Speed Profiles for Deceleration Guidance During Rollout and Turnoff (ROTO)

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Abstract

Two NASA goals are to enhance airport safety and to improve capacity in all weather conditions. This paper contributes to these goals by examining speed guidance profiles to aid a pilot in decelerating along the runway to an exit. A speed profile essentially tells the pilot what the airplane's speed should be as a function of where the airplane is on the runway. While it is important to get off the runway as soon as possible (when striving to minimize runway occupancy time), the deceleration along a speed profile should be constrained by passenger comfort. Several speed profiles are examined with respect to their maximum decelerations and times to reach exit speed. One profile varies speed linearly with distance; another has constant deceleration; and two related nonlinear profiles delay maximum deceleration (braking) to reduce time spent on the runway.

Introduction

Airport congestion is a familiar sight to almost everyone, especially during holidays and in low-visibility weather. Without changes, this problem will only get worse. Consequently, NASA is currently looking at ways to expedite airport traffic by deploying new technology that will allow airplanes to takeoff and land in fog and other poor weather conditions. Among the many influential factors, the one of interest here is the time a landing airplane spends on the runway.

Today, there are no rules or procedures to reduce runway occupancy time (ROT), nor is there any specific on-board system to give pilots guidance in decelerating an airplane to an exit or for steering the airplane off the runway. In particular, in low-visibility weather, the lack of out-the-window speed and location cues and other information readily increases the time required for this rollout and turnoff (ROTO) phase of landing. One goal of NASA's Low Visibility Landing and Surface Operations (LVLASO) program is to address these concerns and enhance safety by creating procedures and guidance systems to maintain a clear-weather runway capacity even in low-visibility weather.

This paper analyzes different speed profiles that could be used in a system to provide the pilot with guidance for decelerating the airplane to the desired turnoff speed (exit speed). The analysis focuses on two fundamental questions for each speed profile:

- What is the maximum deceleration?
- How much time is spent on the runway

Symbols

\( a \) deceleration along speed profile, ft/sec\(^2\)

\( c_1, c_2 \) constants in appendix, eqs. (A2) & (A3)

\( f, g \) functions in appendix, eqs. (A8) & (A9)
k  constant (defined by speed profile)

\( t \)  time along speed profile, sec

\( v \)  speed along speed profile, kts

\( x \)  distance along speed profile, ft

\( \xi \)  dimensionless distance, \( x/x_0 \)

Subscripts:

e  exit condition

0  initial condition

Speed Profile Terminology:

Max Nonlinear  nonlinear speed profile which has a specified maximum deceleration (by parameter choice)

Standard Nonlinear  nonlinear speed profile with parameter chosen by equation (16)

Constant Deceleration  speed profile with constant deceleration

Description of Speed Profiles

One way to provide a pilot with guidance to an exit is to reference the airplane’s motion to a speed profile (speed versus distance). The speed profile begins at some distance on the runway prior to an exit with the current ground speed of the airplane and ends at the exit with the desired exit speed. The airplane’s current speed and position on the runway are known. Ideally, the airplane will reach the exit quickly along the profile, and the deceleration will not be uncomfortable for the passengers. In reference 1, the pilot applies reverse thrust and modulates the brakes to follow a speed profile.

There are three basic speed profiles in this analysis: (1) Linear Speed with Distance; (2) Constant Deceleration; and (3) Nonlinear (in both time and distance). The latter nonlinear speed profile has a specifiable constant parameter to delay maximum deceleration.

The objective in this analysis is to examine the merits of the speed profiles, primarily with respect to maximum deceleration and time spent on the runway (exit time). Perfect tracking of each profile is assumed in the analysis.

Exit Time and Exit Distance

The time an airplane spends on the runway (i.e., runway occupancy time (ROT)) usually means the time from when an airplane crosses the runway threshold to when it completely clears the runway at an exit. In this analysis, however, only the major portion of ROT is considered---basically the time spent on a speed profile, which ends when the aircraft reaches the exit rather than when it clears the runway.

Throughout this paper, exit distance is the distance between the exit location and the location of the start of a speed profile.

Maximum Deceleration

For passenger comfort and safety, the maximum deceleration on a speed profile should not exceed a certain value. In the subsequent analysis, this value is chosen conservatively as 8 ft/sec\(^2\) (approximately 1/4 "g", refs. 2 & 3).

Speed Profile Equations

This section contains equations for the different speed profiles to an exit. Distance is zero at the start of a profile and reaches \( x_e \) at the exit. (To express the equations in runway coordinates, simply replace \( x \) with \( (x - x_0) \) and \( x_e \) with \( (x_e - x_0) \), where \( x_0 \) and \( x_e \) become the start of the speed profile and the exit location, respectively, in the runway axis system.)
Linear Speed with Distance (refs. 2 & 3)

The equation for speed as a linear function of distance is

\[ v = v_0 - kx \]  \hspace{1cm} (1)

where the constant rate of change in speed with distance is

\[ k = \frac{v_0 - v_e}{x_e} \]  \hspace{1cm} (2)

The deceleration along the profile is

\[ a = \frac{dv}{dx} = k \frac{dx}{dt} = kv \]  \hspace{1cm} (3)

and the time to reach the exit is

\[ t_e = -\frac{1}{k} \ln \left( \frac{v_e}{v_0} \right) \]  \hspace{1cm} (4)

In the time domain, speed and distance are

\[ v = v_0 e^{-kt} \]  \hspace{1cm} (5)

\[ x = \frac{v_0 - v}{k} = \frac{v_0(1 - e^{-kt})}{k} \]  \hspace{1cm} (6)

and deceleration varies with time by equations (3) and (5). The speed varies linearly with distance in equation (1), but exponentially with time in equation (5).

Constant Deceleration

For constant deceleration, speed varies as a square-root function of the distance. Specifically,

\[ v = \sqrt{v_0^2 - 2ax} \]  \hspace{1cm} (7)

where the constant deceleration to the exit is

\[ a = \frac{v_0^2 - v_e^2}{2x_e} \]  \hspace{1cm} (8)

The approach here differs from that in reference 4, which tracks the instantaneous constant deceleration needed to reach the desired exit conditions (continues to compute the deceleration using current speed and distance to the exit---whereas, the approach here is to track the speed profile computed one time at \( x_0 \)). Initially, there is a tendency to "over control" a distant error, whereas this is not the case when tracking a local speed profile.

Time to reach the exit on this profile is simply the initial distance to the exit at the start of the profile divided by the average speed. Thus,

\[ t_e = \frac{x_e}{\frac{v_0 - v_e}{2}} \]  \hspace{1cm} (9)

In the time domain, the variation of speed and distance is linear and quadratic, respectively, as

\[ v = v_0 - at \]  \hspace{1cm} (10)

\[ x = v_0t - \frac{1}{2}at^2 \]  \hspace{1cm} (11)

Sometimes this speed profile is simply referred to as Constant Deceleration.

Nonlinear (with Distance and Time)

Linear speed with distance implies nonlinear speed with time, and constant deceleration implies nonlinear speed with distance. The nonlinear speed profile in this section is nonlinear in both time and distance. Specifically, the equation for the speed is

\[ v = v_0 - (v_0 - v_e)e^{-kt(1-\xi)} \]  \hspace{1cm} (12)

where
\[ \xi = \frac{x}{x_e} \]  

is a nondimensional distance and the constant parameter \( k \) is “to be determined”. In other words, while equation (12) matches the initial and the exit conditions, it still has a parameter that can be chosen later. When \( k = 0 \), equation (12) simplifies to linear speed with distance, given by equation (1).

Deceleration along the nonlinear speed profile is given by

\[ a = \left( -\frac{dv}{dt} \right) = \left( -\frac{dv}{dx} \right) \left( \frac{dv}{dt} \right) = \left( -\frac{dv}{dx} \right) v \tag{14} \]

where \( v \) is given by equation (12) and the change in speed with distance is

\[ \left( \frac{dv}{dx} \right) = \left( \frac{v_0 - v_e}{x_e} \right) \left( 1 + k \xi \right) e^{-k(1-\xi)} \tag{15} \]

To compute time histories of the nonlinear speed profile, numerically integrate the speed profile with respect to time, update the speed using equation (12) and (13), and update the deceleration by using equations (14) and (15).

In the analysis, there are two special cases of the nonlinear speed profile—which differ only in the way \( k \) is chosen. For convenience in discussion, these two cases are referred to as the Standard Nonlinear Speed Profile and the Max Nonlinear Speed Profile—or, for short, Standard Nonlinear and Max Nonlinear.

**Standard Nonlinear**

Here, the constant parameter \( k \) in equation (12) is given by a simple empirical equation

\[ k = 1 - \left( \frac{v_e}{v_0} \right) \tag{16} \]

which is a function of the ratio of the desired exit speed to the initial speed at the start of the profile; exit distance does not appear in the equation.

**Max Nonlinear**

Here, \( k \) is chosen to reach the exit on the nonlinear speed profile in minimum time with acceptable deceleration. As with all the speed profiles, the initial speed is \( v_0 \) and the speed at the exit is \( v_e \). But, on the nonlinear speed profile, the speed in-between the two end values always remains greater for the larger of two values of \( k \)—which also means a faster trip to the exit. (This is not difficult to see, because in equation (12), for \( 0 \leq \xi \leq 1 \), the initial speed decays more slowly for larger \( k \).) Then, a natural question is why not use a very large value of \( k \) to reduce the time on the runway? The reason for not doing this is that the deceleration may become excessive. Consequently, the approach here is to choose the largest \( k \) for which the maximum deceleration is acceptable.

The method for computing \( k \) is much more complex than simply using equation (16). The idea is to choose \( k \) so that the maximum deceleration reaches, but does not exceed, a desired maximum (for safety and passenger comfort). The approach consists in the following steps: (fast algorithm given in appendix)

- Iterate \( k \) and integrate the speed profile to the exit
- Record the maximum deceleration
- Choose the largest \( k \) that has the desired maximum deceleration

**Analysis and Discussion**

The purpose of a speed profile is twofold: (1) to reduce time on the runway without using excessive deceleration and (2) to enhance safety, especially in low-visibility weather.
Examples of the Different Speed Profiles

Figure 1. Speed profiles.

(a) $x_e = 4000$ ft.

(b) $x_e = 5000$ ft.

Figure 1. Concluded.

Example Decelerations

Figure 2. Decelerations.

(a) $x_e = 4000$ ft.

(b) $x_e = 5000$ ft.

Figure 2. Concluded.

Figure 1 shows examples of four different speed profiles (as labeled) to exits at 4000 feet and 5000 feet from the start of the profile. Initial speed is 120 knots, and the desired exit speed is 40 knots. Note that the two middle curves are similar and that all of the profiles retain their relative positions in figure 1(b) in which the exit is further away.

Figure 2 shows how the deceleration varies with time along the different speed profiles in figure 1 and also shows the ROT at the end of each curve.

Heavy initial deceleration can lengthen the time an airplane spends on the runway. Clearly,
when the speed varies linearly with distance, the initial deceleration is higher and the time to reach the exit is noticeably longer. Consequently, based on these two factors, there is no further analysis of this profile.

Although an airplane reaches an exit in about the same amount of time using either Constant Deceleration or the Standard Nonlinear, pilots may prefer the nonlinear profiles because heavier deceleration is delayed.

The idea behind Max Nonlinear is to decelerate less initially and more later to allow the airplane to get off the runway sooner. By design, the maximum deceleration is 8 ft/sec² (1/4 "g"). In figure 2, Max Nonlinear clearly has the best (smallest) exit time---but, as shown later, this is not always the case.

**Upper Limit on Initial Speed Imposed by Maximum Deceleration**

![Figure 3. Maximum initial speed corresponding to a desired maximum deceleration of 8 ft/sec².](image)

There is an initial speed above which the deceleration to an exit will exceed a target maximum deceleration. Figure 3 shows this upper limit on the initial speed for two of the speed profiles. (See figure labels.) A 1-curve corresponds to Constant Deceleration (8 ft/sec²), and a 2-curve corresponds to Max Nonlinear (where the maximum deceleration is 8 ft/sec²). As shown, constant deceleration can tolerate higher initial speeds, but the difference is small and decreases with exit speed and exit distance.

The interpretation of an initial speed below a 1-curve in figure 3 is different than that for an initial speed below a 2-curve. Consider specifically, an exit at 4000 feet with an exit speed of 20 knots.

For the 1-curve, the initial speed in this case is approximately 151 knots. If the initial speed were less (below the curve), the computed constant deceleration to the exit would be less than 8 ft/sec²; and, if the initial speed were more, the computed constant deceleration would be more than 8 ft/sec².

For the 2-curve, however, the initial speed is approximately 140 knots. If the initial speed were less, the maximum deceleration would still be 8 ft/sec² by a new choice of $k$ in the nonlinear speed profile. However, if the initial speed were more, the computed maximum deceleration would be greater than 8 ft/sec², even with a new choice of $k$.

**Maximum Deceleration on Speed Profiles**

A key factor in choosing a speed profile is the level of deceleration. If the deceleration is too high, passengers may be uncomfortable, but if it is too low, this will result in unnecessary time on the runway.

**Constant Deceleration**

Figure 4 shows the constant deceleration to exits at 3000, 4000, or 5000 feet for initial speeds of 120, 130, 140, or 150 knots and exit speeds of 20, 30, 40, 50, 60, or 70 knots. As expected, greater constant deceleration levels are required for higher initial speeds and lower exit speeds.

As an example, suppose the exit is 3000 feet
ahead, the initial speed is 120 knots, and the nominal exit speed is 20 knots.

Table 1. Maximum Deceleration on Speed Profiles
(Decelerations > 8 ft/sec$^2$ are shaded.)

(a) Constant Deceleration

<table>
<thead>
<tr>
<th>$x_e$, ft</th>
<th>$v_D$, ft/sec</th>
<th>$v_f$, ft/sec</th>
<th>$v_p$, ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>6.7</td>
<td>120</td>
<td>20</td>
</tr>
<tr>
<td>4000</td>
<td>6.4</td>
<td>130</td>
<td>20</td>
</tr>
<tr>
<td>5000</td>
<td>6.1</td>
<td>140</td>
<td>20</td>
</tr>
</tbody>
</table>

(b) Standard Nonlinear Speed Profile

(c) Max Nonlinear Speed Profile

As an example, suppose the exit is 3000 feet ahead, the initial speed is 120 knots, and the nominal exit speed is 20 knots. Then, the constant deceleration to this exit from the initial speed is approximately 6.6 ft/sec$^2$.

If the initial speed were 140 knots or more, the constant deceleration would be above 8 ft/sec$^2$ for exits with speeds less than 50 knots and closer than 3000 feet.

Table 1(a) shows the same information as figure 1, except in tabular form and with shading to indicate when the constant deceleration exceeds 8 ft/sec$^2$.

**Standard Nonlinear**

Table 1(b) shows the peak deceleration on the Standard Nonlinear speed profile. For example, suppose the initial speed is 120 knots, the exit is 3000 feet away, and the nominal exit speed is 20 knots. Then, the peak deceleration will be 7.9 ft/sec$^2$. A shaded area means that the peak deceleration exceeds 8 ft/sec$^2$.

Corresponding data in tables 1(a) and 1(b) show that the deceleration for Standard Nonlinear always rises above that needed for Constant Deceleration. In fact, when table 1(a) shows a constant deceleration of 7.3 ft/sec$^2$ or higher, the peak deceleration in table 1(b) exceeds 8 ft/sec$^2$. Hence, there are more exits available (unshaded) when using constant deceleration. However, the analysis assumes that the constant deceleration level can be achieved instantaneously at the start of the speed profile.
whereas in reality some time is required its largest value.

On the other hand, some pilots prefer to brake lightly initially to verify proper brake operation and then later increase braking and reverse thrust to achieve a maximum deceleration, and then reduce the braking in approaching the exit, as is characteristic of the Standard Nonlinear Speed Profile

**Max Nonlinear**

Table 1(c) shows that Max Nonlinear has three more exits available (unshaded) than the Standard Nonlinear (table 1(b)), but five fewer than Constant Deceleration (table 1(a)).

Figure 5 is a plot of the values of \( k \) that correspond to the unshaded areas in table 1(c). As an example, suppose the initial speed at the start of the profile is \( v_0 = 120 \) knots; the exit lies ahead at \( x_e = 3000 \) feet; and the desired exit speed is \( v_e = 20 \) knots. Then, using \( k = .9 \) will result in a maximum deceleration of 8 ft/sec\(^2\).

**Distance to Maximum Deceleration**

This section presents the distance along the speed profile to where the deceleration reaches its largest value.

**Constant Deceleration**

Obviously, in this case, there is only one value for the deceleration to the exit.

**Standard Nonlinear**

Figure 6 shows the distances corresponding to the peak decelerations in table 1(b), but only for those peak decelerations that are 8 ft/sec\(^2\) or less. The distance to peak deceleration increases with exit speed and exit distance, but decreases with initial speed.

For the range of conditions in figure 6, notice that peak deceleration occurs prior to 4000 on the way to an exit at 5000 feet, before 3200 feet on the way to an exit at 4000 feet, and before 2400 feet for an exit at 3000 feet.

**Max Nonlinear**

Figure 7 shows the peak deceleration distance as a function of the exit time for all the unshaded conditions in table 1(c). Results of figures 6 and 7 are similar.
Maximum deceleration occurs at the exit when the initial speed is 120 knots and the nominal exit speed is 70 knots for each of the three exit distances shown; that is, for the lowest initial speed and the highest exit speed. This delay is beneficial in reducing the time spent on the runway (however, it may be operationally unacceptable that maximum deceleration occurs at the exit).

**Exit Time Relative to that for Constant Deceleration**

The time to reach an exit on a given speed profile depends on the initial speed, distance to the exit, and the speed at the exit. In this section the time to reach an exit on the Standard and Max Nonlinear speed profiles is shown and discussed relative to that for constant deceleration.

![Figure 7. Peak deceleration distance versus exit time on the Max Nonlinear Speed Profile.](image)

**Standard Nonlinear**

Figure 8 shows the difference in the exit times for the Standard Nonlinear and Constant Deceleration. Shaded symbols indicate when the deceleration exceeds 8 ft/sec².

![Figure 8. Time to reach exit using Standard Nonlinear minus time to reach same exit using Constant Deceleration.](image)

(b) $x_e = 4000$ feet.

Figure 8. Continued.
For the higher-speed exits, there is little difference between the exit times for the two speed profiles. The largest exit-time difference (in magnitude) is only half a second or less for exit speeds of 35 knots or more. So, for the high-speed exits, the pilot can choose a favorite of the two speed profiles. For the lower-speed exits, using constant deceleration is faster by a number of seconds.

**Max Nonlinear**

Figure 9 shows exit-time difference between the Max Nonlinear and Constant Deceleration. For example, Max Nonlinear is faster by 6.6 seconds for an exit at 5000 feet if the initial speed is 120 knots and the exit speed is 20 knots. However, raising the initial speed to 150 knots, results in Constant Deceleration being faster by approximately a second.
Substantial Time Savings by Using Available High-Speed/Closer exits

Understandably, in low visibility weather, pilots are more cautious and spend extra time on the runway looking for an exit, hence, the time spent on the runway increases significantly. With ROTO guidance, a pilot may be more inclined to choose a closer exit and a little higher exit speed. As a result, the savings in the time spent on the runway can be substantial. For example, in figure 10, choosing an exit at 3000 feet with an exit speed of 50 knots instead of an exit at 5000 feet with an exit speed of 20 knots, results in 19.7 seconds less time on the runway. Or, choosing an exit speed of 20 knots for an exit at 3000 feet instead of at 5000 feet results in 13.2 seconds less time spent on the runway.

Concluding Remarks

Deceleration speed profiles that could provide a pilot with guidance in decelerating an airplane to a desired exit turnoff speed after touchdown were analyzed.

The analysis compared four speed profiles with respect to maximum deceleration and time spent on the runway. The profiles were (1) Linear Speed with Distance; (2) Constant Deceleration; (3) Standard Nonlinear; and (4) Max Nonlinear. The first two speed profiles are self-explanatory. The latter two nonlinear speed profiles (which differ only in the choice of a constant) delay maximum deceleration.

The Linear Speed with Distance speed profile was discarded early in the analysis because the initial deceleration is much larger (completely impractical value to initially achieve) than that for the other speed profiles and the time to reach an exit was somewhat longer.

The remaining speed profiles were examined for a range of initial speeds and exit conditions, specifically: (1) three exit distances of 3000, 4000, and 5000 feet; (2) four initial speeds of 120, 130, 140, 150 knots; and (3) six exit speeds of 20, 30, 40, 50, 60, 70 knots. A reasonable maximum allowable deceleration was chosen as $8 \text{ ft/sec}^2$. Each of the speed profiles has its advantages.

The analysis showed (as might be expected) that a constant deceleration speed profile offers more exits than the nonlinear speed profiles—more exits in that near maximum constant deceleration is required. But, instantaneous
achievement of the required maximum deceleration is assumed, whereas in reality some time would be required. (In practice, the next available exit would be chosen if the desired deceleration exceeds a specified limit.)

The Standard Nonlinear Speed Profile has a deceleration characteristic that gradually increases up to a maximum and then decreases. The time to reach an exit is about the same as using constant deceleration; for example, if the exit speed were 35 knots or more, the difference in time to reach the exit would be less than half a second.

The Max Nonlinear Speed Profile has the smallest initial deceleration of the speed profiles examined. By design, the maximum deceleration increases to a maximum of 8 ft/sec², which occurs near the exit. This tends to provide a faster exit time (the exit time can be as large as 6.6 seconds faster than that for the Constant Deceleration Speed Profile) but there are cases where the exit time is not faster. Also, there are cases where the maximum deceleration occurs at the exit location—which may not be operationally practical or acceptable to the pilot.

The use of high-speed exits and exits closer to touchdown were examined relative to the reduction in exit time. For example, choosing an exit at 3000 feet with an exit speed of 50 knots instead of an exit at 5000 feet with an exit speed of 20 knots can result in 20.7 seconds less time on the runway. Or, choosing an exit speed of 20 knots for an exit at 3000 feet instead of at 5000 feet can result in 13.2 seconds less time on the runway.
Appendix

Method for Choosing Constant Parameter in Nonlinear Speed Profile for Desired Maximum Deceleration

The maximum deceleration on the nonlinear speed profile depends on the constant parameter \( k \) in equation (12) in the main text; for example, figure A1 shows the deceleration to an exit that is 4000 feet away for different values for \( k \). The initial speed is 130 knots, and the exit speed is 20 knots. When \( k \) has a value of either 0.24 or 1.338, the maximum deceleration is 8 ft/sec\(^2\).

![Figure A1. Deceleration versus distance along representative Nonlinear Speed Profiles, illustrating the dependency relationship between \( k \) and the maximum deceleration.](image)

Figure A1. Deceleration versus distance along representative Nonlinear Speed Profiles, illustrating the dependency relationship between \( k \) and the maximum deceleration.

Figure A2 shows the maximum deceleration on the nonlinear speed profile as a function of \( k \). As shown, the maximum deceleration first decreases and then increases, as \( k \) increases from zero.

![Figure A2. Choosing \( k \) to attain the desired maximum deceleration on the nonlinear speed profile.](image)

Effect of \( k \) on Deceleration

An airplane decelerates along the nonlinear speed profile from some initial condition on the runway to an exit. This deceleration as a function of distance is

\[
a = c_1(1 + k \frac{v_e}{v_0})e^{-k} \left[ 1 - c_2 se^{kt} \right] e^{kt} \quad (A1)
\]

where the known constants \( c_1 \) and \( c_2 \) are

\[
c_1 = v_0 \left( \frac{v_0 - v_e}{x_e} \right) \quad (A2)
\]

\[
c_2 = 1 - \frac{v_e}{v_0} \quad (A3)
\]

and \( k \) is a specifiable constant.

Equation (A1) is derived using equations (12) to (15) in the main text. Here, assume that \( k \geq 0 \) (will justify later). Moreover, for practical reasons, assume \( c_1 > 0 \) and \( 0 < c_2 < 1 \). The two latter assumptions simply mean that the initial speed is greater than the exit speed.

The initial and exit decelerations follow from
equation (A1) by setting $\xi = 0$ and $\xi = 1$, respectively. These decelerations are

$$a(\xi = 0) = c_1 e^{-k} \quad \text{(A4)}$$

$$a(\xi = 1) = c_1 (1 + k)(1 - c_2) = c_1 (1 + k) \left( \frac{v_e}{v_0} \right) \quad \text{(A5)}$$

As figure A3 illustrates, when $k$ increases, the initial deceleration decreases and the exit deceleration increases. When $k = 0$, the deceleration varies linearly with distance.

**Change of Deceleration with Distance**

The derivative of equation (A1) with respect to $\xi$ can be written as

$$\frac{da}{d\xi} = c_1 e^{-2k} (g - f) e^{k\xi} \quad \text{(A6)}$$

where

$$g = k e^k \left[ 2 + (k\xi) \right] \quad \text{(A7)}$$

$$f = c_2 \left[ 1 + 4(k\xi) + 2(k\xi)^2 \right] e^{(k\xi)} \quad \text{(A8)}$$

The deceleration reaches an extreme (maximum or minimum) or an inflection point when the derivative in equation (A6) is zero; that is, when

$$f = g \quad \text{(A9)}$$

Figure A4 illustrates $f$ and $g$ as functions of $k\xi$ along with possible intersections of the two functions. There are two example curves for $f$. Function $g$ is simply a straight line through $k\xi = -2.0$ with a slope depending on $k$. The function $f$ is the product of a quadratic function and an exponential function and always increases for $k\xi > 0$ (being steeper for larger values of the constant $c_2 > 0$).

There will be no intersection point to the right of the vertical axis ($k\xi = 0$) in figure A4 if the straight line crosses the vertical axis beneath the function $f$. An inequality for $k$ when this will happen is

$$2ke^k + \left( \frac{v_e}{v_0} \right) < 1 \quad \text{(A10)}$$

However, this condition will only occur for very small values of $k$ (clearly true for $k = 0$).

Notice in equation (A8) that for $k\xi \geq 0$, the function $f$ always increases as $k\xi$ increases.
The increase is faster for larger \(c_2\). Hence, on the nonlinear speed profile, there can be at most one intersection point---maybe none. If there is no intersection point, this means that the deceleration continues to decrease along the speed profile.

If the line \(g\) in figure A4 were vertical, there would be no intersection with the function \(f\). Figure A5 shows the slope geometry for \(g\).

![Figure A5. Slope of line \(g\).](image)

The slope of the straight line increases with \(k\), as shown in Figure A6. As \(k\) increases through negative values (shown but not used here) toward \(k = 0\), the slope simultaneously proceeds negatively from zero to approximately \(\theta = -20.2^\circ\) (at \(k = -1\)) and back to zero again. Then, as \(k\) increases positively from zero, the slope increases positively and approaches 90°.

![Figure A6. Variation of the slope of \(g\) with \(k\).](image)

When \(k = 3.5\), the slope is \(\theta = 89.5^\circ\). The largest \(k\) discussed in the main body of this report is approximately 3.01. However, an upper limit on \(k\) is not necessary in the process of iterating to find a value of \(k\) to use to attain a desired level of deceleration, but it does provide a confidence check.

**Algorithm to Find \(k\) for Specified Maximum Deceleration**

Let \(M\) denote a specified maximum deceleration on the nonlinear speed profile. Let \(\varepsilon_1\) and \(\varepsilon_2\) denote small positive numbers. A computer program quickly finds \(k\) via the following steps:

**Step 1:** Iterate \(k\) until the calculated maximum deceleration starts to increase (minimum).

**Step 2:** Revert to the previous iteration of \(k\), prior to reaching the minimum deceleration, and begin iterating forward again until the maximum deceleration exceeds \(M + \varepsilon_1\).

(Note: If more accuracy is desired, duplicate Step 2 with a smaller iteration.)

**Step 3:** Accept \(k\) if maximum deceleration does not exceed \(M + \varepsilon_2\).

(Here, \(M = 8\) ft/sec², \(\varepsilon_1 = 0.004\), and \(\varepsilon_2\) is either 0.0444 or a sufficiently large value to accept all computations.)

**Calculating Speed Profile Deceleration**

For each \(k\)-iteration, the maximum deceleration on the nonlinear speed profile can be computed either by calculating the deceleration along the profile to the exit as a function \(\xi\) in equation (A1), or by integrating the speed profile to exit in time and calculating the deceleration along the way. In either case, record maximum deceleration. The latter
A computer program was written to implement the second approach by using the integration equation

\[ x(t + \Delta t) = \left( \frac{\Delta t}{2} \right)[3v(t) - v(t - \Delta t)] + x(t) \] (A11)

along with equations (12) to (15) in the main text. (An integration step size of \( \Delta t = 0.1 \) seconds is sufficient.)

**Negative \( k \)**

A valid question is why not use negative \( k \) with positive exit distance (\( k \xi \leq 0 \)). To see why, write equation (12) in the main text as

\[ \left( \frac{v - v_0}{v_0 - v_e} \right) = \xi e^{k(\xi - 1)} \] (A12)

where the left-hand side is the nondimensional change in profile speed from the initial speed. At the start of the speed profile this change is zero and at the exit it is unity. Figure A7 shows that the initial deceleration is more if \( k \) were allowed to be negative. In fact, when \( k \) changes to negative and becomes \( k = -2 \), the speed reaches the desired exit speed too early, requiring a subsequent acceleration.
References


**Title:** Speed Profiles for Deceleration Guidance During Rollout and Turnoff (ROTO)  

**Authors:** L. Keith Barker, Walter W. Hankins III, and Richard M. Hueschen

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**ABSTRACT**

Two NASA goals are to enhance airport safety and to improve capacity in all weather conditions. This paper contributes to these goals by examining speed guidance profiles to aid a pilot in decelerating along the runway to an exit. A speed profile essentially tells the pilot what the airplane's speed should be as a function of where the airplane is on the runway. While it is important to get off the runway as soon as possible (when striving to minimize runway occupancy time), the deceleration along a speed profile should be constrained by passenger comfort. Several speed profiles are examined with respect to their maximum decelerations and times to reach exit speed. One profile varies speed linearly with distance; another has constant deceleration; and two related nonlinear profiles delay maximum deceleration (braking) to reduce time spent on the runway.