Selection of optical glasses using Buchdahl's chromatic coordinate

DeVon W. Griffin
M/S 110-3
NASA Lewis Research Center
21000 Brookpark Road
Cleveland, OH 44135
(216) 433-8109 (voice) (216) 433-3793 (FAX)

Interest in reducing the size of glass catalogs has recently resurfaced. Zhang and Shannon used a seven element double Gauss to develop a reduced catalog of nine glasses. Smith varied the flint element in 90 Cooke triplet designs and concluded that a change in Abbe number of ±2 produced a degradation in performance that would be judged acceptable by most lens designers. In contrast to these analyses, Robb developed a method of glass selection based on the goal of minimizing secondary spectrum in a thin lens airs spaced doublet. His development began by expanding the refractive index in a power series about the chromatic coordinate defined by Buchdahl. He then developed an algebraic expression based on the coefficients in the power series that was designed to indicate which glass pairs should be selected for polychromatic correction. This investigation attempted to extend the method to a global glass selection technique with the hope of guiding glass catalog offerings.

Buchdahl's development of optical aberration coefficients included a transformation of the variable in the dispersion equation from wavelength to a chromatic coordinate \( \omega \) defined as

\[
\omega = \frac{\lambda - \lambda_0}{1 + 2.5(\lambda - \lambda_0)}
\]

where \( \lambda \) is the wavelength at which the wavelength is calculated and \( \lambda_0 \) is a base wavelength about which the expansion is performed. The advantage of this approach is that the dispersion equation may be written in terms of a simple power series and permits direct calculation of dispersion coefficients. Robb determined that a pair of glasses should be achromatic for \( n \) wavelengths (\( n \leq 5 \)) if the ratio of the dispersion coefficients calculated using this formalism for the first glass are equal to the ratio of the same order dispersion coefficients for the second glass. For three wavelengths, the ratio of the first-order dispersion coefficient to the second-order coefficient must be equal for the two glasses. If these coefficients, which he called \( n_1 \) and \( n_2 \), are the coordinates used to plot all glasses on a two-dimensional graph, this condition implies that the line connecting both glasses used for achromatization should include the origin. In practice, Robb relaxed that criterion to ±4°. The angular difference between the position vector of the two glasses in a doublet will hereafter be called the achromatic spread. For correction at four wavelengths, the vectors are coordinates in three-dimensional space. While several promising examples were given, a systematic application of the technique to an entire glass catalog and analysis of the subsequent predictions was not performed. The goal of this work was to apply the technique in a systematic fashion to glasses in the Schott catalog and assess the quality of the predictions.

The first step was to verify that the coefficients in the expansion matched those previously published. Coefficients for glasses representing disparate portions of the glass map were found to be essentially identical for the linear coefficient and the disagreement in the second coefficient was never greater than three percent. Following that verification, the ratio of \( n_1 \) to \( n_2 \) for all glasses was computed to produce a list of 78 potential achromats. To assess the quality of the prediction, all glass combinations were optimized using the Zemax software package. The doublets were on-axis broken contact with an aperture of F/3.5. Since minimization of spot size across the visible spectrum is a good measure of both the chromatic and spherochromatic correction, as well as the ultimate usefulness of the lens, it was chosen as the figure of merit. Three wavelengths were used: 486, 589 and 656 nm, with 589 being the design wavelength. The first application was to the FK54/KZFSN2 doublet previously described.
only did the spot size across the visible spectrum match published results, but the slope of the line connecting the two glasses and the origin did as well.

To quantify the performance of the remainder of the doublets selected, all were optimized and then the radius required to encircle all ray intercepts in the polychromatic spot diagram (geometric radius) at the paraxial focal plane was recorded. Applying the conclusion that the achromatic spread is a predictor of optical performance, Figure 1 displays the geometric spot radius in microns as a function of the achromatic spread for the 78 doublets identified using the Robb technique. As can be seen in that figure, no clear relationship exists between the achromatic spread and the geometric spot radius.

Since larger differences in $\eta_1$ between the two glasses predict smaller powers for the individual elements, thus reducing the amount of induced aberration, Figure 2 shows the geometric spot radius plotted as a function of $\Delta \eta_1$. This figure shows two regions of possible correlation between $\Delta \eta_1$ and the spot radius. Using Robb's criteria, all doublets identified contained exclusively crown or flint glass; none contained one of each as would be found in a traditional achromat. In Figure 2, the data points with radii between 2 and 6 microns and $\Delta \eta_1$ between 0.03 and 0.041 contain exclusively crown glass. All other data points represent flint doublets.

After noting that the failure of the method to mix glass types, the achromatic spread was increased in an attempt to use the method to produce a doublet of mixed type. While that goal was achieved, optical performance of the potential achromats was not satisfactory. However, when one of the glasses
was fixed and the analysis performed to identify the best combinations for that particular glass, regardless of the achromatic spread, results were more encouraging. Figure 3 displays the Geometric Spot Radius plotted as a function of $\Delta n$, for doublets that used BK7 as the first lens. Most contained a flint glass as the second element. When $\Delta n$ was greater than about 0.04, the spot size for all doublets was essentially diffraction-limited. However, the analysis does not predict this result since it violates the condition for low $\Delta n$. As shown in Figure 4, the achromatic spread associated with this plot varies from 8.14 to 34.94 degrees, clearly outside of the previously-identified acceptable range. While some correlation between the spot radius and achromatic spread does exist, it is clearly not as strong as that shown in Figure 3. Both of these graphs share the limitation that the potential insight they offer is limited since most of the doublets are nearly diffraction-limited. Hence, potential correlations based on glass selection were difficult to resolve.

To test whether or not the results for BK7 could be generalized, a similar analysis was repeated by fixing one of the glasses as SK2. As before, the vast majority of the doublets contained a flint glass as the second element and the form of the graphs was essentially unchanged. In an attempt to increase the resolution of the results, the eight achromats with the largest values of $\Delta n$, were reoptimized for F/2.5. Figure 5 shows that performance changed almost linearly as a function of $\Delta n^2$, suggesting that perhaps this criteria could be used as a figure of merit in glass selection.

Since one of the goals of Robb’s analysis was to develop a formalism to eliminate secondary spectrum, a comparison with predictions generated by calculating the secondary spectrum directly using Abbe number and partial dispersion was performed. When one of the glasses was not fixed, the same general spread seen in Figures 1 and 2 was evident. Therefore, one glass was fixed as BK7 and the top 35 doublets were identified. Figure 6 displays the geometric spot radius as a function of partial dispersion difference for these doublets. Unlike Figure 3, all of the designs found by this procedure were close to diffraction limited. The ordinate has the same range in both Figures 3 and 6 for ease of comparison.

While the polychromatic spot size is a good indicator of chromatic correction, it does not represent the extent of the longitudinal color. Figure 7 is a plot of the distance in microns between the blue and red focus for the initial 78 doublets as a function of the achromatic spread. While the difference in focal lengths is not exactly longitudinal spherical due to the effects of spherical aberration, the plot is a good indicator of the degree of chromatic correction. As can be seen, there is no correlation between achromatic spread and chromatic correction. Figure 8 repeats this analysis for the BK7 doublets. While an approximate relationship is demonstrated for this case, it is the inverse of that suggested by the earlier work.

In conclusion, Robb postulated a formalism for selecting glasses in airspaced doublets using
Buchdahl's chromatic coordinate. This was applied to the Schott catalog with the goal of developing a systematic technique for reducing the size of the catalog. Analysis of 253 doublets has demonstrated that the technique does not predict which glass combinations will produce the best polychromatic imaging results. Traditional analysis using the Abbe number and partial dispersion better identified the best glass combinations.

References
5. Schott '96 Catalog Optical Glass, Schott Glass Technologies Inc., Duryea, PA.