Fuel Plume Image Mixing Analysis Formulation With Proper Treatment of Non-Constant Velocity Flowfields

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Federal Data Corporation, Hampton, Virginia

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Abstract

A previously developed technique allows an estimate of integral mixing to be obtained from an image of laser scattered light from particle seeded fuel in the hypervelocity flow through a scramjet combustor. This previous mixing analysis formulation contains an assumption of a constant velocity flowfield across the plane of the fuel plume image. For high-speed scramjet combustors, the velocity flowfield is quite uniform and an assumption of constant velocity works well. Applying this same mixing analysis technique to fuel plume images obtained from a mid-speed scramjet combustor makes it desirable to remove the constant velocity assumption. This is due to the non-uniform velocity flowfields present in mid-speed scramjet combustors. A new formulation of the mixing analysis methodology is developed and presented so that the technique can be applied to a mid-speed scramjet combustor without the need to assume a constant velocity flowfield.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>A</td>
<td>Area</td>
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<tr>
<td>f</td>
<td>Fuel-air mass ratio</td>
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<tr>
<td>H</td>
<td>Fuel injector ramp height</td>
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<tr>
<td>I</td>
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<tr>
<td>M</td>
<td>Mass flow rate</td>
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<tr>
<td>N</td>
<td>Number density</td>
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<tr>
<td>N_A</td>
<td>Avogadro’s number</td>
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<tr>
<td>W</td>
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<tr>
<td>X</td>
<td>Distance from fuel injection location</td>
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<tr>
<td>Y</td>
<td>Mass fraction</td>
</tr>
<tr>
<td>(\eta_M)</td>
<td>Mixing efficiency parameter</td>
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<tr>
<td>(\rho)</td>
<td>Mass density</td>
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<tr>
<td>(\tau)</td>
<td>Image exposure time</td>
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<tr>
<td>(\phi)</td>
<td>Fuel equivalence ratio</td>
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Subscripts

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<th>Definition</th>
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<tr>
<td>C</td>
<td>Camera ROI Region of Interest</td>
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<tr>
<td>f</td>
<td>Fuel RX Reactable</td>
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<tr>
<td>L</td>
<td>Laser ST Stoichiometric</td>
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Introduction

The objective of scramjet combustor ground tests is to acquire data useful in quantifying the performance of the modeled scramjet. Key to performance is the fuel mixing achieved within the combustor, which can be deduced from flowfield data. One way to achieve this result is to acquire fuel concentration distributions within the fuel plume structure through the use of fuel plume imaging. Image data obtained in this manner can be processed to yield an estimate of the integral fuel mixing. Initially this technique was applied to high-speed scramjet combustors and the analysis method utilized an assumption of constant velocity across the plane of the fuel plume image [1]. The purpose of the current work is to extend the range of applicability of fuel plume image mixing analysis to include mid-speed scramjet combustors, for which the velocity flowfield is much less uniform and cannot be assumed to be con-
Fuel Plume Imaging

Fuel plume imaging relies on Mie scattering from particle seeded fuel which is illuminated by a sheet of light optically formed from a laser beam. References 1, 2, and 3 contain specific details concerning optimization of operational parameters for the purpose of obtaining clean fuel plume images. Basically, silica spheres are used to seed the fuel prior to injection. It is assumed that the seed particles are uniformly distributed and that they track the flow direction accurately. A pulsed laser is set up so that the beam forms a sheet of light which passes through the combustor test section perpendicular to the direction of flow. The laser light sheet is assumed to be of uniform intensity and thickness. Finally a high resolution digital camera is used to capture an image from the laser light scattered by the seed particles, resulting in a fuel plume image. The assumptions of uniform particle distribution, particle flow tracking, and laser light sheet uniformity are reasonable assumptions which are discussed in more detail in the above mentioned references.

Image Processing

Fuel plume imaging is a non-intrusive procedure. As a result, the camera view angle is such that the raw image will need perspective and optical corrections. As a basis for performing these corrections, a grid block with uniform squares marked out on the surface is typically used. This grid block can be placed at the same location and orientation as the laser light sheet, and then imaged by the same camera which is used to obtain the fuel plume images. By doing this, known image locations from the grid block image can be used to properly alter the perspective and optical distortion of the fuel plume image.

After correcting for perspective and optical distortion, the region of interest must then be extracted from the fuel plume image. Ideally the region of interest would encompass the entire combustor cross section shape. In reality, laser sheet wall and window interference and reflection patterns can render sections of the image unusable. Also, silica particles can agglomerate on the walls and windows creating increased or additional interference patterns. Even so, it is possible at times to obtain fuel plume images which are clean enough to use the entire combustor cross section as the region of interest. Typically an area immediately surrounding a single fuel plume or group of plumes is used as the region of interest.

When the region of interest has been extracted from the perspective corrected fuel plume image, a determination must be made of the intensity range which constitutes presence of zero fuel. This is accomplished by looking at the intensity values in locations clearly outside of the fuel plume structure and identifying the maximum intensity from the ambient background. Once this maximum ambient intensity has been identified, all image intensity values in the extracted region of interest are modified so that values ranging from zero to the ambient background maximum are reset to a value of zero. All other values are proportionally stretched to fill in the range of I=1 to the maximum image intensity. Typical fuel plume images are obtained as 8-bit monochrome images for which the maximum intensity range is I=0 to I=255. A raw fuel plume image will usually not span the entire range of possible intensities, so this image intensity modification will actually increase the apparent dynamic range of the image, but this will not affect the correctness or accuracy of the calculated mixing efficiency. A processed fuel plume image from which a mixing efficiency will be calculated must have the image intensity range modified so that an intensity of zero corresponds to zero fuel.
Each individual fuel plume image will have unique subtleties. In some images there may be small areas of interference possibly caused by particle agglomeration on the optical window. If these areas are small and few, it is possible to utilize either an image filtering process or a photo-retouching process to allow the region of interest to include areas which would otherwise be unusable. In some instances, this could allow information to be extracted from an image which would have otherwise been entirely unusable.

Mixing Analysis

Based on the assumptions outlined above pertaining to uniform particle distribution, particle flow tracking, and laser light sheet uniformity, it can be subsequently assumed that fuel plume image intensity is proportional to seed particle number density, \( N \), when taking particle velocity and image exposure time into account.

\[
I_{\text{IMAGE}} = C_0 N U \tau \quad \text{where} \quad N = \frac{N_A}{W_f} \rho Y_f
\]  

\( C_0 \) is a constant along with the image exposure time, \( \tau \), which represents minimum time duration of either the laser pulse, \( \tau_L \), or camera shutter speed, \( \tau_C \). \( U \) represents a known velocity distribution across the plane of the fuel plume image. Since a non-constant velocity distribution will affect the apparent particle number density, the fuel plume image must be processed to take this into account.

\[
I = \frac{I_{\text{IMAGE}}}{U} \quad \text{so that} \quad I = C_1 N \quad \text{where} \quad C_1 = C_0 \tau
\]  

If the velocity distribution is assumed to be constant or is unknown and must be assumed to be constant, then the following relationships are used.

\[
I = I_{\text{IMAGE}} \quad \text{so that} \quad I = C_1 N \quad \text{where} \quad C_1 = C_0 U \tau
\]  

At this point, either equation set (2) or equation set (3) can be substituted into equation set (1) to yield the same result.

\[
I = C_1 \frac{N_A}{W_f} \rho Y_f
\]  

Since both Avogadro’s number, \( N_A \), and the fuel molecular weight, \( W_f \), are constant, equation (4) can be expressed as follows.

\[
I = C_2 \rho Y_f \quad \text{where} \quad C_2 = C_1 \frac{N_A}{W_f}
\]  

Using the average velocity over the region of interest, \( \bar{U} \), which is constant by definition, yields the following.
\[ I = C_3 \rho \bar{U} Y_f \quad \text{where} \quad C_3 = \frac{C_2}{\bar{U}} \]  

Integrating the fuel plume image intensity over the area of the region of interest results in the following.

\[ I_{\text{TOTAL}} = \int_{\text{ROI}} I \, dA = C_3 \int_{\text{ROI}} \rho \bar{U} Y_f \, dA = C_3 \bar{m}_f \]  

Using the definition of the fuel-air mass ratio yields the following relationship.

\[ \bar{m}_f = f \bar{m}_{\text{air}} = f (\rho \bar{U} A_{\text{ROI}} - \bar{m}_f) \]  

Rearranging equation (8) yields the following.

\[ (1 + f) \bar{m}_f = f \rho \bar{U} A_{\text{ROI}} \quad \text{and} \quad \bar{m}_f = \frac{f}{1 + f} \rho \bar{U} A_{\text{ROI}} \]  

Substituting equation set (9) into equation set (7) produces the following.

\[ I_{\text{TOTAL}} = C_3 \bar{m}_f = C_3 \frac{f}{1 + f} \rho \bar{U} A_{\text{ROI}} = C_3 \rho \bar{U} Y_f A_{\text{ROI}} \]  

The quantity \( \rho \bar{U} \) represents the average mass flux over the region of interest, which is constant by definition. Next, the average intensity over the region of interest is defined as follows.

\[ \bar{I}_{\text{ROI}} = \frac{I_{\text{TOTAL}}}{A_{\text{ROI}}} = C_4 \frac{f}{1 + f} \bar{Y}_f \quad \text{where} \quad C_4 = C_3 \rho \bar{U} \]  

Using equation set (11) allows the constant \( C_4 \) to be redefined using known and obtainable values.

\[ \bar{I}_{\text{ROI}} = C_4 \bar{Y}_f \quad \text{and in general} \quad I = C_4 Y_f \quad \text{where} \quad C_4 = \frac{\bar{I}_{\text{ROI}}}{\left( \frac{f}{1 + f} \right)} \]  

Next, assume a linear relationship between \( \bar{I}_{\text{ROI}} \) and \( I_{\text{ST}} \) such that at \( \phi = 1 \), \( I_{\text{ST}} = \bar{I}_{\text{ROI}} \).

\[ \bar{I}_{\text{ROI}} = \phi I_{\text{ST}} \quad \text{or} \quad I_{\text{ST}} = \frac{\bar{I}_{\text{ROI}}}{\phi} \]  

The implication inherent in this assumption is that the processed fuel plume image intensities have been modified so that \( I = 0 \) corresponds to zero fuel.
Over the region of interest, for \( I \leq I_{ST} \), \( \int \text{IdA} \) is proportional to that part of \( m_f \) which is fuel lean and entirely reactable. For \( I > I_{ST} \) in the region of interest, the reactable mass fraction is estimated by the following relationship:

\[
I_{RX} = C_4 Y_{RX} = C_4 f_{ST}(1 - Y_f) \quad \text{where} \quad Y_f = \frac{I}{C_4}
\]

Therefore, in the region of interest, for \( I > I_{ST} \), \( \int I_{RX}\text{dA} \) is proportional to that part of \( m_f \) which is fuel rich and can react only to the extent of available air. Previously [1], this integration has been performed as a linear approximation between two points, \( I = I_{ST} \) and \( I = I_{MAX} \). With this formulation, it is recommended to perform the integration at each discrete intensity value. One additional consideration for this integration is that the upper limit intensity can be no more than a value of \( I = C_4 \). Any fuel plume image intensities above this value would numerically supply negative reactable fuel to the integration process. This would indicate some non-physical aspect to the processed fuel plume image.

The definition of mixing efficiency is the following [1,4].

\[
\eta_M = \frac{\int \rho U Y_{RX}\text{dA}}{\bar{m}_{RX}} \quad \text{where} \quad Y_{RX} = \begin{cases} Y_f & \text{for } \phi \leq 1 \\ f_{ST}(1 - Y_f) & \text{for } \phi > 1 \end{cases} \quad \text{and} \quad \bar{m}_{RX} = \begin{cases} \bar{m}_f & \text{for } \phi \leq 1 \\ f_{ST}\bar{m}_{air} & \text{for } \phi > 1 \end{cases}
\]

By definition, \( \phi = f/f_{ST} \) and \( f_{ST} = f/\phi \), therefore \( f_{ST}\bar{m}_{air} = (f/\phi)\bar{m}_{air} = \bar{m}_f/\phi \). Substituting this relationship into the appropriate portion of equation set (15) provides the following.

\[
\bar{m}_{RX} = \begin{cases} \bar{m}_f & \text{for } \phi \leq 1 \\ \bar{m}_f/\phi & \text{for } \phi > 1 \end{cases}
\]

Using equation set (7) along with the definition of mixing efficiency contained in equation sets (15) and (16) allows mixing efficiency to be directly calculated from the information contained in a fuel plume image.

\[
\eta_M = \begin{cases} \left( \frac{\int \text{IdA} + \int I_{RX}\text{dA}}{\text{LEAN ROI}} \right) \text{ for } \phi \leq 1 \\ \phi \left( \frac{\int \text{IdA} + \int I_{RX}\text{dA}}{\text{LEAN ROI}} \right) \text{ for } \phi > 1 \end{cases}
\]

In arriving at equation (17), note that it is not necessary to be able to obtain a known value for the constant \( C_3 \). This is due to the fact that it appears in both the numerator and denominator of equation (17). An additional implication of this is that the spanned intensity range of the processed fuel plume image does not affect the calculation of a mixing efficiency as long as \( I = 0 \) corresponds to zero fuel.
Results and Discussion

The first two fuel plume images were acquired from a Mach 2 scramjet combustor test which represented a mid-speed combustor at a flight enthalpy of Mach 5 [5]. These images were both taken at the X/H=6 location, where the full test section size is 6H wide by 4H tall. Figures 1 and 2 show the unprocessed fuel plume images, which were both acquired during one test. It should be noted that these images do not encompass the entire test section, but do include the entire fuel plume. The result of applying the image processing steps to these raw fuel plume images is shown in Figures 3 and 4. These processed fuel plume images will be referred to as image 1 and image 2.

![Figure 1. Unprocessed fuel plume image 1](image1.png)  ![Figure 2. Unprocessed fuel plume image 2](image2.png)

![Figure 3. Processed fuel plume image 1](image3.png)  ![Figure 4. Processed fuel plume image 2](image4.png)

A CFD solution is used to provide velocity field information for use with these fuel plume images [5]. The velocity field at the X/H=6 location is shown in Figure 5 with the outlined area representing the extent of the processed fuel plume images. Figures 3 and 4 contain the result of applying the velocity correction to images 1 and 2 respectively. It is found that within the fuel plume, velocity ranges from approximately 100 to 1400 m/sec. As a result, it appears that an assumption of constant velocity would not work well for mixing analysis of these images. To gain more insight into this matter, it is necessary to look at the range of particle travel distances captured in the images. Acquisition of these images involved a pulsed laser which formed a sheet approximately 1 mm thick. The limiting factor for the image exposure time was the laser pulse duration such that $\tau = \tau_L = 7$ nanoseconds. As a result, the particle travel distances captured in these fuel plume images range from 0.0007 to 0.0098 laser sheet thickness.
nesses. Velocity non-uniformity is not an issue in these fuel plume images, which are essentially instantaneous images rather than time averaged images. Consequently, even though these fuel plume images are taken from a mid-speed combustor with significant velocity non-uniformity, an assumption of constant velocity is appropriate.

![Velocity field at X/H=6 with outline of processed fuel plume image area](image)

**Figure 5.** Velocity field at X/H=6 with outline of processed fuel plume image area

![Processed fuel plume image 1 with velocity correction applied](image)

**Figure 6.** Processed fuel plume image 1 with velocity correction applied

![Processed fuel plume image 2 with velocity correction applied](image)

**Figure 7.** Processed fuel plume image 2 with velocity correction applied

Mixing analysis of images 1 and 2 was performed both with and without the constant velocity assumption as a basis of comparison between the two methods. For image 1, $\eta_M = 23.85\%$ assuming constant velocity and $\eta_M = 23.77\%$ with the velocity correction applied. For image 2, $\eta_M = 23.90\%$ assuming constant velocity and $\eta_M = 23.37\%$ with the velocity correction applied. Applying the constant velocity assumption, these two images exhibit closer agreement of the resultant mixing efficiency.

The next set of fuel plume images was obtained from a similar test in the same facility [2]. From this test, one image taken at X/H=6, image 3, will be analyzed along with an image taken at X/H=2, image 4. Computational velocity information is also available for these images. Figures 8-11 contain the processed fuel plume images both with and without velocity correction applied. Note that for this set of images, the raw fuel plume images did include the entire combustor cross section. And as a result, the
processed fuel plume images encompass the entire 6H by 4H test section, a much larger area than that of the previous set of processed images.

Images 3 and 4 were acquired using a continuous laser which formed a sheet that was 200 μmeters thick. The limiting factor for the image exposure time for these images was the camera exposure setting such that \( \tau = \tau_C = 0.8 \) seconds. For image 3, at \( X/H=6 \), velocity within the fuel plume ranges from 100 to 1000 m/sec. In image 4, at \( X/H=2 \), velocity within the fuel plume ranges from 1400 to 2200 m/sec. In terms of particle travel distances captured in these images, image 3 ranges from 0.4 million to 5.0 million laser sheet thicknesses, while image 4 ranges from 5.6 million to 8.8 million laser sheet thicknesses. These images provide time averaged representations of the fuel plume, and as a result, it is desirable to apply a velocity correction prior to calculating a mixing efficiency.

Mixing analysis of images 3 and 4 was also performed with and without the constant velocity assumption so that a comparison can be made between the two methods. For image 3, \( \eta_M = 15.02\% \) assuming constant velocity and \( \eta_M = 14.02\% \) with the velocity correction applied. For image 4, \( \eta_M = 5.63\% \) assuming constant velocity and \( \eta_M = 5.80\% \) with the velocity correction applied. Note that at \( X/H=2 \) there is less variation of velocity across the fuel plume than at \( X/H=6 \), and the resulting mixing efficiency quantities are closer in agreement at \( X/H=2 \) than those at \( X/H=6 \).
Table 1 provides a summary of results obtained from the mixing analysis of the four images presented above. Note that the fuel equivalence ratio is lower for images 3 and 4. As a result, it is not possible to make direct comparisons among images 1, 2, and 3 at X/H=6.

<table>
<thead>
<tr>
<th>Image</th>
<th>Type of Image</th>
<th>X/H</th>
<th>Velocity Correction</th>
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<tr>
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<td>Time Averaged</td>
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**Conclusions**

When processing fuel plume images to calculate an estimate of mixing efficiency, a known velocity field is typically not available. Consequently, the assumption of a constant velocity must then be used. For fuel plume images obtained from tests of a mid-speed scramjet combustor, this constant velocity assumption appears to be a potential source of great error due to the non-uniform velocity fields present in mid-speed scramjet combustors. However, in looking closely at the mixing analysis formulation, it is based on integrations of the general form \( \int \text{IdA} \). Also, as part of the image processing, areas with no fuel present will have an intensity value of zero. As a result, an integration of image intensity over area will accumulate nothing in areas of zero fuel. Effectively, this means that an assumption of constant velocity is really only an assumption of constant velocity within the fuel plume. Non-uniformity of velocity outside the fuel plume does not enter into the calculation of mixing efficiency.

If the velocity field is known when processing fuel plume images, mixing efficiency can be calculated both with and without the constant velocity assumption to verify the resulting difference in these values. A typical expectation is that the difference between these two values would be greater at locations closer to the fuel injectors. Certainly this would seem logical since the non-uniformity of velocity is greater across planes that are closer to the fuel injection location. In reality, it does not matter how uniform or non-uniform the velocity is across the entire plane. Locations closer to fuel injection provide greater velocity uniformity across the fuel plume. For mid-speed or high-speed scramjet combustor tests, fuel plume image mixing analysis with a constant velocity assumption will be more accurate for locations closer to fuel injection.

It is important to understand whether a fuel plume image represents an instantaneous image or a time averaged image of the fuel plume. An instantaneous image does not require a velocity correction to be applied since it has effectively been applied already due to the method of image acquisition. On the other hand, accuracy of fuel plume image mixing analysis can be improved for time averaged images by taking the velocity field into account. This accuracy improvement is greater in situations where the velocity non-uniformity across the fuel plume is greater.
Summary

For accurate mixing analysis of fuel plume images obtained from mid-speed scramjet combustor tests, two methods can be used to properly take into account the non-constant velocity flowfield. Fuel plume images can be acquired as instantaneous images and processed using the constant velocity assumption to arrive at an accurate estimate of mixing. And for time averaged fuel plume images, the presented mixing analysis formulation for non-constant velocity can be used to obtain an accurate estimate of mixing if flowfield velocity information is available.

References

A previously developed technique allows an estimate of integral mixing to be obtained from an image of laser scattered light from particle seeded fuel in the hypervelocity flow through a scramjet combustor. This previous mixing analysis formulation contains an assumption of a constant velocity flowfield across the plane of the fuel plume image. For high-speed scramjet combustors, the velocity flowfield is quite uniform and an assumption of constant velocity works well. Applying this same mixing analysis technique to fuel plume images obtained from a mid-speed scramjet combustor makes it desirable to remove the constant velocity assumption. This is due to the non-uniform velocity flowfields present in mid-speed scramjet combustors. A new formulation of the mixing analysis methodology is developed and presented so that the technique can be applied to a mid-speed scramjet combustor without the need to assume a constant velocity flowfield.