TRANSPORT OF LIGHT IONS IN MATTER


*NASA, Langley Research Center, Hampton VA 23681-0001, U.S.A.
**Old Dominion University, Norfolk, VA 23581, U.S.A.
***Hampton University, Hampton, VA 23668, U.S.A.
†GSI, Darmstadt, Germany

ABSTRACT

A recent set of light ion experiments are analyzed using the Green's function method of solving the Boltzmann equation for ions of high charge and energy (the GRNTRN transport code) and the NUCFRG2 fragmentation database generator code. Although the NUCFRG2 code reasonably represents the fragmentation of heavy ions, the effects of light ion fragmentation requires a more detailed nuclear model including shell structure and short range correlations appearing as tightly bound clusters in the light ion nucleus. The most recent NUCFRG2 code is augmented with a quasielastic alpha knockout model and semiempirical adjustments (up to 30 percent in charge removal) in the fragmentation process allowing reasonable agreement with the experiments to be obtained. A final resolution of the appropriate cross sections must await the full development of a coupled channel reaction model in which shell structure and clustering can be accurately evaluated.

INTRODUCTION

The need for accurate transport methods and corresponding atomic/nuclear database for protection against radiations in space was demonstrated in a recent review of issues in space radiation protection (Wilson et al., 1995). Although the earliest efforts in code validation experiments were placed on the relatively light ion beam of neon because of the potential importance of that beam to radiation therapy (Schimmerling et al., 1989; Shavers et al., 1990; Shavers et al., 1993; Wilson et al., 1994), the first beam studied specifically for space radiation protection was the iron beam which is the single most important species in long term space exposures (Shinn et al., 1994; Cucinotta et al., 1995). Among the specific issues in the iron beam experiments are the production cross sections for light fragments which were in doubt and in fact the reason for development of the NUCFRG code (Wilson et al., 1987). Recent experiments at the GSI heavy ion synchrotron (SIS) in preparation for medical therapy has recently provided data on the transport in water of the light ions of carbon, nitrogen and oxygen (Schall, 1994; Schardt et al., 1995; Sihver et al., 1995). Aside from the fact that the most abundant ions in the space environment with charge of three or more is the CNO group (produced in abundance in star interiors). These are also interesting ions for which the NUCFRG code (based on a liquid drop model) is expected to be least appropriate (Wilson et al., 1995; Cucinotta and Dubey, 1994b; Cucinotta, 1994a). Hence, the current data set is an important test for the fragmentation database used in space studies.

In our prior solutions of the Boltzmann equation, we assumed the nuclear cross sections were energy dependent (Shinn et al., 1994). There is a general decline in the nuclear cross sections with increasing energy approaching a minimum near 300A MeV (A is the ion atomic mass) followed by an increase in the cross section above pion production threshold. Above the first isobar resonance there is little variation in
the cross sections and constant cross sections can be assumed above 1A GeV. These intermediate energy variations are most dramatic for light ions with heavy ion cross sections being dominated by the ion's nuclear size. It was found essential to treat the energy dependence in the analysis of the 630A MeV Ne transport data obtained from studies at the Bevalac facility (Shavers et al., 1993; Wilson et al., 1994). Even so the main dependence was through the primary attenuation and first collision source terms with higher order terms adequately represented by energy independent terms so long as the extinction coefficient (energy averaged inverse attenuation length) is used in place of the constant cross section (Chun et al., 1994). Herein we use a simpler form of the Green's function in which the energy dependence is treated exactly to second order and higher order terms are treated using nonperturbative theory using extinction coefficients.

In the present report, we give a brief review of the current state of the transport formalism and application to the analysis of the GSI beam data. The initial results from the NUCFRG2 database are found to be in error by about 30 percent. It is surmised that the fault lies in the liquid drop assumption. The nuclear interaction viewed as a combination of inelastic, quasielastic, and nonelastic events leads us to assume the main quasielastic event in which specific particles or clusters are removed from a reasonably stable core cannot be accommodated within a liquid drop formalism (Cucinotta and Dubey, 1994b; Cucinotta et al., 1994a) while the highly nonelastic events in which the mass removed from the projectile is fully dissociated is better represented by the NUCFRG2 code. One can see these dynamic differences in the $^{12}$C/$^{13}$C ratio in the fragmentation of $^{40}$Ar (ratio of 1) in comparison with the value for fragmentation of $^{16}$O (ratio of 2.3) as measured by Tull (1990); Olson et al. (1983) respectively. The cluster effect for $^{16}$O projectiles is taken herein from the work of Cucinotta and Dubey (1994b); Cucinotta et al. (1994a) some additional ad hoc adjustments are made to the liquid drop model cross sections representing the non elastic events until a more complete cluster code is available. The resultant database is an improvement over the NUCFRG2 code but Li and Be fragments remain untested by the GSI experiments. In the absence of such data the further development of the cluster model is our best hope of resolving these cross sections.

**TRANSPORT METHODS**

In the domain of a spatially uniform beam, the Boltzmann transport equation assuming a straight-ahead approximation is useful in evaluating many field quantities in high-energy ion transport. Even for narrow and directed ion beams, angular and spectral corrections have been applied successfully as multiplicative factors (Shavers et al., 1990; Shavers et al., 1993; Wilson et al., 1994). The ion flux of type $j$ at $x$ with energy $E$ (AMeV) is given as (Wilson et al., 1977)

$$
\phi_j(x,E) = \frac{S_j(E_j) P_j(E)}{S_j(E) P_j(E)} \phi_j(0,E_j) + \sum_k \int_E^{E_k} dE' \frac{A_j P_j(E')}{{S_j(E) P_j(E)}} \int_{E'}^{E} dE'' \sigma_{jk}(E',E'') \phi_k[x + R_j(E) - R_j(E'),E''],
$$

where

$$
\phi_j(x,E) = \frac{S_j(E_j) P_j(E)}{S_j(E) P_j(E)} \phi_j(0,E_j)
$$

is the stopping power, $P_j(E)$ is the nuclear attenuation given as

$$
P_j(E) = \exp \left[ -\int_0^E \sigma_j(E') A_j dE' / S_j(E') \right]
$$

where $S_j(E)$ is the extinction coefficient.

$$
E_j = R_j^{-1}[x + R_j(E)] = E.
$$

We may rewrite the solution in terms of the Green's function as

$$
G_{jm}(x) = \phi_m(0,E_j) \frac{S_j(E_j) P_j(E)}{S_j(E) P_j(E)} \phi_j(0,E_j) + \sum_k \int_E^{E_k} dE' \frac{A_j P_j(E')}{{S_j(E) P_j(E)}} \int_{E'}^{E} dE'' \sigma_{jk}(E',E'') G_{jk}(x),
$$

where the leadi.
Transport Properties of Light Ions

\[ \phi_j(x,E) = \sum_k \int_0^\infty G_{jk}(x,E,E_o) \phi_k(0,E_o) dE_o \quad (3) \]

where \( G_{jm}(x,E,E_o) \) satisfies an integral equation similar to Eq. (1) as

\[ G_{jm}(x,E,E_o) = \frac{S_j(E)}{S_j(E) P_j(E)} G_{jm}(0,E,E_o) + \sum_k \int_E^{\infty} dE' \frac{A_j P_j(E')}{S_j(E) P_j(E)} \int_0^{\infty} dE'' \sigma_{jk}(E',E'') G_{km}[x + R_j(E) - R_j(E'),E'',E_o] \quad (4) \]

where

\[ G_{jm}(0,E,E_o) = \delta_{jm} \delta(E - E_o) \quad (5) \]

We use the Neuman expansion as a perturbative series

\[ G_{jm}(x,E,E_o) = \sum_{i=0}^{\infty} G_{jm}^{(i)}(x,E,E_o) \quad (6) \]

where the leading term is

\[ G_{jm}^{(0)}(x,E,E_o) = \frac{S_j(E)}{S_j(E) P_j(E)} \delta_{jm} \delta(E - E_o) \quad (7) \]

and the higher order terms are given by

\[ G_{jm}^{(i)}(x,E,E_o) = \sum_k \int_E^{\infty} dE' \frac{A_j P_j(E')}{S_j(E) P_j(E)} \int_0^{\infty} dE'' \sigma_{jk}(E',E'') G_{km}[x + R_j(E) - R_j(E'),E'',E_o] \quad (8) \]

The first iterate of Eq. (8) is given as

\[ G_{jm}^{(1)}(x,E,E_o) = \sum_k \int_E^{\infty} dE' \frac{A_j P_j(E')}{S_j(E) P_j(E)} \int_0^{\infty} dE'' \sigma_{jk}(E',E'') \frac{S_k(E'') P_k(E''_k)}{S_j(E'') P_j(E'')} \delta_{km} \delta(E' - E_o) \quad (9) \]

where

\[ E' = \frac{R_k^{-1}}{R_k^{-1}}[x + R_j(E) - R_j(E') + R_k(E'')] \quad (10) \]

If we make the usual assumption that the interaction is dominated by peripheral processes then

\[ \sigma_{jm}(E',E'') = \sigma_{jm}(E'') \delta(E' - E'') \quad (11) \]

for which the second term of the Neuman series becomes...
for values of $E$ such that

$$\frac{\nu_m}{\nu_j} [R_m(E_o) - x] < R_j(E) < \frac{\nu_m}{\nu_j} R_m(E_o) - x$$

(13)

where

$$E' = R_j^{-1}\left[\frac{\nu_m}{\nu_j} [x + R_j(E) - R_m(E_o)]\right]$$

(14)

and $\nu_j$ is the range scaling parameter ($\nu_i = Z_i^2 / A_j$). It was shown by Chun et al. (1994) that the second term is approximately a linear function of energy. We approximate the higher order terms as

$$G^{(i)}_{jm}(x, E, E_o) = \frac{1}{2} \left[ G^{(i)}_{jm}(x, E, E_{o \text{ min}}) + G^{(i)}_{jm}(x, E, E_{o \text{ max}}) \right]$$

$$+ \frac{G^{(i)}_{jm}(x, E, E_{o \text{ max}}) - G^{(i)}_{jm}(x, E, E_{o \text{ min}})}{E_{o \text{ max}} - E_{o \text{ min}}} (E - \frac{E_{o \text{ max}} + E_{o \text{ min}}}{2})$$

(15)

where $E_{o \text{ max}}$ and $E_{o \text{ min}}$ are associated with the allowed range in relation (13). Eq. (15) allows a simple numerical evaluation of the ion flux spectra (Wilson et al., 1994; Chun et al., 1994). The higher order terms we evaluate using the nonperturbative method discussed elsewhere (Wilson et al., 1994) except the assumed constant cross section is replaced by the extinction coefficient.

**COMPARISON WITH EXPERIMENT**

Experiments were performed at the GSI accelerator using beams of $^{12}$C, $^{14}$N, and $^{16}$O at energies of 674(±2)A MeV in which the transmitted flux of charge 5 to 8 were measured behind a water target of variable thickness (Schall, 1994; Schardt et al., 1995; Silverson et al., 1995). The measured transmitted fluence of all ions of the same charge as the beam (open circles) is shown in Figure 1 along with the solution for the primary beam fluence (dashed curve) and the calculated fluence of all ions of charge equal to the initial beam (solid curve). It appears that the total absorption and neutron removal cross sections of the NUCFRG2 are reasonably correct. The measured fluence of all ions with a single charge removed (open circles) is shown in Figure 2. The NUCFRG2 code (filled circles) tends to overestimate the single charge removal cross section for $^{12}$C and $^{16}$O projectiles and underestimates this cross section for $^{14}$N projectiles. This unsystematic behavior is indicative of structure dependent effects and perhaps results from the fact that the carbon and oxygen nuclei consist of integral numbers of highly stable alpha particles and nitrogen is a more loosely bound open shell structure. Results for the revised NUCFRG cross sections are also shown as the first collision term (dashed) and the complete solution (solid) which is in reasonable agreement with the experiments. The measured fluence for removal of two charge units from the initial beams of $^{14}$N and $^{16}$O (open circles) is shown in Figure 3 with the NUCFRG2 results (filled circles). NUCFRG2 underestimates the charge 2 removal from $^{16}$O and overestimates for $^{14}$N. The effects of alpha clustering is most apparent in the alpha knockout process for $^{16}$O collisions (Figure 3, left panel). The carbon isotope distribution in highly nonelastic collisions are equally distributed between $^{12}$C and $^{13}$C as can be seen in $^{40}$Ar fragmentation (Tull, 1990). In distinction, the fragmentation of $^{16}$O shows the single alpha knockout cross section to cause an excess of $^{12}$C fragments being produced (Olson et al., 1983). The addition of the alpha knockout cross section leaving the $^{12}$C core in the ground state to the NUCFRG2 nonelastic cross section (solid curve) brings good agreement with the GSI oxygen beam data. The carbon...
As a result of the present comparison the NUCFRG2 code has been modified to include Cucinotta's alpha knockout cross sections for $^{16}$O projectiles on all targets giving satisfactory agreement with the measurements of Olson et al. (1983) as shown in Figure 5. The final NUCFRG2 cross sections are shown in comparison with those measured at GSI in Table I. The use of these cross sections in evaluation of the transport result are shown in Figures 2–4 as dashed curve for the first collision flux and the solid curve including all the higher order collision terms using nonperturbative theory. Still, the Li and Be production cross sections are not represented in these latest measurements and are left uncertain. Further development of the cluster model calculations will be helpful in resolving these cross sections and such results will hopefully be available in the near future. Most important in this respect is the strong energy dependence in the cluster knockout cross sections as seen in Figure 6 for several targets. There is expected to be a large $\alpha$ knockout cross section for other $4n$ nuclei such as $^{20}$Ne, $^{24}$Mg, and $^{28}$Si which are all important contributors to galactic cosmic ray exposures (Cucinotta, 1994a). Also, the knockout of other light clusters will become important in heavy ion fragmentation for all nuclei which have large spectroscopic constants for clusters outside closed subshells in the ground state of the projectile or target. There is strong energy dependence from the nuclear form factors and the effects of pion production as clearly shown in the few hundred MeV to one GeV region in Figure 6. Other structure dependent effects are expected to show strong energy variations. Fortunately the energy dependence is less severe in light targets and low energy which is helpful in developing medical therapy beams.

Fig. 1. Attenuation of primary beam particles and secondaries formed by neutron removal.

Fig. 2. Secondaries produced by removal of one proton.
Fig. 3. Secondaries produced by removal of two protons including alpha clusters.

Fig. 4. Secondaries produced by removal of three protons.

Fig. 5. Fragmentation of $^{16}$O at 2.1 A GeV forming a $^{12}$C fragment.
Table 1. Augmented NUCFRG2 Charge Changing Cross Sections (mb) and Experimental Values in a Water Target (Schäff, 1994)

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Fragment Charge</th>
<th>NUCFRG2</th>
<th>Experiments</th>
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<tbody>
<tr>
<td>676A MeV $^{12}$C</td>
<td>6</td>
<td>103</td>
<td></td>
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<tr>
<td></td>
<td>5</td>
<td>225</td>
<td>215 ± 3</td>
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<td>4</td>
<td>129</td>
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<tr>
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<td>95</td>
<td></td>
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<td>303A MeV $^{14}$N</td>
<td>7</td>
<td>87</td>
<td></td>
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<td>6</td>
<td>362</td>
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<tr>
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<td>5</td>
<td>147</td>
<td></td>
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<tr>
<td></td>
<td>4</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>674A MeV $^{14}$N</td>
<td>7</td>
<td>84</td>
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</tr>
<tr>
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<td>6</td>
<td>337</td>
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<td>137 ± 6</td>
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<td>4</td>
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<tr>
<td></td>
<td>3</td>
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<td>300A MeV $^{16}$O</td>
<td>8</td>
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<td>7</td>
<td>284</td>
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<td>672A MeV $^{16}$O</td>
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CONCLUDING REMARKS

The value of having transport experiments to guide the development of models used to generate nuclear databases for estimation of shielding properties is aptly demonstrated in the present paper. The resulting revisions in the NUCFRG2 code will increase its usefulness in future studies. It is clear from the present study that a final database generator will require cluster models for the light ions. Partial results of such models was instrumental in correcting some of the deficiencies in the revised NUCFRG2 code presented herein. Clearly future versions should use exclusively cluster models for the light ions.

REFERENCES


