A Preliminary Model for Spacecraft Propulsion Performance Analysis based on Nuclear Gain

and Subsystem Mass-Power Balances

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Space Flight Requirements

- Omniplanetary space flight requires new high-performance propulsion systems based on nuclear energy.

- Over the last several decades, many propulsion concepts have discussed one-month missions to Mars and one-year missions to the outer planets.

- Such missions entail large mission velocities and vehicle accelerations, which in turn require both high exhaust velocities (and therefore, specific impulses) and extremely low mass-power ratios, e.g.:

\[
I_{sp} \geq 10^4 \text{ to } 10^5 \text{ sec} \quad \alpha \leq 10^{-2} \text{ kg/kW}
\]
Spacecraft Energy “Gain”

- High performance electric propulsion appears capable of enabling multi-month transits to Mars and the near-earth asteroids; however, the mass-power ratio ($\alpha$) of these “power-limited” systems appears too high to achieve large accelerations for outer planet missions.

- Higher accelerations demand energy “gains” from nuclear reactions in the propellant.

- Such energy “gains” must account for power required to “drive” nuclear reactions — this type of system is “gain-limited” in that driver power can be a significant fraction of total power produced.
The concept of energy “gain” for propulsion systems implies that an approach analogous to control theory may be useful in evaluating the performance of such systems — in effect, treating propulsion systems as “power circuits”.

First, derive expressions for mission trip time and distance as functions of parameters including (required) $I_{sp}$ and $\alpha$.

Next, derive expressions for $\alpha$ for both power- and gain-limited systems.

Last, connect the mission relations to the power systems relations.
Mission Assumptions

- Treat $I_{sp}$ and $\alpha$ as independent parameters that characterize a propulsion system
  - Treat vehicle acceleration as a parameter dependent upon $I_{sp}$ and $\alpha$.

- Assume round-trip mission between points A and B
  - Goal is to minimize trip time $\tau_{RT}$ for the distance $D_{AB}$
  - Assume accelerations far greater than local acceleration of the sun
  - Assume constant thrust accelerations
  - Assume zero velocity at points A and B
    - These points permit assumption of a straight-line trajectory where $D_{AB} = D_{BA}$
- If we begin with $\Delta \tau = \frac{m_{\text{propellant expended}}}{m}$, we can obtain trip times in each direction:

$$\tau_{AB} = \frac{gI_{sp}}{T/m_{A2}} \frac{m_{A2}}{m_B} \left( \frac{m_B}{m_{A1}} - 1 \right)$$

$$\tau_{BA} = \frac{gI_{sp}}{T/m_{A2}} \left( \frac{m_{A2}}{m_B} - 1 \right)$$

- Using $D_{if} = \frac{1}{\bar{m}} \int_{m_i}^{m_f} V dm$, we can obtain the distance in each direction:

$$D_{AB} = \frac{(gI_{sp})^2}{T/m_{A2}} \frac{m_{A2}}{m_B} \left( \sqrt{\frac{m_B}{m_{A1}}} - 1 \right)^2$$

$$D_{BA} = \frac{(gI_{sp})^2}{T/m_{A2}} \left( \sqrt{\frac{m_{A2}}{m_B}} - 1 \right)^2$$
Trip Times = f(D_{AB}, I_{sp}, T/m_{A2})

- With straight-line trajectories, $D_{AB} = D_{BA}$, and the mass ratios can be eliminated to yield both round-trip and one-way trip times as functions of $I_{sp}$ and $D_{AB}$:

$$\tau = \frac{D_{AB}}{gI_{sp}} \cdot (h + kU)$$

where $(h,k) = \begin{cases} (4,4) & \text{for Round Trip} \\ (3,2) & \text{for One Way} \end{cases}$ and $U = \frac{gI_{sp}}{\sqrt{(T/m_{A2})D_{AB}}}$
Vehicle acceleration, $T/m_{A2}$

- The acceleration $T/m_{A2}$ is related to the system mass-power ratio $\alpha$. We can use the following expression for final (burnout) mass $m_{A2}$, together with relations for power output and propellant mass:

$$m_{A2} = m_{pay} + \alpha \cdot P_{out} + \beta \cdot m_{prop}$$

$$m_{pay} = m_{A2} \lambda_{pay} ; P_{out} = TV_e / 2 ; m_{prop} = T\tau / V_e$$

- Substitution enables solving for the acceleration in terms of $\gamma$:

$$\frac{1}{T/m_{A2}} = \frac{1}{1 - \lambda_{pay}} \left( \alpha \frac{gI_{sp}}{2} + \beta \frac{\tau}{gI_{sp}} \right)$$
Trip Times = f(I_{sp}, D_{AB}, \alpha)

This leads to a generalized expression relating trip time and \( I_{sp} \):

\[
\tau = \frac{1}{I_{sp}} \cdot \left[ X \pm \sqrt{Y + Z \cdot I_{sp}^3} \right]
\]

\[
X =\left( \frac{D_{AB}}{2g} \right) \left( 2h + k^2 \frac{\beta}{1 - \lambda_{pay}} \right)
\]

\[
Y =\left( \frac{kD_{AB}}{2g} \right)^2 \left[ 4h + k^2 \frac{\beta}{1 - \lambda_{pay}} \right] \frac{\beta}{1 - \lambda_{pay}}
\]

\[
Z = \frac{k^2 \frac{gD_{AB}}{2}}{1 - \lambda_{pay}} \alpha
\]
An optimal $\tau$ is obtained by taking the derivative with respect to $I_{sp}$ and solving:

$$\left[\left(I_{sp}\right)_{OPT}\right]^3 = \frac{1}{Z} t_{OPT}^3$$

$$t_{OPT}^3(X,Y) = \left(\frac{2X}{9}\right) \left[\left(X - \frac{3Y}{X}\right) + \sqrt{X^2 + 3Y}\right]$$

Substitution of this optimized $I_{sp}$ yields the optimal trip time:

$$\tau_{OPT} = \frac{Z^{1/3}}{t_{OPT}} \left[X + \sqrt{Y + t_{OPT}^3}\right]$$

Note that $Z$ is proportional to $\alpha$: this means that $\tau_{OPT}$ varies as $\alpha^{1/3}$. 
Minimizing Trip Times

![Graph showing the relationship between distance from Earth (AU) and round trip time (days). The graph includes lines for different values of specific impulse (Isp) and mass ratio (α) in kilogram per kilowatt (kg/kW).]
Fast missions require low \( \alpha \), high \( I_{sp} \)

- The previous two figures emphasize that fast missions to the outer planets necessitate very low \( \alpha \) and very high \( I_{sp} \) values:
  - One year to Jupiter: \( \alpha \sim 10^{-1} \text{ kg/kW}; I_{sp} \sim 70000 \text{ sec} \)
  - One year to Pluto: \( \alpha \sim 10^{-3} \text{ kg/kW}; I_{sp} \sim 300000 \text{ sec} \)

- These values are beyond the limits of power-limited systems — including even high-performance electric propulsion

- Instead, consider the influence of gain-limited systems, with spacecraft gain illustrated by the following power system schematic (or “power circuit”) ...
Power System Schematic

Waste Heat Disposal ($h$)

Supply ($s$)

Driver ($d$)

Gain

Power Processing ($p$)

Thruster ($t$)

\[
\frac{1 - \eta_d}{\eta_d} (1 - e)
\]

\[
(1 - e)
\]

\[
\frac{P_{out}}{P_{in}} = (1 - f) \eta_h \eta_p G
\]

\[
(1 - f) \eta_p G
\]

\[
f \eta_p G
\]
Power flows as fractions of $P_{in}$

- The input power to the nuclear process — $P_{in}$ — is obtained from two sources:
  - Fractional power from an onboard source: $e$
  - Fractional power from a driver powered from system: $1 - e$
    - If $e = 1$ — solely power-limited
    - If $e = 0$ — solely gain-limited

- The power flows in the schematic are represented as fractions of this input power:
  - Fraction of power needed to power driver: $f$
  - Subsystem mass-power ratios: $\hat{\alpha}_D, \hat{\alpha}_P, \hat{\alpha}_T, \hat{\alpha}_S, \hat{\alpha}_H$
  - Subsystem component efficiencies (always < 1): $\eta_D, \eta_P, \eta_T$
Conservation of Mass/Power

- Using a similar equation for conservation of mass as used previously, we can substitute the mass-power ratio $\alpha$ multiplied by the fractional power to yield the mass for each subsystem.

  Example: [mass of power supply subsystem]: $m_S = (\hat{\alpha}_S)(e)P_{in}$

- Summing all subsystem masses and dividing through by output power yields an equation for the overall system mass-power ratio, $\alpha$:

$$\alpha = \frac{\left\{ \hat{\alpha}_S \eta_D e + \hat{\alpha}_D (1-e) + \hat{\alpha}_P \eta_D G + \hat{\alpha}_T \left[ G \eta_P \eta_D - (1-e) \right] + \right\}}{\left\{ \hat{\alpha}_H \left[ (1-\eta_D)(1-e) + \eta_D (1-\eta_P)G \right] \right\} \eta_T \left[ G \eta_P \eta_D - (1-e) \right]}$$
This equation is capable of modeling the $\alpha$ of either power-limited or gain-limited systems.

For solely power-limited systems ($e = 1, G = 1$):

$$\alpha_{P-L} = \alpha_{POWER-LIMITED} = \frac{\hat{\alpha}_S + \hat{\alpha}_P \eta_P + \hat{\alpha}_H (1 - \eta_P)}{\eta_T \eta_P} + \hat{\alpha}_T$$

For solely gain-limited systems ($e = 0$):

$$\alpha_{G-L} = \alpha_{GAIN-LIMITED} = \frac{\hat{\alpha}_D \eta_D + \hat{\alpha}_P \eta_D \eta_P G + \hat{\alpha}_H \left[ (1 - \eta_D) + \eta_D (1 - \eta_P) G \right]}{\eta_T (\eta_P \eta_D G - 1)} + \hat{\alpha}_T$$
Limits on Values of Gain

- In order to have a net positive input power for thrust production and driver operation, the denominator of this equation must have a positive value. This condition results in:

\[ G > G_{MIN} \quad ; \quad \text{where} \quad G_{MIN} = \frac{1}{\eta_P \eta_D} \]

- Progressively higher values of \( G \) above this minimum result in successively lower mass-power ratios, \( \alpha \).

- In the limit where gain goes to infinity, there is a minimum of \( \alpha \):

\[ \alpha_{G_{\infty}} = \hat{\alpha}_T + \frac{\hat{\alpha}_P \eta_P + \hat{\alpha}_H (1 - \eta_P)}{\eta_T \eta_P} \]
A Simplified form of \( \alpha_{G-L} \)

In the limit where gain goes to zero, the value of \( \alpha \) has no physical significance:

\[
\alpha_{G_0} = \alpha_T - \frac{\hat{\alpha}_D \eta_D + \hat{\alpha}_H (1 - \eta_D)}{\eta_T}
\]

However, substitution of \( \alpha_{G_0} \) and \( \alpha_{G_\infty} \) simplifies the \( \alpha_{G-L} \) power balance into a more compact form emphasizing gain-driven and gain-independent parameters:

\[
\frac{G}{G_{MIN}} = \frac{\alpha_{G_\infty} - \alpha_{G_0}}{G_{MIN}} - 1
\]
Given $\alpha$, calculate needed Gain

- Inverting the compact $\alpha_{G-L}$ equation for $G$ yields an equation stating the $G$ required for a given $\alpha_{G-L}$ (or $\alpha$):

$$G = G_{MIN} \frac{\alpha - \alpha_{G0}}{\alpha - \alpha_{G\infty}}$$

- The lower the value of $\alpha_{G\infty}$ — implying lower subsystem $\alpha$'s and higher $\eta_p$ — the smaller the value of gain $G$ required to meet a mission.
Summary so far ...

- For very fast missions with straight-line trajectories, it has been shown that mission trip time is proportional to the cube root of $\alpha$.

- Analysis of spacecraft power systems via a power balance and examination of gain vs. mass-power ratio has shown:
  - A minimum gain is needed to have enough power for thruster and driver operation.
  - Increases in gain result in decreases in overall mass-power ratio, which in turn leads to greater achievable accelerations.

  - However, subsystem mass-power ratios and efficiencies are crucial: less efficient values for these can partially offset the effect of nuclear gain.

  - Therefore, it is of interest to monitor the progress of gain-limited subsystem technologies.

  - It is also possible that power-limited systems with sufficiently low $\alpha$ may be competitive for such ambitious missions.