np \rightarrow nn\pi^+$</td>
<td>$\frac{1}{2}(\sigma_{11} + \sigma_{01})$</td>
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<tr>
<td>$np \rightarrow pp\pi^-$</td>
<td>$\frac{1}{2}(\sigma_{11} + \sigma_{01})$</td>
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Table 3. Isospin Cross-Section Parameters

[From ref. 9]

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<tr>
<th>Parameter</th>
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<td>$\alpha$</td>
<td>6.030</td>
<td>3.772</td>
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<td>$\beta$</td>
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<td>$m_{\pi}$, MeV</td>
<td>1203</td>
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<td>$\Gamma$, MeV</td>
<td>134.3</td>
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Table 4. Meson Production Thresholds

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<th>Channel</th>
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<td>$NN \rightarrow NN\pi$</td>
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<td>$NN \rightarrow NN\pi\pi$</td>
<td>0.602</td>
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<td>0.934</td>
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<tr>
<td>$NN \rightarrow K\Lambda$</td>
<td>1.12</td>
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Figure 1. Comparison of isotopic cross sections $\sigma$ for one-pion production versus laboratory kinetic energy of incident nucleon.

Figure 2. Comparison of isotopic cross sections $\sigma$ for one-pion production versus laboratory kinetic energy of incident nucleon using connection of low- and high-energy fit equations.
Laboratory kinetic energy, $T_{\text{lab}}$ GeV versus collisions versus laboratory nucleon kinetic energy.
Figure 4. Comparisons of model fits to experimental data (ref. 11) for angular distributions; $T_\pi$ is kinetic energy of pion; $T_p$ is kinetic energy of proton.
Figure 5. Secondary $\pi^+$ spectrum in $p + p \rightarrow \pi^+ + d$ reactions for several proton beams.

Figure 6. Secondary deuteron $d$ spectrum in $p + p \rightarrow \pi^+ + d$ reactions for several proton beams.
Figure 7. Diagrams for one-pion production through $\Delta$ or nucleon resonance $N^*$ formation and decay in nucleon-nucleon (NN) collisions.
Figure 8. Comparison of model to experiments (refs. 10 and 15) for invariant distribution in $pp \rightarrow n\Delta^{++}$ reactions.
(a) $p + p \rightarrow \pi^+ + n + p$.

(b) $p + p \rightarrow \pi^0 + p + p$.

beam energies.
Figure 10. Calculations of pion (one-pion contribution) energy spectrum in neutron-proton (np) collisions at several beam energies.
Figure 10. Concluded.

(c) $n + p \rightarrow \pi^- + p + p$.
## Parameterizations of Pion Energy Spectrum in Nucleon-Nucleon Collisions

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National Aeronautics and Space Administration
Washington, DC 20546-0001

The effects of pion (π) production are expected to play an important role in radiation exposures in the upper atmosphere or on the Martian surface. Nuclear databases for describing pion production are developed for radiation transport codes to support these studies. We analyze the secondary energy spectrum of pions produced in nucleon-nucleon (NN) collisions in the relativistic one-pion exchange model. Parametric formulas of the isospin cross sections for one-pion production channels are discussed and are used to renormalize the model spectrum. Energy spectra for the deuteron related channels (NN → dπ) are also described.

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