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# STOCHASTIC MODELING OF AIRLINES' SCHEDULED SERVICES REVENUE

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## ABSTRACT

Airlines' revenue generated from scheduled services account for the major share in the total revenue. As such, predicting airlines' total scheduled services revenue is of great importance both to the governments (in case of national airlines) and private airlines. This importance stems from the need to formulate future airline strategic management policies, determine government subsidy levels, and formulate governmental air transportation policies. The prediction of the airlines' total scheduled services revenue is dealt with in this paper. Four key components of airline's scheduled services are considered. These include revenues generated from passenger, cargo, mail, and excess baggage. By addressing the revenue generated from each schedule service separately, air transportation planners and designers are able to enhance their ability to formulate specific strategies for each component. Estimation results clearly indicate that the four stochastic processes (scheduled services components) are represented by different Box-Jenkins ARIMA models. The results demonstrate the appropriateness of the developed models and their ability to provide air transportation planners with future information vital to the planning and design processes.

## INTRODUCTION

Airlines are under tremendous pressure to generate more revenues to lessen government provided subsidies (in case of national airlines). An airline's total revenue basically consists of two major segments. The first segment comprises revenue generated from scheduled services such as passenger, cargo, mail, and excess baggage. The second segment comprises revenue generated from chartering and other non-scheduled services and activities. The revenue generated from scheduled services accounts for the major bulk of the total airline revenue.

The basic difference between the two segments is that the first segment consists of services that are scheduled. As such, revenue generated from these services can be studied over a period of time and for the most part are predictable. The revenue generated from the second segment however can be categorized as being uncertain (fluctuates widely over time). The importance of future scheduled services revenue stems from the need to formulate future strategic management policies, determine government subsidy levels, and formulate governmental transportation policies.

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By far, air passenger revenue constitutes the major bulk of the total revenue. Because of fierce competition, airlines are on the move to capture more passengers partly by means of opening new efficient routes and by improving their quality of service. It is therefore not surprising to see air passenger demand forecasting evoking the attention of many researchers.

Different techniques have been applied in the literature to predict future airline passenger traffic demand. Waheed, McCullough, and Crawford (1985) implemented the Box-Jenkins methodology to forecast airline passenger demand and assess future airport needs. An aggregate oriented data set was used for that purpose. Ashford and Benchemam on the other hand, developed an airport choice model on the basis of a disaggregate data set (1987). The estimated model highlighted the major factors that influence airline passenger decisions to choose a particular airport. Although the developed model was not intended to forecast airline passenger traffic demand, it nevertheless has the ability to predict air traffic demand. Recently, Rengaraju and Arasan developed a city-pair model to estimate domestic air travel demand. The specified model was calibrated with a cross-sectional aggregate data taken from 40 city pairs (1992). Other studies in this area include the work by Moore and Soliman (1981), Skinner (1976), Ozoka and Ashford (1988), and Harvey (1987).

Nearly most of the previous work done in this area was geared toward the prediction of air passenger traffic. Recognizing air passenger revenue as being the major contributor to the total airline revenue, it is also equally important to consider other sources of revenues in proper future strategic planning. Since each revenue component is affected by different external factors it is expected that each component will have its own characteristics and structure. Furthermore, addressing each component separately enhances air transportation planners, designers, as well as airlines to formulate specific strategies for each component.

To this end, no attempts were made to model the revenue generated from the airlines' scheduled services. This paper will explicitly address the total revenue generated from scheduled services, namely passenger, cargo, mail, and excess baggage. These stochastic processes will be represented by time-series models. The specified models will be estimated with the use of data obtained from the Royal Jordanian Airlines (RJA).

## MODEL DEVELOPMENT

A number of mathematical techniques can be used to model the airline's total scheduled services revenue. These include multiple regression, econometric, and time-series analysis. Both multiple regression (special case of econometric models) and econometric models require variations in a number of economic factors to forecast the revenue generated from each airline's scheduled services. Time series models on the other hand require only the time-lagged values of each scheduled service revenue (past behavior of each revenue). Furthermore,

utilizing time series models would explicitly account for patterns in the past variations of the scheduled services revenues, thus making them more widely used particularly in circumstances where information on variations in economic factors is lacking or unavailable.

Let  $Y_i(t)$  represent the yearly generated revenue from the  $i$ th scheduled airline service (passenger, cargo, mail, or excess baggage). Since the collection of activities in each service is ordered in time, the process is called a stochastic process. A number of stochastic processes can be used to model the revenue generated from the aforementioned services. These processes include; Autoregressive Integrated Moving Average (ARIMA), pure Autoregressive (AR), pure Moving Average (MA), and random walk. When the mean, variance, and the covariance of the process is time invariant, then this process is considered as a stationary stochastic process. This implies that the fluctuation of revenue is stable over time. However, most encountered time series are nonstationary in nature, particularly those that deal with a passenger's choice of an airline and demand for air travel. Hence techniques are sought to overcome the nonstationarity. Of particular interest is the Box-Jenkins (1976) ARIMA models.

The success of this method can be attributed to the fact that this methodology is capable of dealing with different forms of time series (stationary, nonstationary, with or without seasonal elements). Furthermore, many computer packages available on the market have full documentation of this method.

The general polynomial representation of the Box-Jenkins Integrated model for nonstationary scheduled service revenue time series with seasonality ARIMA (p,d,q) x (P,D,Q)<sub>s</sub> can be written as,

$$\phi_p(B)\theta_p(B)(1-B)^d(1-B^s)^D Y_{i(t)} = \rho_q(B)\chi_Q(B)\tau_t + \delta$$

where  $\theta_p(B)$  and  $\rho_q(B)$  are polynomials representing regular autoregressive and moving average of order  $p$  and  $q$  respectively.  $\phi_P(B)$  and  $\chi_Q(B)$  are polynomials representing seasonal autoregressive and moving average of order  $P$  and  $Q$  respectively,  $\tau_t$  is the random error component, and  $\delta$  is the trend parameter. The trend parameter should be included in the model if the differenced series has a significantly large mean value. The Box-Jenkins methodology is used to convert the nonstationary time series into a stationary one. This conversion can be achieved partly by differencing the time series. The order of differencing is determined by studying the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) of the original and of each of the differenced time series derived from it. In some cases however, a power transformation might be needed to stationarize a time series. Once stationarity has been achieved, identification of the stationary time series is sought. This is done by studying the ACF and PACF of the converted series.

In terms of the original time series which represent the revenue generated from scheduled services, the models below are called integrated models; ARI (p,d,0), IMA (0,d,q), ARIMA (p,d,q) respectively,

$$W_{i(t)} = \phi_1 W_{i(t-1)} + \phi_2 W_{i(t-2)} + \dots + \phi_p W_{i(t-p)} = \tau_{i(t)}$$

$$W_{i(t)} = \theta_1 \tau_{i(t-1)} + \theta_2 \tau_{i(t-2)} - \dots - \theta_q \tau_{i(t-q)} = \tau_{i(t)}$$

$$W_{i(t)} = (\phi_1 W_{i(t-1)} + \phi_2 W_{i(t-2)} + \dots + \phi_p W_{i(t-p)}) - (\theta_1 \tau_{i(t-1)} + \theta_2 \tau_{i(t-2)} + \dots + \theta_q \tau_{i(t-q)}) = \tau_{i(t-q)}$$

where

$$W_{i(t)} = Y_{i(t)} - Y_{i(t-1)} \quad \text{for } t = 2, 3, \dots, N$$

where  $\phi_1, \dots, \phi_p$  denote autoregressive coefficients,  $\theta_1, \dots, \theta_q$  denote moving average coefficients, and  $\tau$  is the disturbance term.

The idea then is to specify and estimate all univariate time-series model for revenue generated by each of the four scheduled services.

### DATA DESCRIPTION

Royal Jordanian Airlines (RJA) is the only national carrier in Jordan. Established in 1963, then under the name of Alia, RJA made great strides over the years. The airline started with three aircraft and made scheduled flights to three destinations Beirut, Kuwait, and Cairo. Now RJA has a fleet consisting of 19 aircraft (mainly Airbus and Boeing), with 39 destinations in total, and around 117 flights per week. Furthermore, RJA's route network has expanded rapidly over the years. Although, the RJA provide domestic air services (Amman-Aqaba route), international services dominate the RJA operations. Figure 1 shows the RJA route map. The total RJA revenue has registered a somewhat continuous growth.

The data set was obtained from the Royal Jordanian Airlines. The data consists of four stochastic processes. The data set represents the components of the scheduled services revenue, namely passenger revenue, cargo revenue, mail revenue, and excess baggage revenue. The data set represents the time period from 1964 until 1993 inclusive.

Figure 2 shows the evolution of RJA total scheduled services revenue over time. Figure 3 on the other hand shows the contribution of each component of the schedule services to RJA's total revenue. The figure clearly shows that on the average passenger revenue constitutes over 70 percent of the total revenue. Both mail and excess baggage revenues seem to be relatively uniform over time. The availability of air cargo aircraft that were capable of handling various types of products have positively influenced the revenue generated from cargo traffic.

Figure 4 shows the contribution of both chartering and other non-scheduled services to the RJA's total revenue. Unlike the scheduled services revenue, both revenues have pronounced peaking and fluctuate widely over time. For example, in 1990 revenue generated from both chartered services and other non-scheduled services amounted to 16.3 percent and 21 percent from the total RJA's total revenue respectively. However, such contributions fell to 5.6 percent and 7.2 percent respectively in 1992. This specific fluctuation can be attributed to



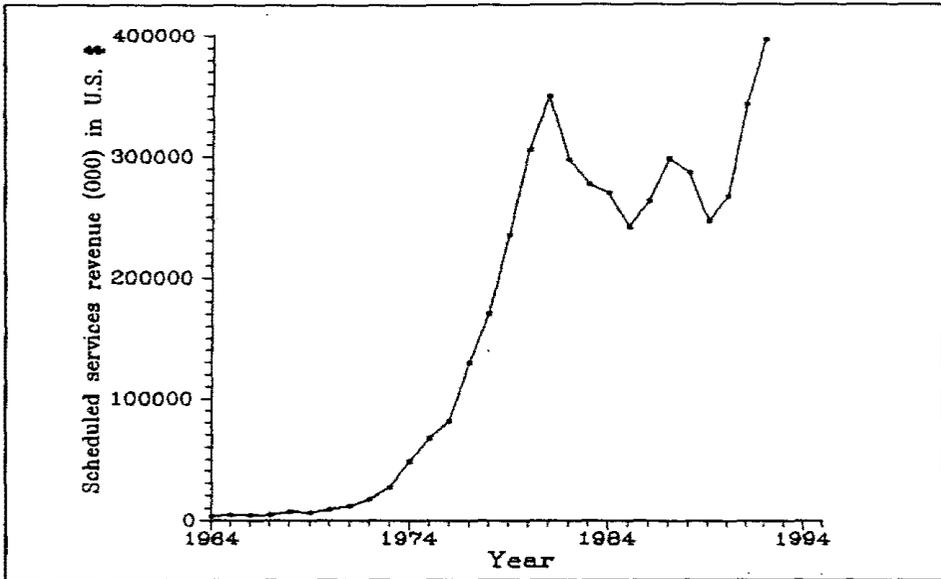


Figure 2. Total scheduled services revenue of RJA, 1964-1993

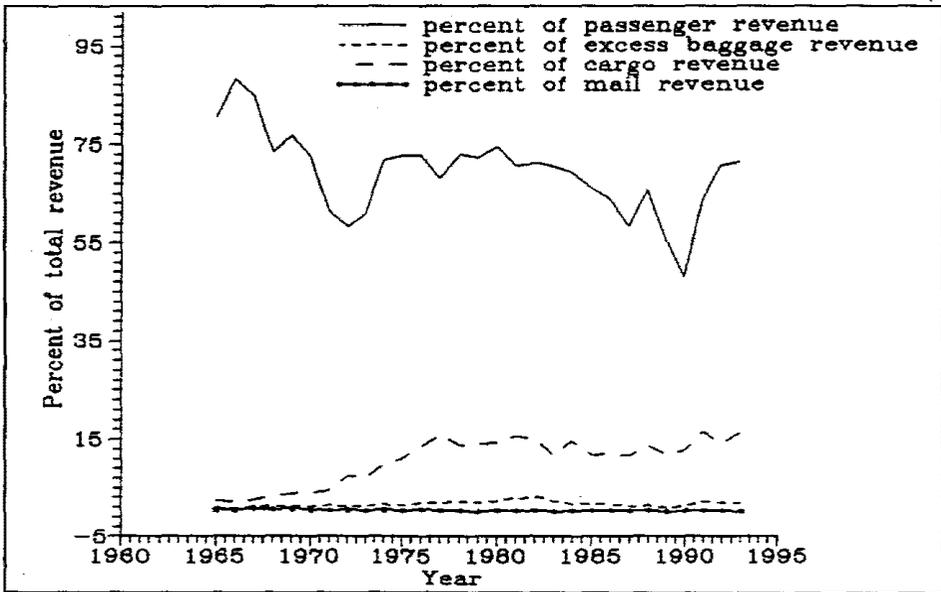


Figure 3. Scheduled services revenue components of RJA, 1964-1993

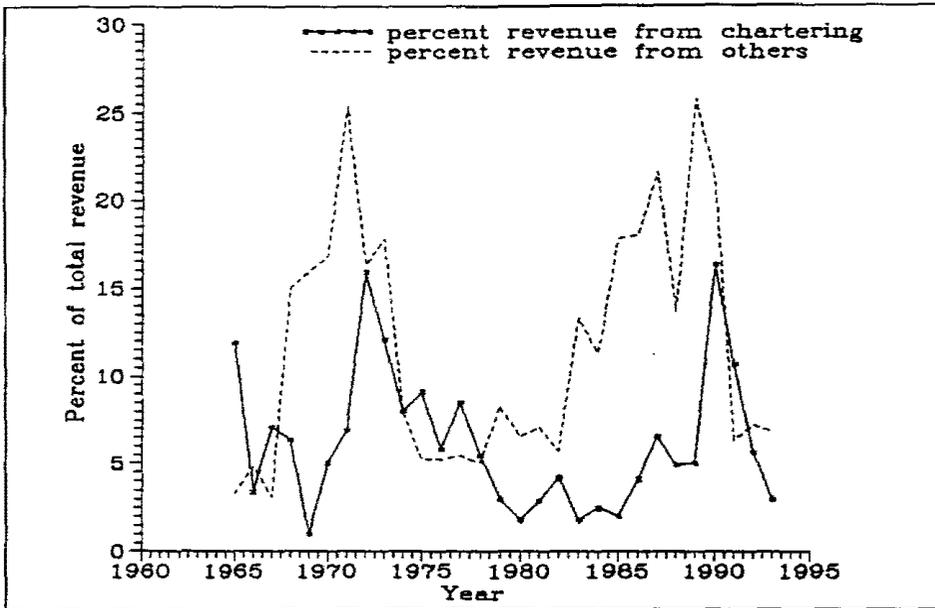


Figure 4. Yearly revenue from chartering and other non-scheduled services for RJA, 1964-1993

the Gulf Crisis where RJA had to compensate for the dramatic fall in passenger revenue through chartering and other activities. Even prior to 1990, Figure 4 clearly shows significant fluctuations in these two revenues.

### EMPIRICAL RESULTS

The ACF and PACF for each schedule services revenue are shown in Figures 5 through 8. Without exception, all ACFs of the original series show slowly declining sinusoidal (slow damping off) with a large spike at lag 1. In fact, Figure 5 shows a large spike approaching unity at lag 1. The PACF plots for all time series show a similar pattern. In that a large spike approaching unity in some cases is present at lag 1. The ACF and PACF patterns clearly indicate all time series are nonstationary in the mean. A number of regular differencing of the original series has been carried out. It was found that for all time series, one regular differencing ( $d = 1$ ) was enough to produce a stationary time series. Figures 5-8 show time-series plots for each of the above measures. All plots show strong evidence of nonstationarity in the mean. Generally, time series representing all the four schedule services revenue appear to have an upward trend. The decline in revenue in some years (e.g. 1967, 1970, and 1988) can be attributed to regional instability as a whole, the unprecedented low growth of Jordan's economy and the Gulf Crisis.

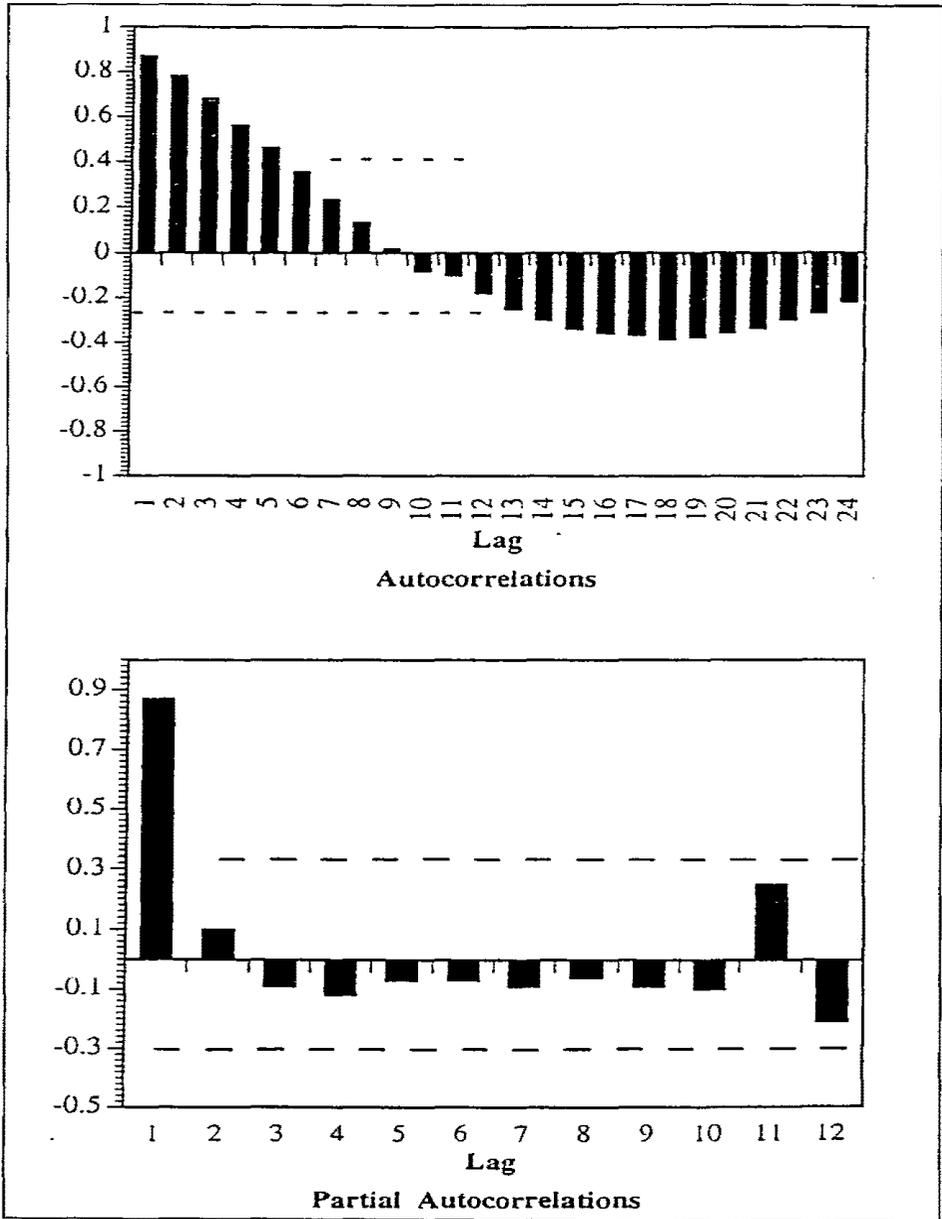


Figure 5. Estimated autocorrelations and partial autocorrelations of original passenger revenue time-series of RJA, 1964-1993

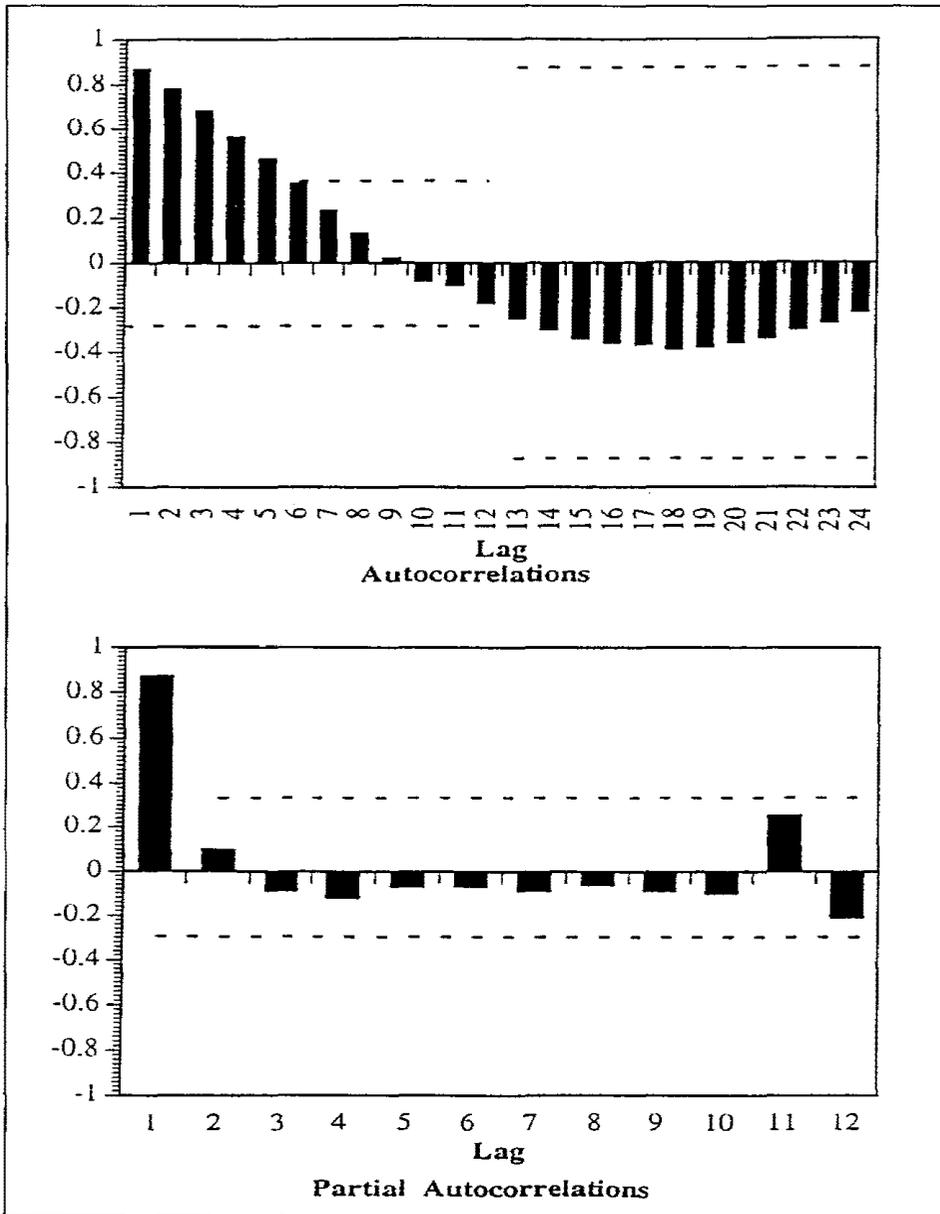


Figure 6. Estimated autocorrelations and partial autocorrelations of original cargo revenue time-series of RJA, 1964-1993

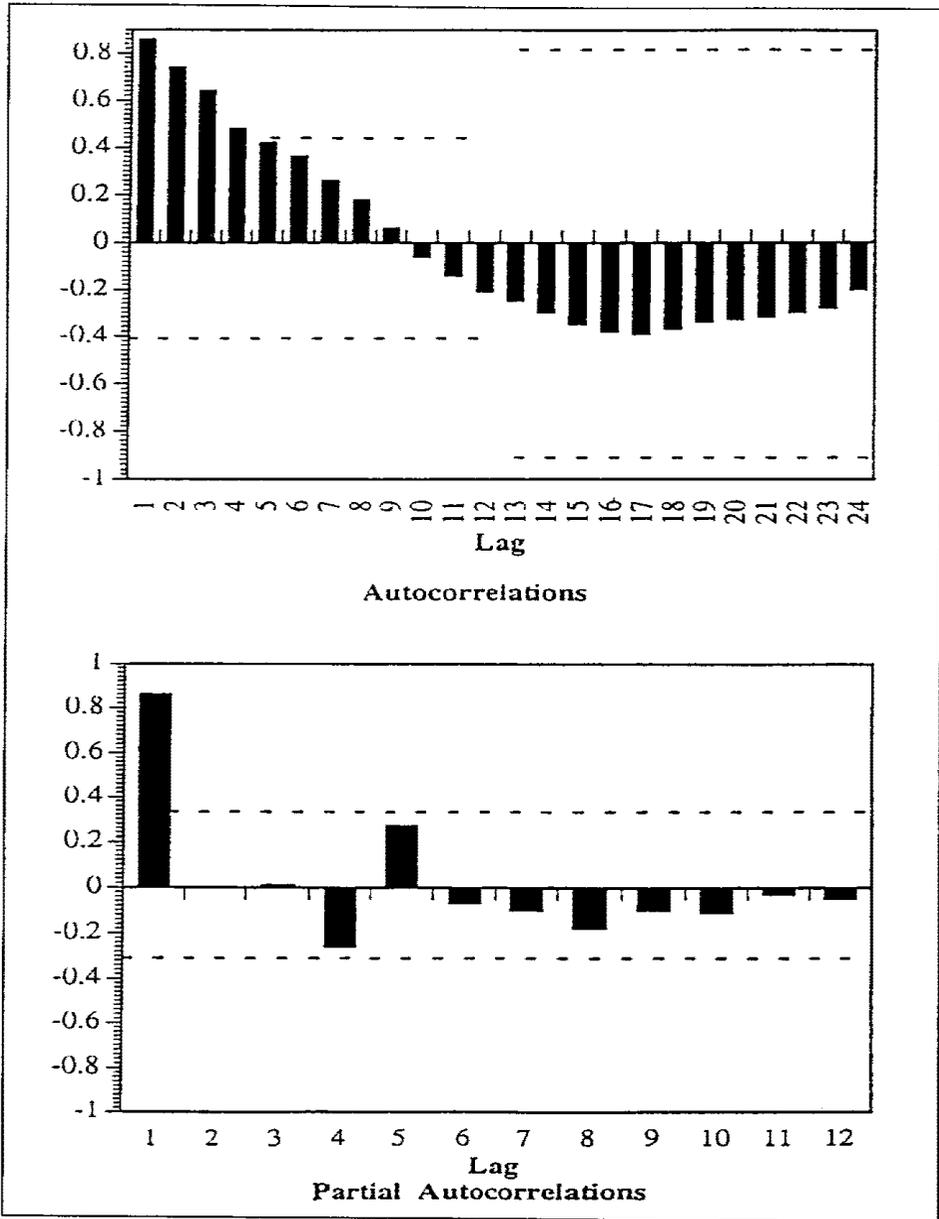
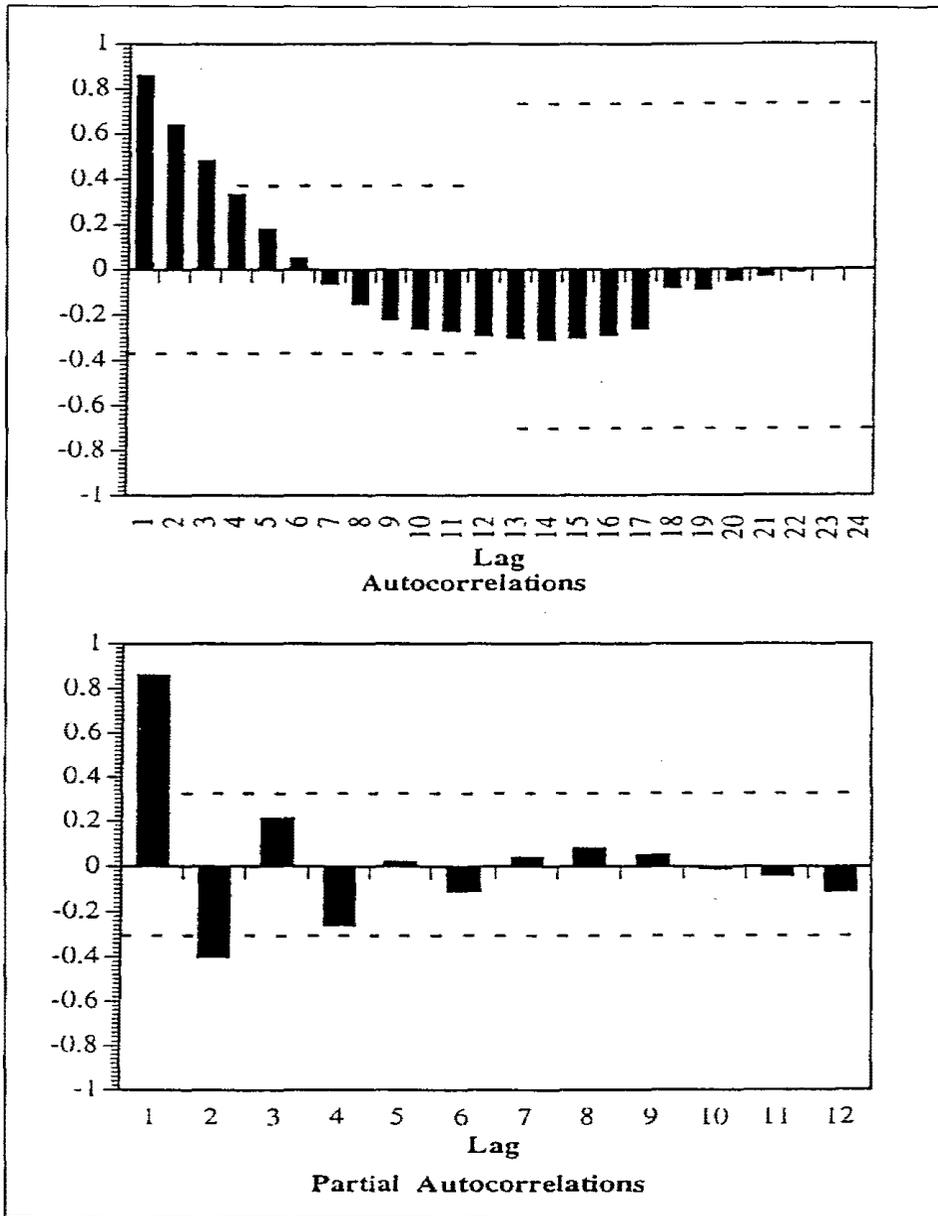


Figure 7. Estimated autocorrelations and partial autocorrelations of original mail revenue time-series of RJ, 1964-1993



**Figure 8. Estimated autocorrelations and partial autocorrelations of original excess baggage revenue time-series of RJA, 1964-1993**

The initial model selection for each series was based on the inspection of the ACF and PACF patterns. A number of models were considered for each time-series in an attempt to avoid overfitting or underfitting. The selected model estimation results are shown in Table 1. The Akaike Information Criterion (AIC), which is a measure of the precision of the estimate and the degree of parsimony in the parameterization of the stochastic model, was used to determine the best coefficient values. After considering several models, the model that provided the lowest AIC value was chosen (see Table 2).

**Table 1**  
**Scheduled Services Revenue Models Estimation Results, t-statistics in Parentheses**

<i>Scheduled Services Revenue Component</i>	<i>Model Structure</i>	<i>Best Coefficient Value</i>
1. Passenger revenue	IMA (0,1,1)	0.713 (5.279)
2. Cargo revenue	ARI (1,1,0)	0.270 (1.294)
3. Mail revenue	ARIMA (1,1,1)	
	AR(1)	0.517 (2.832)
	MA(1)	0.923 (18.490)
	trend constant( $\delta$ )	21.208 (2.175)
4. Excess baggage revenue	ARI (1,1,0)	0.338 (1.834)

Estimation results clearly show the stochastic process generating each scheduled service revenue is different in its structure. For example, passenger revenue turned out to be best represented by the ARIMA model (0,1,1) with no trend, cargo revenue by the ARIMA model (1,1,0) with no trend, mail revenue by the ARIMA model (1,1,1) with a trend, and the excess baggage revenue by the model (1,1,0) with no trend.

With the exception of the mail revenue time series, the remaining time series did not incorporate a constant trend. The trend constant in the mail revenue series turned out to be significantly different from zero at the 5 percent level. Furthermore, the t-statistics show that all parameter estimates are statistically significant at the five percent level. The difference in model structure and best coefficient values support our claim that each revenue component has its own characteristics and patterns.

Below is a formal representation of the ARIMA models (0,1,1) with no trend constant (model 1), (1,1,0) with no trend constant (model 2), (1,1,1) with a trend constant ( $\delta$ ) (model 3), and (1,1,0) with no trend constant (model 4), respec-

tively. These models represent the passenger, cargo, mail, and excess baggage revenues respectively.

$$Y_{1(t)} = Y_{1(t-1)} + \tau_{1(t)} - 0.713\tau_{1(t-1)}$$

$$Y_{2(t)} = 1.271Y_{2(t-1)} - 0.271Y_{2(t-2)} + \tau_{2(t)}$$

$$Y_{3(t)} = 1.517Y_{3(t-1)} - 0.517Y_{3(t-2)} + \tau_{3(t)} + 0.923\tau_{3(t-1)} + 21.208$$

$$Y_4 = 1.338Y_{4(t-1)} - 0.338Y_{4(t-2)} + \tau_{4(t)}$$

Figures 9 through 12 show the actual and fitted airline scheduled services revenue.

The residual analysis was based on the assumption stated earlier that the residuals of the best model are approximately white noise. The estimated residual autocorrelations and partial autocorrelations were not significant.<sup>1</sup> This clearly support the hypothesis that the residuals came from a population whose mean is zero and whose values are random.

<sup>1</sup>The significance of the residuals autocorrelations were checked by comparing with approximate two standard error bounds  $\pm 2 / \sqrt{N} = (\pm 0.371)$  where N is the number of observation used in computing the estimate (N=29 in our case)

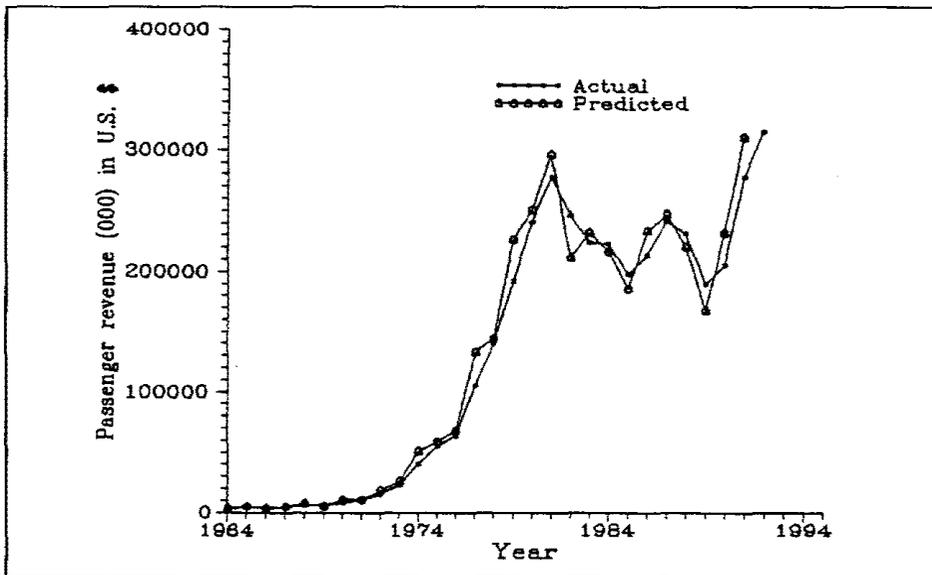


Figure 9. Actual and fitted yearly passenger revenue for RJA, 1964-1993

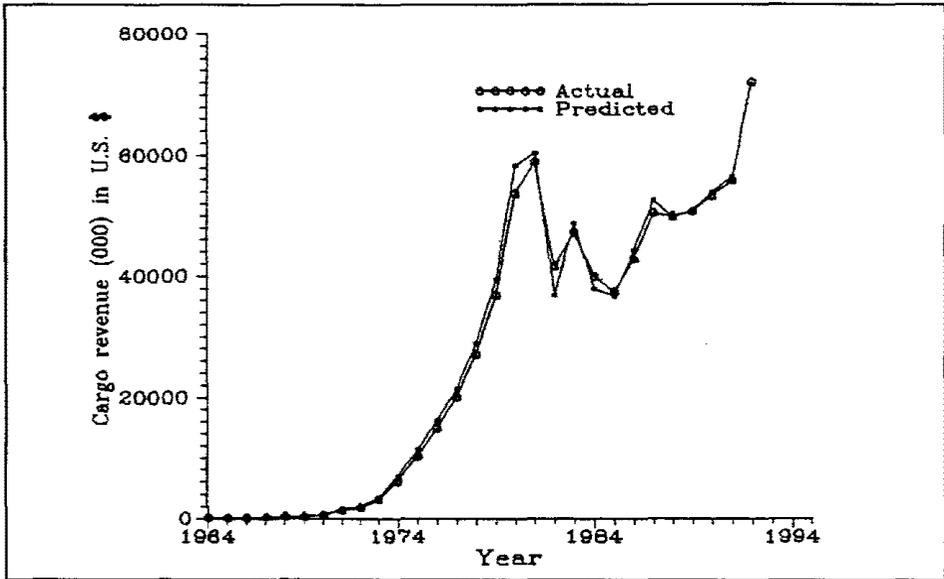


Figure 10. Actual and fitted yearly cargo revenue for RJA, 1964-1993

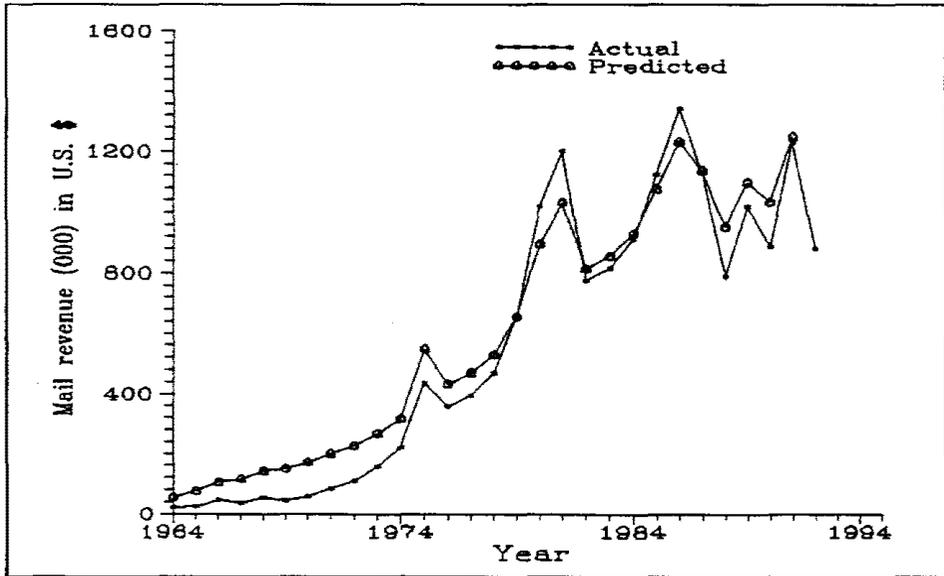


Figure 11. Actual and fitted yearly mail revenue for RJA, 1964-1993

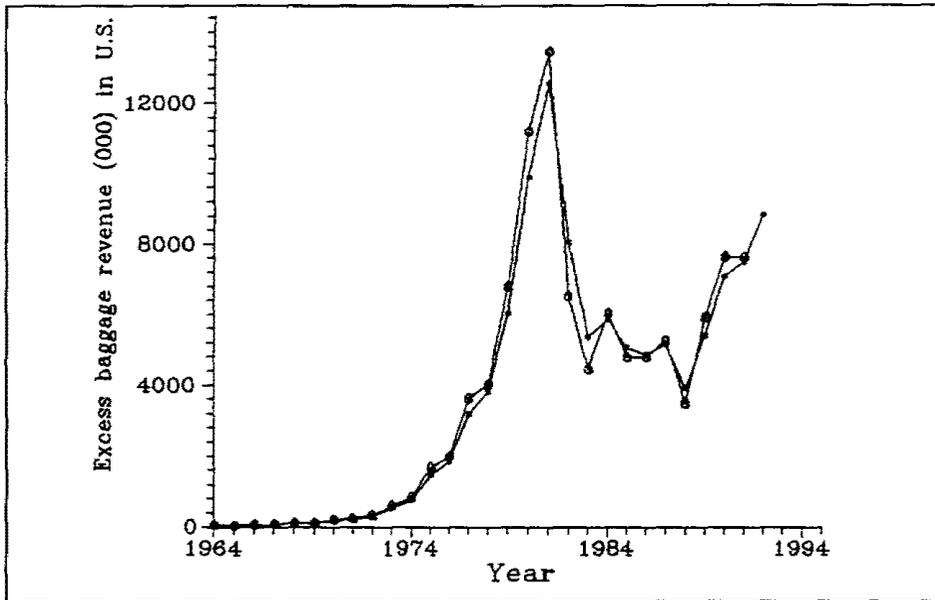


Figure 12. Actual and fitted yearly excess baggage revenue for RJA, 1964-1993

Table 2 shows summary statistics computed after the best coefficient values have been estimated. The Box-Pierce test statistics for all estimated models show no existence of serial correlation pattern in the residuals. The Q value for each revenue model is well below the critical 95 percent level (chi-squared critical value). Hence the selected models are appropriate for the purpose of forecasting.

The yearly fitted revenue from each schedule services time series model were added up to generate the total predicted scheduled services revenue. Figure 13 shows that the total yearly scheduled services revenue conforms very well to the

Table 2  
Scheduled Services Time-series Models' Summary Statistics

<i>Revenue Component Model</i>	<i>AIC*</i>	<i>Box-Pierce Statistics**</i>
1. Passenger revenue	20.24	9.20
2. Cargo revenue	17.70	9.80
3. Mail revenue	10.65	16.50
4. Excess baggage revenue	14.68	8.50

\*AIC: Akaike information criterion

\*\* Q values for k=24 lags

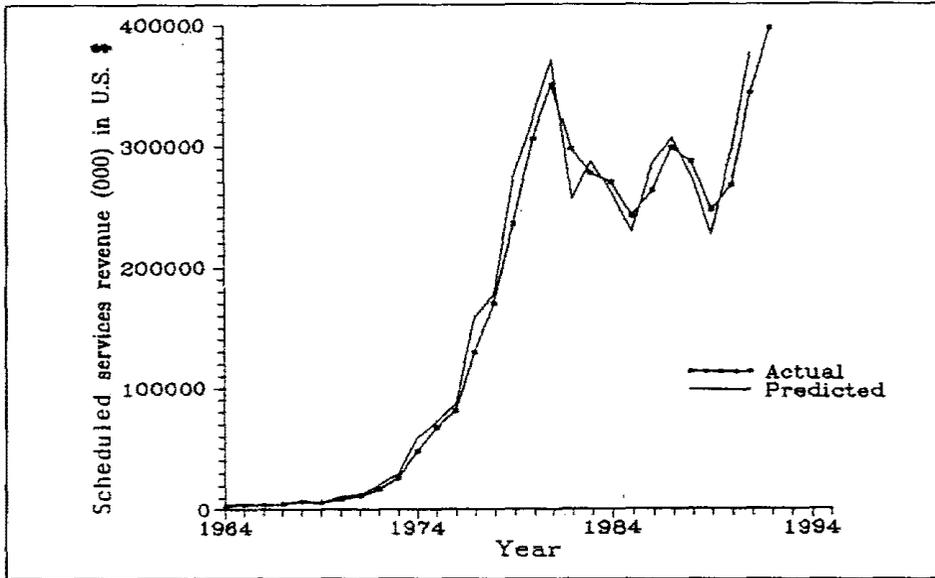


Figure 13. Actual and fitted total scheduled services revenue for RJL, 1964-1993

total actual scheduled services revenue. Both the direction and magnitude of forecasting values are correct.

### CONCLUDING REMARKS

A set of stochastic ARIMA models were specified and calibrated in this paper to forecast the airlines' total scheduled services revenue. Four key components of the airlines' schedule services were considered. These are passenger revenue, cargo revenue, mail revenue, and excess baggage revenue.

Results showed that the four stochastic processes are represented by different Box-Jenkins ARIMA models. This clearly suggests that each component has its own characteristics and structure and as such should be considered separately. The yearly forecasts from each scheduled services revenue component were added up to produce the airline's total scheduled services revenue. The generated forecasts turned out to be reasonable and efficient in terms of both the magnitude and direction of forecasts. With the use of such forecasts, airlines can plan their operations and expenditure according to the expected scheduled services revenue. Furthermore, since forecasts are available for each service, air transportation planners and designers can enhance their skills to evaluate future capacity expansions, predict changes in airline's equipment, and formulate future strategies concerning each service.

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