Thermoelastic Stress Analysis:  
The Mean Stress Effect in Metallic Alloys

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ABSTRACT

The primary objective of this study involved the utilization of the thermoelastic stress analysis (TSA) method to demonstrate the mean stress dependence of the thermoelastic constant. Titanium and nickel base alloys, commonly employed in aerospace gas turbines, were the materials of interest. The repeatability of the results was studied through a statistical analysis of the data. Although the mean stress dependence was well established, the ability to confidently quantify it was diminished by the experimental variations. If calibration of the thermoelastic response to mean stress can be successfully implemented, it is feasible to use the relationship to determine a structure's residual stress state.

Key Words: Thermoelastic Stress Analysis, TSA, Thermoelastic Constant, Residual Stress, Mean Stress, Stress Pattern Analysis by Thermal Emission, SPATE:

1. INTRODUCTION

Thermoelastic stress analysis (TSA) is a full-field, non-contacting technique for surface stress mapping of structures. TSA is based on the fact that materials experience a temperature change when compressed or expanded (i.e., experience a change in volume). If the load causing the volumetric change is removed and the material returns to its original temperature and shape, the process is deemed reversible. This reversibility is achieved when a material is loaded elastically at a high enough rate so as to eliminate significant conduction of heat. The thermoelastic temperature change, in steel for instance, as a result of an applied cyclic load of 1.0 MPa (145 psi) is on the order of 0.001 °C. Lord Kelvin first quantified an analytical relationship between the change in temperature and the change in stress. The formulation is as follows

\[ \Delta T = -\frac{\alpha T \Delta \sigma}{\rho C_p} \]  

where \( \Delta T \) is the cyclic change in temperature; \( \alpha \) is the coefficient of linear thermal expansion; \( T \) is the absolute temperature of the specimen; \( \Delta \sigma \) is the change in the sum of the principal stresses; \( \rho \) is the material density; \( C_p \) is the specific heat at constant pressure; and \( K \) is the thermoelastic constant. Note that the formulations of equations (1) and (2) indicate that \( \Delta T \) is independent of the mean stress. Therefore, \( \Delta T \) is assumed to remain constant for a given stress range, \( \Delta \sigma \), and absolute temperature, \( T \), regardless of the applied mean stress.
In recent years, experimental data as well as theoretical formulations have shown a mean stress dependence concerning the thermoelastic constant, \( K \). Such stress dependence was illustrated experimentally by Machin et al.\(^{3}\) and Dunn et al.\(^{4}\). The theoretical explanation for this non-linear response was developed by Wong et al.\(^{5}\). Further experimental verification of the mean stress effect on \( K \) was dictated by Wong et al.\(^{6}\). In Wong et al.\(^{5}\), the reformulated theory shows that the mean stress dependence of the thermoelastic parameter (earlier referred to as a constant) is fully accounted for by the temperature dependence of the elastic moduli of the material. In the study, the effective \( K \) for a homogeneous Hookean material subjected to a uniaxial stress under adiabatic conditions is given as

\[
K = \left( \alpha - \frac{1}{E} \frac{\partial E}{\partial T} \sigma_{\text{mean}} \right)^{\frac{1}{2}} \rho C_e^{-1}
\]  

(3)

where \( E \) is the Young's modulus, \( C_e \) is the specific heat at constant strain, and \( \sigma_{\text{mean}} \) is the mean stress.

In this paper, further experimental verification of the mean stress effect is provided as well as a comparison with theory for three metallic alloys. The alloys of interest are the titanium base alloys TIMETAL 21S and Ti-6AI-4V, as well as a nickel base alloy, Inconel 718. These alloys are heavily utilized by the aviation industry in propulsion components due to their relative lightweight, high strength and stiffness, as well as their property retention at elevated temperatures.

It should be noted that the past thermoelastic data (see above references) were produced utilizing TSA systems with IR cameras based on a single detector. This single detector incorporated a network of scanning mirrors. Furthermore, the scanning devices were disengaged and only the information from a single point on the specimens was analyzed. For this study, the TSA system uses an IR camera with a 128 x 128 focal plane array (FPA) of detectors. As a result, the reported IR detector response at a given mean stress is based on the average of an array of detectors representing a larger two-dimensional area on the specimen surface.

In addition, assessments are made concerning the best method of differentiating the thermoelastic parameter’s experimentally measured mean stress dependence. This involves the analysis of the data to help define the most feasible method of comparison that minimizes the test variations. The results of this preliminary investigation preview the experimental issues that need to be addressed for accurate measurements of the mean stress effect. Therefore, the main objectives of this study are to perform enough tests to locate trends in the data, determine the experimental issues that need to be resolved for future studies, and assess the general mean stress sensitivity of the TSA method for the various tested materials.

As mentioned above, the metallic alloys in this study are used by the aviation industry in various propulsion components. Certain components, which are subjected to a cyclic fatigue environment, are fabricated so as to contain compressive residual stresses on the surface. These compressive stresses inhibit the nucleation of cracks. As a result of overloads and elevated temperature excursions, the induced residual stresses dissipate while the component is still in service, in turn, lowering its resistance to crack initiation.

Once confidence is achieved concerning reliable TSA measurements of the mean stress effect, research can focus on the application of the method to residual stress assessment. Such measurements will assist in the characterization of materials in the laboratory as well as in-situ monitoring of the current residual stress state in actual structural components during fabrication and service.

2. EXPERIMENTAL METHODS

Three metallic alloys with various pre-test heat treatments were analyzed for this study. Two were titanium base and one was a nickel base. The first titanium alloy, sheet TIMETAL 21S, was aged at 621 °C (1150 °F) for 8 hours. The second titanium alloy, Ti-6AI-4V in sheet form, was annealed at 788 °C (1450 °F) for 15 minutes and then air cooled. The nickel base alloy Inconel 718 sheet plates were solution treated at 1038 °C (1900 °F) and quick cooled. The material was then aged at 718 °C (1325 °F) for 8 hours. Next, it was cooled at a rate of 56 °C/hour (100 °F/hour) to 621°C (1150 °F) and held for 8 hours. This was followed by a quick cool.

The Ti-6AI-4V and Inconel 718 plates, measuring 17.8 cm x 17.8 cm (7 in. x 7 in.), were cut into straight-sided specimens using the EDM (Electrical Discharge Machining) process. The overall length of the specimens was 17.8 cm (7 in.) and the
width was 2.54 cm (1 in.). The Ti-6Al-4V had a nominal thickness of 0.203 cm (0.08 in.), while the Inconel 718 had a nominal thickness of 0.178 cm (0.07 in.). The TIMETAL 21S was supplied as round, reduced gage section coupons machined from 0.79 cm (0.31 in.) thick, flat plates. The coupons had a constant diameter gage length of 1.91 cm (0.75 in.) and a nominal gage diameter of 0.635 cm (0.25 in.). For each material three specimens were tested.

The liquid nitrogen cooled infrared camera of the TSA system employs a 128 x 128 focal plane array of InSb detectors (3 – 5 μm sensitivity). The system operates by recording a periodic temperature change, as viewed by the IR camera, of a specimen subjected to a cyclic mechanical load (see Figure 1). A reference signal from the load cell is used by the software to allow it to monitor only the true change in temperature and to disregard any noise and environmental effects that do not correlate with the reference signal. To further improve the signal to noise ratio, hundreds or thousands of cycles are collected and averaged. When an image is captured, the computer display depicts a dimensionless, digitized value of the average camera signal range corresponding to the cyclic load range for each of the 16384 pixels. For structural stress measurements, the dimensionless digital IR values are correlated to a known stress (e.g., as measured by a strain gage) and then utilized to map the surface stresses of the entire structure. Further details concerning the TSA system and the specific settings are given in the Appendix.

For this study, direct experimental measurements were conducted for analyzing the mean stress effect. This was accomplished by comparing the camera IR value at various discrete mean stresses while maintaining a constant cyclic stress range. The test platform employed for inducing the mechanical excitation was a digitally controlled (± 50 kN dynamic / ± 100 kN static) servo-hydraulic test system. The experiments were conducted in load control with a 10 Hertz sinusoidal waveform. Each of the three materials were subjected to a cyclic stress range of 70 MPa (10 ksi). The round TIMETAL 21S specimens had mean stresses varying from -276 MPa (-40 ksi) to 276 MPa (40 ksi). For the straight sided Ti-6Al-4V specimens, the mean stress ranged from 0 to 276 MPa (40 ksi). Finally, the Inconel 718 was subjected to discretely varying mean stresses from 0 to 345 MPa (50 ksi). At least three repetitions were conducted for each individual specimen (except for the TIMETAL 21S, where only one repetition was performed). A test repetition involved stepping through each of the discrete mean stresses and recording an IR signal. The mean stress stepping order was randomized for each test. The camera distance and angle were held constant throughout the experiments. The camera's line of sight was perpendicular to the specimen face, while the distance from the camera lens to the specimen face was maintained at 12.38 cm (4.88 in.). Lastly, it should be noted that all loads were well within the materials elastic regimes, hence, no plastic deformation occurred.

Test preparation included painting the specimens with an ultra-flat black paint to improve surface emissivity. In addition, K-type thermocouples were spot welded to the back face of each specimen so as to allow temperature monitoring throughout the tests. Since the IR flux is dependent on the source (i.e., specimen) temperature, empirical equations were used to correct the camera signals to values corresponding to a specimen temperature of 23°C (73.4°F). This empirical equation was obtained from a Stress Photonics, Inc. internal report. In the document, a temperature correction curve was developed for 6061-T6 aluminum. The temperature dependent relationship, which is physically a function of the specimen's IR flux, is assumed to be independent of the subject material. The same empirical relationship was used here in the following form

\[ \text{where } S_{23^\circ C} = \frac{S}{0.365 + 0.0593T} \]  

3. RESULTS

Equation (3) shows that the thermoelastic parameter is a linear function of the mean stress, \( \sigma_{\text{mean}} \). Wong, et al. expressed a measure of the normalized mean stress dependence as

\[ \frac{1}{K_o} \frac{\partial \alpha}{\partial \sigma_{\text{mean}}} = -I \frac{\partial E}{\alpha E^2 \partial T} \]  

where

\[ \text{NASA/TM-1999-209376} \]
Equation (5) simply gives the slope of the thermoelastic constant (in a dimensionless, normalized form) in respect to the mean stress, \( \sigma_{\text{norm}} \). This form allows for easy comparison with experimental data, where dimensionless digital values are used in the TSA output. Note that Equation (6) is the zero mean stress solution of Equation (3). Table I presents the values of the relevant constants for the materials tested as well as the theoretical values for the thermoelastic parameter at zero mean stress. The calculated values of the normalized mean stress effect using Equation (5) are given in Table II. The experimental results are also given in Table II and are discussed below.

Figure 2 displays a typical thermoelastic image as captured by the software. The dimensionless IR camera signal value used throughout this study is an average of an image box in the gage area of the specimen. For the TIMETAL 21S, the box consisted of 300 pixels representing the individual IR detectors (10 wide x 30 long) which were focused on the center portion of the specimen. The boxed area of interest was 0.318 cm x 0.953 cm (0.125 in. x 0.375 in.). For the larger Ti-6Al-4V and Inconel 718 specimens, the image box consisted of 3500 detectors (50 x 70), representing a 1.56 cm x 2.15 cm (0.616 in. x 0.846 in.) area.

Figures 3 and 4 show the thermoelastic responses for the TIMETAL 21S specimens as a function of the mean stresses. Seen in the figures is the significance of both the mean stress effect and the temperature correction procedure. Note that an adjustment of the mean stress from \(-276\) MPa \((-40\) ksi\) to 276 MPa \((40\) ksi\) induces, on average, a 21\% increase in the IR signal. The data of the three specimens were pooled and then fitted with a linear first order regression. The constants of the regression fit are presented in Table III. Note that the standard error represents a statistical measure of the goodness-of-fit of the model to the data, while R-value measures the strength of the relationship between the IR signal and the mean stress. The regression constants were used to develop the experimentally determined measure of the mean stress dependence as presented in Table II. This was accomplished by normalizing the regression slope by the intercept, which corresponds to the IR signal at zero mean stress. Table II compares the experimentally measured normalized mean stress effect with the theoretical value. Also, a comparison was conducted concerning the means of the pooled data at the various discrete cyclic mean stresses. Using the student t-test (unpaired) with a 95\% confidence level, the smallest statistically significant observable difference in mean stresses for the pooled TIMETAL 21S specimens was calculated as 207 MPa (30 ksi).

Concerning the temperature effect indicated in Figure 3, it can be shown that an input of 25°C into Equation (4) produces a multiplication factor of 0.895, i.e., a 10.5\% adjustment of the IR signal. The typical test temperatures for this study ranged from 23 to 26 °C, hence the significance of the temperature effect. Herein, only the corrected IR signals are discussed.

Figures 5 through 7 show the thermoelastic responses for the Ti-6Al-4V and Inconel 718 specimens as functions of the mean stresses. A minimum of three repetitions was conducted for each specimen. In Figures 6 and 7, the symbol shade corresponds to the individual specimen, while the symbol shape is indicative of the three test repetitions. Because no significant difference existed between the specimens at the various discrete mean stresses, the data were pooled for each material type as shown in Figures 6 and 7. The regression constants and values for the normalized mean stress effect are given Tables II and III, respectively. For the Ti-6Al-4V specimens, an increase of 276 MPa \((40\) ksi\) in the mean stress alters the IR signal by 8\%. The smallest statistically significant difference in mean stresses for the pooled specimens was calculated as 69 MPa (10 ksi).

Because of the especially large variations in the data (see Figure 7), no statistically significant difference in mean stresses existed for the Inconel 718 material within the tested range of mean stresses. It should be noted from Figure 7, that there exist two high value outliers at the 0 mean stress, which do not appear to fit the pattern of the data. The increased signals for these points may have been caused by a slight buckling of the relatively thin specimens during the negative load portion of the fully reversed cycle. If the camera was focused on the compressive side of the bend, the IR signal would have an increased value as a consequence of the additive compressive bending stress. After removing the two outliers, the smallest statistically significant difference in mean stresses for the pooled specimens was 241 MPa (35 ksi). The updated normalized mean stress effect was $1.35 \times 10^4$ MPa$^{-1}$. Overall, a 5\% change in the IR signal was realized for a 345 MPa (50 ksi) increase in the mean stress value.
4. DISCUSSION

It was evident from the data that significant mean stress effects were present for the TIMETAL 21S and Ti-6Al-4V specimens, while a modest mean stress influence was apparent in the Inconel 718 material. It was most prominent when viewing the results of a single repetition (e.g., Figure 3). For most of the tests in this study, the $R^2$ values for the linear regressions of individual repetitions were 0.90 and better. Note that the linear regression constants for individual specimens were not discussed in this study because the specimens were pooled. The pooled, average IR signals experienced changes up to 20% (e.g., Ti-6Al-4V) as a result of varying the mean stresses.

Concerns arose when viewing the rather large test-to-test variations. In fact, the test variations of the pooled data greatly reduced the ability to discern between the IR signal means at the various discrete mean stresses. In addition, the variations dramatically diminished the $R$ values of the pooled linear regression constants. As a result, a relatively large number of tests needed to be run at each condition to achieve any sort of statistical confidence.

It is well known that an object's absolute temperature has a significant influence on the IR flux. Because of this, the temperature was accounted for by an empirical correction equation. Some of the test error was probably introduced because of the inaccuracies of this equation. For improved data, a more accurate empirical equation or an in-depth theoretical formula needs to be utilized. Another source of temperature error was due to the inherent inaccuracies of the K-type thermocouples used to monitor the specimens. An error of $\pm 1^\circ C$ ($\pm 1.8^\circ F$) or more is typical for this type of thermocouple. Hence, more precise temperature monitoring methods need to be employed for future studies.

Discrepancies were noticed when comparing the experimentally obtained normalized mean stress effect and the theoretical predictions as shown in Table II. The experimental values were between 20 and 40% lower. The material properties used to calculate the theoretical values were obtained from various published reports, as opposed to direct measurement of the test material used here. Slight changes in these numbers can significantly change the value for the normalized mean stress effect. Combining these possible errors in the theoretical solution with the experimental ones discussed above explains a large portion of the discrepancies.

5. CONCLUSIONS AND FUTURE DIRECTIONS

The effect of mean stress on the thermoelastic parameter was demonstrated for TIMETAL 21S, Ti-6Al-4V, and Inconel 718 materials. Although the effect was well established, the ability to confidently quantify it was diminished by the experimental variations. Certain steps can be taken to reduce the experimental variations as discussed earlier. Research will continue concerning the direct measurement of the mean stress effect with the experimental modifications implemented.

Another direction to be taken concerns a more in-depth analysis of the thermal response. In Wong et al., it was shown that the dynamic temperature response of a cycled specimen contained a second harmonic even when a pure sinusoidal load was applied. This second component can be significant when large stress amplitudes are present. In addition, theoretical formulations show the second harmonic to be dependent on the stress range while the first harmonic is dependent on both the range and the cyclic mean stress. By combining the theoretical formulation and experimental data, direct measurements can be made concerning the mean stress.

The significance of establishing the mean stress dependence of the thermoelastic parameter is based on two main factors. First, thermoelastic analysis techniques must take this effect into account when viewing certain materials, or substantial errors can occur. Secondly, the potential for using this relationship for the measurement of residual stresses is now apparent.

APPENDIX

The thermoelastic stress analysis was conducted using the DeltaTherm 1000 system manufactured by Stress Photonics Incorporation. The following software settings were utilized during IR data acquisition: Accumulation Time = 4.7 sec; Gain = 2; and AC Channel Accumulation Time constant = 300 seconds.
REFERENCES


### TABLE I. MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$, g/cm$^3$</th>
<th>$E_{25^\circ C}$, GPa</th>
<th>$\partial E/\partial T$, MPa/°K</th>
<th>$a$, $10^{-6}$/°K</th>
<th>$C_v$, J/g °K</th>
<th>$K_o$ (Eq. 6) $10^6$/MPa</th>
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<tbody>
<tr>
<td>TIMETAL 21S</td>
<td>4.95</td>
<td>114</td>
<td>-56.0</td>
<td>7.20</td>
<td>0.495</td>
<td>2.94</td>
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<tr>
<td>Ti-6Al-4V</td>
<td>4.43</td>
<td>120</td>
<td>-61.8</td>
<td>9.06</td>
<td>0.544</td>
<td>3.76</td>
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<tr>
<td>Inc 718</td>
<td>8.22</td>
<td>205</td>
<td>-57.7</td>
<td>12.2</td>
<td>0.430</td>
<td>3.45</td>
</tr>
</tbody>
</table>

*Properties obtained from references 8 - 11.*

### TABLE II. NORMALIZED MEAN STRESS EFFECT

<table>
<thead>
<tr>
<th>Material</th>
<th>Theory, Eq. (6), Mpa$^{1}$</th>
<th>Experimental, Mpa$^{1}$</th>
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</thead>
<tbody>
<tr>
<td>TIMETAL 21S</td>
<td>6.01x10$^{-4}$</td>
<td>3.53x10$^{-4}$</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>4.75x10$^{-4}$</td>
<td>2.88x10$^{-4}$</td>
</tr>
<tr>
<td>Inc 718</td>
<td>1.12x10$^{-4}$</td>
<td>8.92x10$^{-5}$</td>
</tr>
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</table>

*Experimental value includes data labeled as outliers (see text).*

### TABLE III. LINEAR REGRESSION CONSTANTS

<table>
<thead>
<tr>
<th>Material</th>
<th>Intercept, Signal (Stnd Error)</th>
<th>Slope, Signal/Mpa (Stnd Error)</th>
<th>R</th>
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<tr>
<td>TIMETAL 21S</td>
<td>6498 (32.2)</td>
<td>2.29 (0.17)</td>
<td>0.948</td>
</tr>
<tr>
<td>Ti-6Al-4V</td>
<td>7997 (59.5)</td>
<td>2.30 (0.35)</td>
<td>0.731</td>
</tr>
<tr>
<td>Inc 718</td>
<td>7603 (62.0)</td>
<td>0.678 (0.28)</td>
<td>0.383</td>
</tr>
</tbody>
</table>

*Constants include data labeled as outliers (see text).*
Figure 1 Schematic diagram of TSA system (DeltaTherm 1000 User's Manual).

Figure 2 Typical IR test image for a TIMETAL 21S specimen. The rectangular box within the specimen indicates the area where the average signal was obtained. The scale displays the dimensionless digital values of the IR camera signal.
Figure 3 Typical mean stress dependence for a TIMETAL 21S specimen. Also shown is the influence of the signal correction procedure (Specimen 2).

Figure 4 Comparison of the mean stress dependence for multiple TIMETAL 21S specimens. Single repetition for each specimen.
**Figure 5** Comparison of the mean stress dependence for multiple repetitions of a single Ti-6Al-4V specimen (Specimen 3).

**Figure 6** Results of pooling three Ti-6Al-4V specimens (a total of 10 repetitions).
Figure 7 Results of pooling three Inconel 718 specimens (a total of 9 repetitions).
**Title and Subtitle**
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